P.S.S.C. PHYSICS

TEACHER'S RESOURCE BOOK AND GUIDE

PART TWO

D. C. HEATH AND COMPANY

INDIAN EDITION

NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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INTRODUCTION TO PART II

Part II has a threefold purpose: to examine how light behaves, to consider a model for this behavior, and to explore the behavior of waves.

The choice of light as the first field of physics to be examined in detail has advantages. Much of the subject can be learned by the students from laboratory work that is stimulating and yet does not require much experimental maturity or sophisticated apparatus. With light, we can start with simple phenomena and progress to rather subtle ideas about waves, thus bringing the student to an awareness of the nature of waves before delving into the mechanics of particles in Part III. Since most of us have better intuitive ideas about particle behavior than about wave characteristics, this sequence should cause students to be more responsive to the ideas in Part IV that matter particles are in some degree wavelike. Finally, the study of light provides an especially effective context for considering, in parallel with the characteristics of light, how a physical theory is developed. The organization of Part II leads in a somewhat contrapuntal style toward the subtleties of modern physics.

Chapters 11 through 14 present a picture of how light behaves without going behind the behavior to look for an explanatory model. These chapters derive the principles of reflection and refraction from observation and experiment in the laboratory and from the analysis of experiments in the text. The application of these principles in lenses and simple optical instruments is shown.

Chapter 15 proposes a model for light, the particle model. This model is adequate to describe most of the characteristics of light but fails to give a simple explanation of the relationship between refraction and the speed of light. Because of this and other flaws, a simple particle model is temporarily abandoned in favor of a search for a more satisfactory model. This chapter highlights the characteristics of light by way of searching for a model that fits these characteristics, and also provides an example of how scientific hypotheses grow out of, and are confirmed or denied, by experiment.

Chapters 16 through 18 lay the groundwork for a wave model for light by considering how waves reflect and refract, first in one dimension (waves on a coil spring), then in two dimensions (water waves). Diffraction and interference are introduced. As wave characteristics are explored, parallelism with the behavior of light is apparent.

Chapter 19 examines the wave explanation of interference and diffraction in light, and the resultant grounds for belief in the wave-like nature of light.

RELATED MATERIALS FOR PART II

Laboratory. Laboratory work always helps to show that physical ideas are rooted in reality, and that while they are often imaginative, they are not sourceless. The laboratory experiments for Part II are of vital importance. The laboratory should be the fundamental resource for learning about waves. The yellow pages of this Guide include recommendations on scheduling and handling each of the experiments, and answers to questions in the students' Laboratory Guide.

Films. "Introduction to Optics" is an introduction to Part II of the course. The film displays many of the phenomena that must be explained by a model for light, and directs the students' attention to some of the more significant questions which class and laboratory work will have to answer. This film can be used on the first day of classwork on Part II.

"Pressure of Light" fits with Chapter 15, Section 4. In exploring a particle model, the text notes that light pressure is predicted by such a model. While this is not simple to confirm in the laboratory, the film, using sensitive apparatus, shows the existence of light pressure.

"Speed of Light" also relates to the work of Chapter 15. The film shows not only a straightforward measurement of the speed of light, but a side by side comparison of the

speed of light in air and in water. This is useful in connection with an experimental examination of the prediction from the particle model of the relation between the speed of light in air and in a material of greater refractive index. The techniques employed are interesting from the standpoint of being a real measurement, by clearly apparent methods of a speed that is outside the range of common experience.

"Simple Waves" is related to the beginning work on waves in Chapter 16. Using somewhat more elaborate apparatus than is ordinarily available in secondary school laboratories, and slow motion photography, the film clearly shows the characteristics of waves traveling in one dimension. Some teachers prefer to use this film early in their development of wave concepts. Most teachers prefer to use it near the end of their work on Chapter 16, to give a more detailed look at phenomena that students have first seen directly with simple apparatus and at first hand.

Science Study Series. As the Science Study Series grows, you will find an increasing number of titles that deal with subjects related generally to Part II. Currently available titles that deal with ideas growing out of those in Part II are: Echoes of Bats and Men, by Donald R. Griffin; Waves and the Ear, by Willem A. van Bergeijk, John R. Pierce and Edward E. David Jr; and Horns, Strings and Harmony, by Arthur H. Benade. Among titles expected to be available soon are Michelson, A Biography, Vision, The Red Shift, and Spectroscopy.

SCHEDULING PART II

The following schedule is consistent with the information on scheduling in the Introduction to the Guide for Part I, and suggests plans for covering Part II of the course in one of two periods: 9 weeks, or 14 weeks. The shorter schedule is appropriate for a one year course. While these schedules are meant only as approximate guides, they are based on teaching experience.

	14-week schedule for Part II			9-week schedule for Part II		
Chapter	Class Periods	Lab Periods	Exp't	Class Periods	Lab Periods	Exp't
11	2	-		2	-	
12	6	4	II-1 II-2	5	3	II-1 II-2
13	5	2	II-3	3	1	II-3
14	5	2	II-4	4	1	II-4
15	4	2	II-5 II-6	3	1	II-5
16	4	1	II-7	2	1	II-7
17	4	6	II-8 II-9 II-10 II-11	2	5	II-8 II-9 II-10 II-11
18	4	3	II-12 II-13	2	2	II-12 II-13
19	5	4	II-14 II-15 II-16 II-17	3	2	II-14 II-15

Chapter 11 - How Light Behaves

The intent of this chapter is to present a quick and qualitative view of the subject of light. The topics which are briefly introduced here become the central themes in the remainder of Part II. This introductory picture of light might be obtained by making the reading of the chapter a reading assignment and then using class time for the discussion of "Home, Desk and Lab" problems as a way of determining the background of the students.

An introductory discussion, suitable for use before you assign the chapter, is suggested below. This discussion is intended to help overcome possible disinterest of students who think they are familiar with light and some of its properties. Because light is so common, some special effort may be required to bring students to an appreciation of its mystery and beauty. Students who can imagine they are learning about light for the first time and who can read the chapter with a fresh point of view, obtain important, lasting insights. The film "Introduction to Optics" can be used effectively for this purpose.

CHAPTER SUMMARY

This chapter can be put in proper perspective by recalling the aims of Part II of this course, the study of light. In general, from this part of the course, one might hope to understand what light is and how it behaves. It is good scientific practice to defer the question of what light is until after having examined what light does under a variety of conditions. Thus, the content of this chapter might be thought of as a set of preliminary answers to the following questions. (The answers, at this stage, are necessarily crude; superior answers from which important conclusions can be drawn will be reached in later chapters after the necessary detailed experiments and observations have been described.)

(a) How does light get started? From what kinds of material does it come?
Section 1 - Sources of Light

(b) What happens when light hits an object? Does all the light enter? Does it go through? Does it bounce?
Section 2 - Transparent, Colored, and Opaque Materials
Section 3 - Reflection

(c) Are there devices, other than eyes, which are sensitive to light? Are all these devices equally sensitive or can some detect what others miss?
Section 4 - Light Sensitive Devices
Section 5 - Invisible Light

(d) Does light go in straight lines? Does it sometimes bend?
Section 6 - How Light Travels
Section 7 - Diffraction

(e) Does light appear simultaneously at different places or does it travel with some definite speed?
Section 8 - The Speed of Light

INTRODUCTORY DEVELOPMENT FOR ENTIRE CHAPTER

In order to help give students a fresh orientation, just before you assign this chapter you might hold a class discussion in which students are challenged to describe light to a race of men who know nothing at all about light. Blind people would not be a suitable example because they have lived with people who constantly discuss what they see. The description should be directed to imaginary beings whose ignorance of light is like man's ignorance of radio waves before they were discovered.

Ask students to give examples of how they would tell such people about light. Probably the most noteworthy difference would be that whereas the "lightless" would have to touch an object to determine its size, shape, and texture, we can determine these properties at a great distance, merely by looking. Our ability to make such observations undoubtedly would seem unbelievable to the "lightless" (perhaps just as the notion of seeing and hearing (a television broadcast) from halfway around the world might have seemed absurd in

1800). Let various students try to explain light to the "lightless".

A possible student response might be: Well, the way we can tell about things that we can't touch is that we use stuff called light. Light is strange; you can't feel it or touch it. (That is, you usually can't feel it, but sometimes if it's strong, it feels warm.) The way you know that there is light is that you see it. Now seeing means that.........."

"Well, human beings have little bulges in their head, and when light enters these bulges, they know it. You don't exactly feel light; you see it. You might think this light hurts when it hits the eye, but it doesn't (unless it is too bright)."

"The thing is, that when you see the light, you know about the object the light came from. Think of it this way: if you throw a ball and hear a crash, then you know there is (or was) something where you threw the ball. Better still, if you throw a ball and it comes back, you know it must have hit a fairly flat object, like a wall. Now light works a lot better than listening for crashes or trying to retrieve balls that have bounced. You don't have to throw light out (which is fortunate because you can't throw it unless you carry a flashlight, a special gadget which throws light). Usually, the sun, or an electric light bulb throws the light; it doesn't make much difference where the light comes from originally, because it spreads out and some usually gets to you."

"Now it may seem funny that light bounces off everything, and goes in all directions and yet you can tell about what its coming from, but the reason is...."

"Another interesting thing about light is that it seems to appear everywhere all at once. For example, if you press a button to turn on a light, the light seems to be everywhere, even before you hear the click; it's not like a ball traveling or even like an echo."

"....and another thing is that light comes in different colors. Now, by color I mean.."

Some of your students may supply better explanations than those given above. Classes in the past have had lively discussions and criticisms of proposed statements to the "lightless".

Perhaps you can find a better way to get the students to back off, as it were, and examine light as though they were noticing it for the first time. If you can get them to read this chapter in that fresh frame of mind, you will contribute greatly to accomplishing the aims of the chapter.

After you have introduced the chapter, before assigning it, you might suggest that the students check as they read, and list the mentioned properties of light which are new to them. It will be a rare student who does not find at least one new property of light if he reads the chapter carefully. Most students will learn many new things.

After the students have read the chapter you may want to discuss it briefly as a whole either by continuing (on a more sophisticated level) the explanation to the lightless or by reviewing the new things about light which different students learned.

Another interesting question which can be discussed before the students read the text, is whether colored glass adds or subtracts something from white light. (See Section 2 and the suggested development for that section in this Guide.)

SCHEDULING CHAPTER 11

Because Chapter 11 is introductory, its scheduling is relatively simple. It should take about two class hours, independent of whether you plan to use 9 or 14 weeks on the entire volume. If you use a preliminary discussion such as the above, a part of one hour can best be spent before the students read the chapter. Several developments are suggested below in the detailed discussions of the sections; however, you will only want to select the one or two that would be most interesting for your class. If you can schedule the film "Introduction to Optics" at the time you are covering this chapter, the film will do a great deal in setting the stage for Part II of the course.

RELATED MATERIALS FOR CHAPTER 11

Laboratory. None of the experiments were designed for this chapter.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 11-9; detailed comments and solutions on page 11-10 to 11-14 .

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1	1	2, 3*		1, 2, 3	1
2, 3		4, 5, 6*		4, 5	
4, 5	7, 8*	9, 10*		7, 8*, 9, 10*	10
6, 7		6*		6*	
8	11*, 12				

Films. "Introduction to Optics", by Elbert P. Little of PSSC. This film is not intended to "explain" light, but rather to identify the major characteristics of light behavior which a theory must explain. The remainder of Part II is concerned with analyzing these characteristics and searching for a satisfactory theory. Running time: 23 minutes.

The film considers the approximation that light travels in straight lines. Demonstrations show the four ways that light can be bent – diffraction, scattering, refraction, and reflection. Refraction is illustrated with underwater photography showing how objects above water appear to a submerged observer.

ADVANCED WARNING FROM ALL PREVIOUS TEACHERS

In Chapter 17, you will begin work with ripple tanks. This is one of the most interesting and instructive activities in the entire course. But, YOU NEED TO GET YOUR RIPPLE TANKS OPERATING NOW TO GAIN SOME EXPERIENCE WITH THEM BEFORE STUDENTS START TO USE THEM! No matter how much too busy you are, or what else must be deprived of time, it is essential that you get your ripple tanks going SOON.

Section 1 - Sources of Light

PURPOSE To indicate what makes objects visible.

CONTENT Luminous objects emit light. Most things we see are non-luminous objects which reflect light.

DEMONSTRATIONS You may want to show students the change in color of an incandescent light as it becomes dimmer, and the constancy of the color emitted by fluorescent and neon lights independent of brightness. This is not the occasion to explain why these light sources differ. It is enough to observe that incandescent sources are materials which give off light characteristic of the temperature rather than the material. The fluorescent and "neon" lamps have colors which depend on the material and its atoms or molecules. Students are often familiar with terms such as "red hot" or "white hot" but they might be interested in the fact that color can be and is used as a semiquantitative measure of temperature. (For explicit examples, look up "Temperature, Color scale of" in a Handbook of Chemistry and Physics.)

DEVELOPMENT You can emphasize the importance of reflected light by asking students what the view would be if nothing reflected light. Emphasize the blackness by asking where lights would be required in going about everyday affairs. Most students will be somewhat surprised when they realize that, unless everything were painted with fluorescent paints, a world without reflectors would not be very different from the world of the blind. For example, all doors and passageways would have to be marked with lights. You could not drive, nor walk, unless roads were lined with lights and all objects were outlined by lights. Lights could not illuminate; they would merely be signals. You could never see anyone else or yourself, and you could not use a mirror.

Note: "Luminous" and "non-luminous" are used in the text in their technical senses; most dictionaries define as luminous any object which gives off light independent of whether the object is an actual source or a reflector.

Some students may not remember that "Celsius" (on page 179) is the new, internationally accepted term for "centigrade".

Section 2 - Transparent, Colored, and Opaque Materials
Section 3 - Reflection

PURPOSE To indicate what happens when light hits different objects.

CONTENT a. Some objects mainly transmit light, others mainly absorb it, and still others mainly reflect it.

b. Even objects which appear transparent usually absorb and reflect some light.

c. A colored, semi-transparent material absorbs some colors, and appears to be the color of those it transmits.

d. Light is bent as it enters or leaves a transparent substance.

EMPHASIS Note that reflection will be studied in detail in Chapter 12, as soon as you finish this chapter. Refraction is described in Chapters 13 and 14.

DEVELOPMENT Examples of some of these phenomena are particularly well introduced in the film "Introduction to Optics".

You can probably get a lively class discussion by asking the students how they would show whether colored glass adds or subtracts something from white light. Have a source of white light and some colored pieces of glass available so that the suggestions given by students can be tried.

If the students have not read the chapter, this prior discussion will emphasize the simple but convincing reasoning used in the text. Even if the students have read the text, you can start a discussion by calling on less vocal students first, and on some argumentative ones next. If no one wants to argue, ask for criticisms of this hypothesis:

> The red filter adds red, the green filter adds green, but these added colors just cancel each other when the two filters are used together.

The students should realize that if only the added colors were cancelled, white light or at least some bright light should remain.

If you are fortunate, you may have a student who dreams up some complex mechanism. Encourage him, but see if his arguments have obvious flaws. If not, admit that his suggestion might be correct, but ask which explanation seems more plausible or simplest. If this happens, you will have a good chance to point out that when two explanations seem to account for the same set of facts, physicists usually tentatively accept the simpler explanation while they constantly try to think of experiments which might distinguish between the alternatives.

You can close this discussion by asking for an experiment even more decisive than the ones quoted in the text. See whether anyone can suggest using different single colors from a "rainbow" (or a spectrum) to test the colored glasses. (Of course, if this suggestion comes too early, defer it.) Have a demonstration ready to prove this if you have the equipment available.

Is Figure 11-4 right-side up? The problem posed by Figure 11-4 tantalizes many students. Most students realize that the picture is right-side up, but a few convince themselves that it is upside down. One of the most obvious clues that the picture is right side up is that the white strips (probably snow or roads) and the distant mountains appear in the upper half but not in the lower reflection. Furthermore, the definition of the outline of the mountain on the right is better in the upper half. The chief reason that students give for believing the picture is upside down is the higher contrast between the clouds and the sky in the reflection. If the students ask about this, restrict your answer and indicate only that the blue light from the sky is reflected more poorly than is the light which comes from the cloud. There is no need to get side tracked now with a discussion of the fact that the blue light, which results from scattering is partially polarized while the light reflected diffusely from the clouds is not. Many students observe this same type of increased contrast when they wear polaroid sunglasses.

Another lively discussion of the properties of light discussed in these sections can be begun by asking what properties an "invisible man" would have to have. Clearly he must not reflect light or absorb it, but one which many students will miss is that he must also not bend, or refract, it.

Note: Some students who try to reproduce the "floating coin" effect shown in Figure 11-2 are disappointed because they do not see as much enlargement as shown in the picture. Most will realize that the coin under water appears closer and hence seems to be floating. But they cannot get the maximum enlargement for water (1.33) unless they put their eye just above the water surface of a tall glass.

COMMENT In such wide open discussions as have been recommended here, there is real danger that students will become embroiled in arguments about overly precise definitions. "How transparent does a substance have to be to be called transparent?" Simply do not allow such discussion. It is pointless. These terms are not meant to be defined precisely, they are merely generally descriptive.

Section 4 - Light Sensitive Devices
Section 5 - Invisible Light

PURPOSE To indicate that the eye is not the only light detector and that some "light" is invisible.

CONTENT a. A variety of substances respond to light; in a photographic film, light produces a lasting chemical change while in a photocell, light affects the electrical current.

b. The heat radiation (infrared) given off by hot objects, and ultraviolet light are similar to visible light but cannot be seen.

CAUTION Avoid discussion of the electromagnetic spectrum, wavelength, frequency or the wave nature of light. The development of the next few chapters, and the success of Chapter 15, The Particle Model of Light, partially depends on students having open minds on the nature of light. If students raise questions about the electromagnetic waves, assure them that they will have a full answer before the course is finished.

Section 6 - How Light Travels
Section 7 - Diffraction

CONTENT a. Light usually travels in straight lines. It bends very slightly, close to the edge of an obstacle, but the bending is so small that it can be disregarded usually, as a good first approximation.

b. Light can travel through empty space, and it can travel great distances.

c. The eye is aware of light only when that light enters the eye directly.

EMPHASIS The straight line propagation of light will be treated in detail at the beginning of Chapter 12. Diffraction will be clarified in Chapter 19; for the present the students do not even have to remember the term, diffraction. Diffraction is introduced here simply to point out one of the interesting phenomena we have to explain. The film "Introduction to Optics" introduces diffraction in a way that will interest students.

COMMENTS Some students may ask you for help in seeing the diffraction effects mentioned on page 186. A common student difficulty is not looking at long objects. Suggest a cord shade in front of a window or the dividing strips in a window. The fingers should be close to the eye and should form a slit parallel to the object being viewed. Slits can also be produced easily by adjusting the wires of paper clips, or rotating a pen or pencil until the slit between clip and barrel is very narrow.

Diffraction is perhaps the most complicated subject treated in Part II. Do not get into an extensive discussion here. Treat it only as a case where light does not travel in a straight line. Avoid mentioning now that this sort of phenomenon "proves" the wave theory of light. That will "give away" some of the plot of Part II. If you leave your students dissatisfied, so much the better. They will learn the details later.

The text refers to diffraction as causing a "fuzziness" around a shadow, and indeed the illustrations show fuzzy looking shadows. Students should not get the idea that the fuzziness they <u>ordinarily</u> see around shadows is diffraction. Ordinarily this fuzziness comes from the fact that the luminous source has a finite size so there is a region at the edge of each shadow which is illuminated by only part of the source. The illumination falls off gradually and gives rise to a fuzzy edge. Probably none of your students has ever seen a shadow edge carefully enough displayed to see genuine diffraction effects. The light must come from a <u>very</u> small source--pinhole or slit--and the shadow must be formed by a very sharp edge – such as a razor blade. It is probably best not to get involved in demonstrations at this stage.

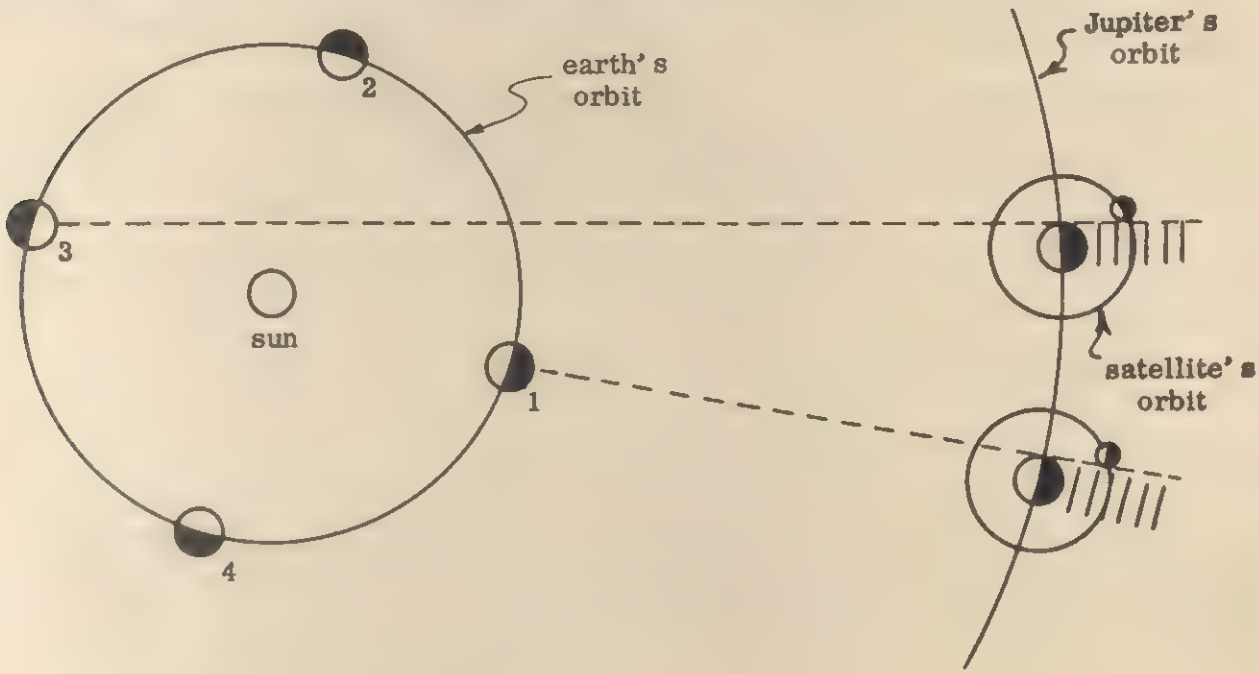
Section 8 - The Speed of Light

CONTENT The speed of light in vacuum or air is so much greater than that of observable objects that it is difficult to comprehend; the value is 3×10^8 m/sec.

CAUTION In order to develop the story of light as this volume intends, it is best at this stage, not to discuss the speed of light in materials other than vaccum or air. Some students may already know that 3.00×10^8 m/sec is an upper bound speed for anything, and therefore, if light travels at a different speed in other media, it must be slower. Also, occasionally there will be a student who knows the connection between the speed of light and the refractive index of the medium. Try to persuade such students to keep their knowledge to themselves for the time being inasmuch as it is not immediately relevant.

If the class presses for some kind of discussion on the speed of light, you might challenge them to devise methods other than the one indicated on page 188 for measuring the speed of light either in a vacuum or in a transparent substance. However, do not discuss alternate methods in detail until Chapter 15.

IMFORMATION Students who are familiar with the motion of the planets and their moons within the solar system may get a fair understanding of Roemer's method of measuring the speed of light. Other students will get only a vague notion. A precise understanding is not needed. However, someone may ask you for an out-of-class explanation. The following notes do not tell the whole story, but they may help.



With reference to the previous diagram:

When the earth is in position 1, the time between two successive eclipses of Jupiter's satellite can be measured, giving the period of rotation as 42 1/2 hours. This measurement can be made so accurately that the exact time of a particular eclipse seen some six months later can be predicted. In six months, the earth has moved from 1 to 3. When the observation of the eclipse is made, it is found to occur 16 minutes 20 seconds late. Roemer's measurements were not so accurate. He obtained 22 minutes. He correctly surmised however, that this discrepancy was caused by the time it took light to cross the diameter of the earth's orbit.

In order to understand a detailed explanation, it is necessary to note that during the 42 1/2 hours between two successive eclipses of the satellite, when the earth is near position 1, the earth has moved some 2.84 million miles along its orbit, but gets very little farther away from Jupiter. Therefore, the time measured between eclipses gives an accurate measure of the satellite's true period of rotation.

Three months later, when the earth is at position 2, the situation is quite different. Between two eclipses, the earth moves 2.84 million miles, now almost directly away from Jupiter. It takes light 15.3 seconds to travel this extra distance, and when the earth is near position 2, one revolution of the satellite appears to take 15.3 seconds longer than at positions 1 or 3. Near position 2, every 42 1/2 hours, the satellite appears to fall 15.3 seconds further behind schedule. By the time the earth has reached position 3, it is 16 minutes 20 seconds behind schedule. At other positions along the path from 1 to 2 to 3, the period seems to be greater than 42 1/2 hours by amounts less than 15.3 seconds. The exact value depends on how far away from Jupiter the earth moves in 42 1/2 hours.

While the earth is traveling from position 3 to position 4 and back to position 1, one revolution of the satellite seems to take less than 42 1/2 hours. By the time the earth is back at 1, all the "lost" time has been made up. It is possible, then, to use an average

period of revolution of the satellite to predict eclipses. This can be measured more accurately than a single period of revolution.

The foregoing analysis does not take into account the relatively small motion of Jupiter during the earth's revolution around the sun. Jupiter moves around the sun in 12 years. Hence, it travels only 1/12th of its orbit in an "earth" year. Jupiter's motion does add a sizeable correction to the previous analysis, but this refinement does not change the general argument.

Roemer's method assumes, among other things, that a moon of Jupiter has a constant period of rotation. In Part III, students will learn what was known, circa 1676, that led to the belief that this period was indeed constant.

Chapter 11 - How Light Behaves

For Home, Desk and Lab — Answers to Problems

Most of the problems for this chapter are qualitative, and intended more for class discussion than for homework. Problems 1, 7, and 10 should be accompanied by demonstrations if possible. Problems 11 and 12 are the only quantitative problems; they involve kinematics and the speed of light. Problem 9 makes a simple but interesting out-of-class exercise.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 11-10 to 11-14 .

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1	1	2, 3*		1, 2, 3	1
2, 3		4, 5, 6*		4, 5	
4, 5	7, 8*	9, 10*		7, 8*, 9, 10*	10
6, 7		6*		6*	
8	11*, 12				

SHORT ANSWERS

1. Color becomes redder as brightness decreases; flashlight is incandescent.
2. See diagrams given under detailed discussion on page 11-10.
3. (a) A black sky, brilliant stars, and a bright halo around a dark earth.
   (b) The sky and stars would be the same; the earth would appear nearly "full".
4. Clear water absorbs some light; experiments may by needed.
5. Plane reflects sunlight which is coming from below the horizon.
6. Light is coming mainly from beyond the cloud, and background of cloud is light.
7. A positive image or print.
8. Objects at different distances are not focused simultaneously.
9. Distance varies; it is about 4 inches for high school students but more for older people.
10. Gray square appears (a) dark gray, (b) green, and (c) very light gray.
11. (a) 500 seconds.
    (b) 4.1×10^{16} meters.
    (c) Larger unit makes expression simpler.
12. (a) 1.6×10^{-2} seconds.
    (b) 4.0×10^{8} meters.

COMMENTS AND SOLUTIONS

PROBLEM 1

Take a flashlight with a rather old battery, turn it on, and observe what happens to the brightness and to the color of the light that it gives off as the battery grows weaker. Does only the brightness change, or do the brightness and color change together? Is the bulb in a flashlight an incandescent source or not?

A flashlight bulb is an incandescent source. As the battery weakens, the filament gets cooler, the brightness decreases rapidly, and the color becomes redder.

PROBLEM 2

Remembering that we see the moon by reflected light, can you show that:

(a) A moon rising in the east at midnight cannot be a full moon.

(b) A new moon cannot be seen for long after dark.

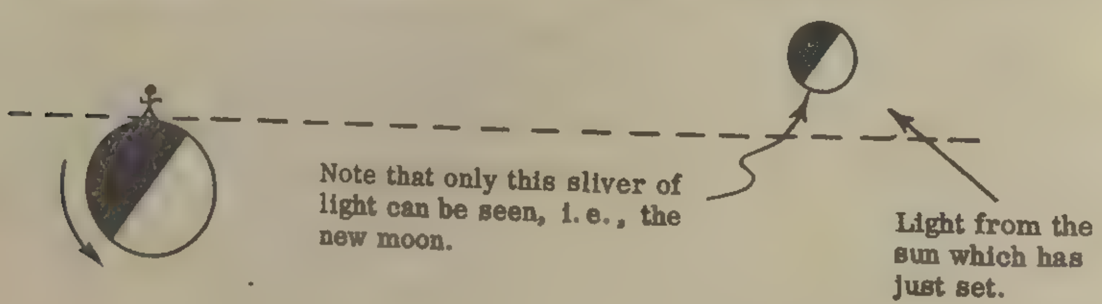
Hint: Make diagrams showing the positions of the sun, earth, and moon at different phases of the moon.

This problem is trivial to some students and challenging to others, dependent on their previous contacts with these ideas. Be sure that students realize that the sun, the earth, and the moon remain approximately in the same plane.

a) If we look down on the earth and moon from the direction of the north pole, we see:

A man at the position shown sees the moon just coming over the horizon. The projection of the horizon out into space is shown as a dotted line. As the earth turns as shown by the arrow, the horizon rotates, and the moon appears to rise above the horizon. Where is the sun? Since it is midnight, the sun is halfway between sunset and sunrise and is therefore a long way straight down toward the bottom of the page. The lower half of the moon is therefore illuminated as shown. To our man the moon appears to be half illuminated with the bright half toward the horizon; it is therefore a half moon and not a full moon.

b) Later in the month, when there is a new moon, the arrangement is shown below.

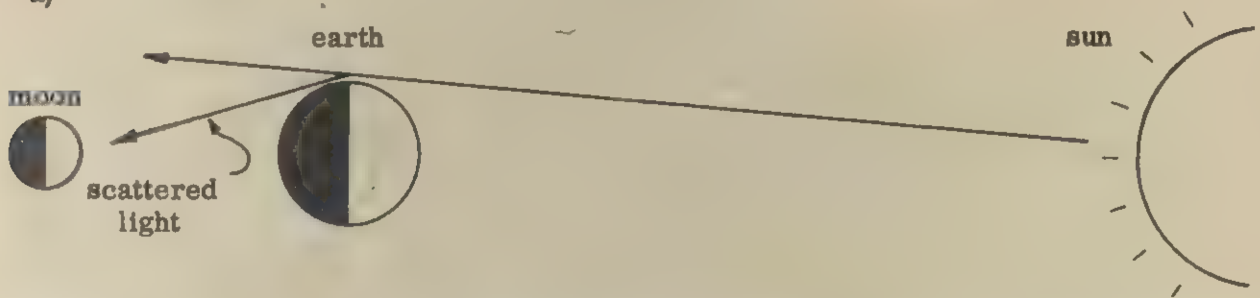


PROBLEM 3

The moon, unlike the earth, has no atmosphere.
(a) If you were standing on the moon, at a place where the earth appeared to be directly overhead, what would you see in the sky, assuming that the moon is seen as full from the earth at the time?
(b) Answer the same question for the time when the moon is seen as a new moon from the earth.

This is a problem which involves a number of concepts in light propagation. It would make a good class discussion.

a)



The man on the moon would see a black sky since the blue of our sky is light scattered by the atmosphere. The stars would be brilliant. If the earth were exactly on the line between the moon and the sun, the moon would be in eclipse as seen from the earth, i.e., the moon would be in the earth's shadow. It would not be completely dark on the moon because the earth's atmosphere would scatter light around the earth to the moon. (Have you ever seen the moon in eclipse? It does not completely disappear.) The earth would thus appear as a dark ball surrounded by a brilliant ring of light. This is not the same as the sun's corona seen during a solar eclipse.

If the moon were not eclipsed but only "very full", the sun and earth would appear side by side in the sky. If the man shielded his eyes from the brilliant light of the sun, he could presumably see a bright sliver of earth on the side toward the sun (a "new earth" instead of a new moon), and the bright ring of light around the earth due to scattering of light by the earth's atmosphere. Remember, there is no "blue sky" on the moon.

b) The observer on the moon would see the earth as nearly "full". The diagram for Problem 2b would be appropriate.

PROBLEM 4

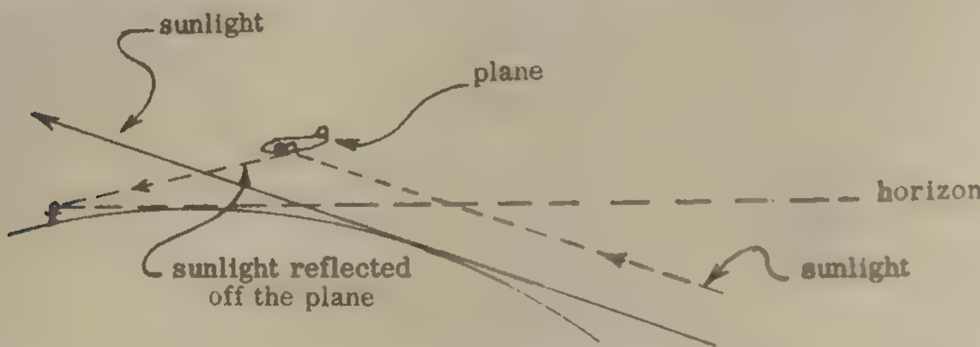
We have seen that glass, although transparent, does not transmit all of the light that enters it, some of the light being absorbed. Is this also true of clear water? Be prepared to discuss what evidence you would look for to support your answer.

All known substances absorb some light. Pure water does not absorb much. Deep in the ocean it is dark; even deep in the water of a sun-lit swimming pool the light is greatly reduced. The absorption of light in these cases may be due to impurities, so that an answer from common experience may not be a good one. If "clear" water is interpreted as "pure" water, a careful measurement with distilled water would be necessary to support a conclusion.

PROBLEM 5

Occasionally one sees an airplane in the sky shortly after sunset or shortly before sunrise and is surprised to see it appear to be very bright, more like a star or planet than like a plane as seen in daylight. Can you explain why this happens? Hint: Draw a diagram showing the earth, the sun, and the airplane.

The diagram would be very nearly like that for the new moon, **in Problem 2**, and the reason for seeing the plane the same. Against the darkening background of the evening sky, the plane reflects sunlight to the observer.



PROBLEM 6

Fig. 11-12 shows clearly that smoke scatters light toward our eyes, even though the light was originally traveling in such a direction that it would not reach us. In view of this, why does a dense cloud of smoke overhead appear dark, rather than light? Write briefly. Be prepared to discuss.

Whether something appears light or dark depends both on the sources of illumination and on the background against which the object is seen. Thus the smoke of Figure 11-8 appears bright against a dark background. The same smoke, however, absorbs some of the light hitting it. If smoke is illuminated from behind, it will then absorb some of the light and appear darker than its background. (Most smokes are carbon smokes, and therefore black. Even cigarette smoke is quite dark, though it is so fine that it is an excellent scatterer of light, and hence often looks light blue. Most white industrial "smokes" are steam.)

PROBLEM 7

We have seen that when light from a white object falls on a photographic film it produces a deposit of black silver after development. This results in the familiar *negative* image on the film. If you shine light through the negative onto another photographic film, after development what kind of an image will you have?

A suitable class discussion should include a demonstration, if possible.

A positive image or "print" is made by allowing light to pass through the negative to another piece of film or photographic paper. The two should be in contact to avoid blurring the final image. Since more light passes through the clear portions of the negative then through the black portions, a dark region on the negative will become light on the positive or print. Thus a white object produces a dark image on the negative and a light image on the positive. (A Polaroid camera does not produce a conventional negative.)

PROBLEM 8

One of the remarkable properties of the eye is its ability to adjust so that objects at different distances are seen clearly. To demonstrate this, place yourself about 3 or 4 feet from a window and concentrate your attention on the window itself. You will find that the window is seen very clearly, that the lines where the glass meets the frame are seen sharply, and that even such details as spots on the glass are apparent. Still concentrating your attention on the window, note the appearance of objects in the distance, outside. Are they sharp or fuzzy? Can you observe their details? Next, concentrate your attention on the far-away objects until you see them sharply. Can you now see the details of the window clearly?

This demonstration problem is intended to show students that their eyes are optical instruments which must be focused.

Most students will readily believe that near and far objects cannot be seen sharply simultaneously. However, to a few students this will be a surprise. They have never consciously thought about focusing their eyes. Some students may not be able to see this effect because they refocus their eyes very rapidly and effortlessly. You might ask them if they can simultaneously focus on the detail of a finger held a few inches from their eyes and read the print from a page a foot or so away.

PROBLEM 9

Move this book toward you, with one eye closed or covered. When the book has just reached the point at which the print blurs, have someone measure the distance of the book from your eye. Repeat the experiment with the other eye. Are the two distances approximately equal? Try this with a few people of different ages, and record the results along with their ages.

(a) Is there a limit to the ability of the eye to adjust itself for clear vision?

(b) Is this limit the same for all persons, or for the two eyes of any one person?

(c) Does it vary, in general, with the age of the person? Compare your answers with those found by other people in the class.

This problem again brings to the student's attention the fact that he must focus his eyes.

The shortest distance for clear vision is about 4 inches for high school students, and may be several feet for older persons. Usually the two eyes are more or less alike, but not in all cases. The distance varies with eye fatigue. See Appendix 5, Supplementary Information on the Eye (at the back of this volume), for additional details.

PROBLEM 10

It was stated in Section 11-2 that color is a complicated phenomenon, depending on many factors, not just on the kind of light reaching a surface and on the nature of the surface. You can readily convince yourself of this. Cut a small square of light gray paper or cardboard, about 1 inch on a side, and place it in the center of a large piece of white paper. View it from a distance of about 2 feet and under bright illumination.

(a) Describe its color and brightness.

(b) Move the gray square to the center of a large piece of bright-red paper and view it under the same conditions as before. Again describe its color and brightness. Are they the same as before?

(c) Finally, move the gray square to a piece of black paper. What happens now?

This laboratory problem tests some of the properties of the eye. It is fun but not fundamental. For most people the gray square appears black or dark gray on a white background, green on a red background, and rather light gray on the black background.

PROBLEM 11

(a) How long does it take light to reach the earth from the sun?

(b) If the light from the nearest star takes 4.3 years to reach us, how far away is the star?

(c) Why is it convenient to express distances to stars in terms of light years, rather than in meters, kilometers, or miles?

a) The distance of the earth from the sun is about 1.5×10^{11} meters.

$$t = \frac{d}{v} = \frac{1.5 \times 10^{11}\ \text{m}}{3 \times 10^{8}\ \text{m/sec}} = \underline{500\ \text{seconds}}.$$

b) 4.3 light years can be expressed as 4.3 years $\times$ 365 days/year $\times$ 24 hours/day $\times$ 3600 sec/hour $\times$ 3.00×10^{8} m/sec = $\underline{4.1 \times 10^{16}\ \text{meters}}$ = 2.5×10^{13} miles.

c) The light year which is 5.89×10^{12} miles is convenient as a unit because the distances of stars are so great that they are more easily expressed in these large units than in the astronomically tiny units of miles. It is similar to the convenience of expressing the distances between cities in miles rather than in inches.

PROBLEM 12

Radio waves travel at the same speed as light in empty space or in air.

(a) How long does it take a radio signal to travel from New York to San Francisco, a distance of about 4.8×10^3 km?

(b) A radar transmitter, which sends out radio signals of a particular type, when pointed at the moon receives a reflection 2.7 sec after the signal is sent. What does this experiment give as the distance of the moon from the earth?

a) $$t = \frac{d}{v} = \frac{4.8 \times 10^{3}\ \text{km} \times 10^{3}\ \text{m/km}}{3 \times 10^{8}\ \text{m/sec}} = \underline{1.6 \times 10^{-2}\ \text{seconds}}.$$

b) The total light path d is $d = vt = 3 \times 10^{8}$ m/sec $\times$ 2.7 sec = 8.1×10^{8} meters. The distance to the moon is $\frac{d}{2}$, or $\underline{4.0 \times 10^{8}\ \text{meters}}$.

Chapter 12 - Reflection and Images

This chapter discusses the straight line propagation of light, ray diagrams, the laws of reflection, and image formation. Two simple concepts, straight line propagation and the laws of reflection, summarize a vast amount of experimental observation. Students will have to know this material well in order to understand Chapters 13 and 14.

CHAPTER SUMMARY

Section 1 uses the straight line propagation of light as an aid in analyzing shadows. This gives students experience in using ray diagrams before the complications of reflection and image formation are introduced.

Section 2 explains the idealization of a light ray, and justifies the use of ray diagrams by pointing out that crossing light beams do not affect one another.

Sections 3 through 5 are best treated as a unit so that students get a coherent picture of the formation of images by plane mirrors. Section 3 shows how, in principle, we can determine the position of an object by the light rays from it. Section 4, introduced by Experiment II-1 (Reflection from Plane Mirrors), discusses the laws of reflection. Finally, Section 5 makes use of both these ideas to discuss what we see in a mirror in terms of the concept of an image. The idea of how light rays are used to locate an object and hence an optical image of that object, is extremely important for the understanding of this and subsequent chapters. The laws of reflection are also of utmost importance and serve as one of the finest illustrations of the way in which a simple law can explain great amounts of information.

Section 6 shows how a combination of plane mirrors can be used to obtain focusing. It then introduces the parabola as that surface which provides perfect focus for incident light parallel to the axis. Preliminary to image formation in Sections 8 and 9, this section locates the path of rays reaching the mirror parallel to the axis.

Section 7 by introducing the reversibility of light, extends the ideas on the parabolic mirror to include its use as a searchlight. Preliminary to image formation in Sections 8 and 9, this section locates the path of rays reaching the mirror after having passed through the principal focus.

Sections 8 and 9 together show how concave mirrors produce images. In Section 8, the focusing of non-axial, parallel light is described. Although the discussion centers on astronomical telescopes, the main idea is that concave mirrors form images of points by concentrating light at points. This sets the stage for Section 9 in which real images are discussed in some detail.

Before Section 9 is studied, students should see the effects in laboratory. Experiment II-2, or at least parts of it, should be done at this stage. Students should then study and discuss image formation by concave mirrors. Teachers should stress ray diagrams and their usefulness, even when mathematical analysis is to be used for precision.

Section 10 summarizes some information on real and virtual images.

SCHEDULING CHAPTER 12

In terms of ideas, both the laws of reflection and image formation are extremely important. However, the laws themselves do not take much time to develop. Their underlying importance can be communicated best by having students draw a ray diagram with a curved mirror, using arbitrary rays and the laws of reflection rather than special rays which eliminate the need for constructing an angle of reflection equal to the angle of incidence.

On the other hand, ray diagrams, the location of the apparent origin of a group of rays, and image formation all require some explanation and practice.

If you are rushed and must hurry through some parts of the material, stress ray diagrams rather than formulas. Be sure that students never lose sight of the laws of reflection even if you have to treat more lightly such topics as parabolic mirrors, searchlights, beams and pencils, etc.

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part II.

	14-week schedule for Part II			9-week schedule for Part II		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2	1	0	-	1	0	-
Secs. 3, 4, 5	1 1/2	2	II-1	1 1/2	1 1/2	II-1
Secs. 6, 7	1/2	0	-	1/2	0	-
Secs. 8, 9	3	2	II-2	2	1 1/2	II-2
Sec. 10	0	0	-	0	0	-

RELATED MATERIALS FOR CHAPTER 12

Laboratory. Experiment II-1 - Reflection from a Plane Mirror. In this experiment students locate an image in a plane mirror. Rays are defined by sighting along pins stuck into soft cardboard.

The equipment for this experiment is very simple and might be distributed in your regular classroom with this laboratory providing the basis for an immediate class discussion. It should be performed rather early in the study of the chapter, most desirably right after Section 3.

Experiment II-2 - Images Formed by a Concave Mirror. The focal point of a small concave mirror is determined. A small bulb is used as an object and the position of the image determined by parallax. After several object distances are used, the formula $S_o S_i = f^2$ can be verified.

This experiment is best performed just before the discussion of Section 9. Concave mirrors might be distributed to the class much earlier to help ensure that students have a concrete basis for understanding Sections 6 through 9. See yellow pages for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 12-21; detailed comments and solutions on page 12-22 to 12-38.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1, 2	1*, 2	3, 21		1*, 2, 3	[redacted]1
3, 4, 5		5, 6*, 7	4, 8 9 10	5, 6*, 7, 8	5, 6
6, 7	11			11	
8, 9	13*, 14 15	12*, 16* 17*, 18*, 19		12*, 16*, 17*, 18*	18*
10		12*	20, 22	12*, 20	

Films. None of the PSSC films relate particularly to this chapter.

Section 1 - Shadows
Section 2 - Light Beams, Pencils, and Rays

PURPOSE To introduce the ray diagram as a useful technique for understanding effects caused by light.

CONTENT a. The gradations of darkness in a shadow can be understood in detail by drawing rays from the light source (or sources) to the shadow.

b. Light rays are idealizations of very narrow pencils.

c. Because beams of light are not affected when they cross other beams, individual rays can be traced independent of other rays.

EMPHASIS Ray diagrams are basic to the later sections of this chapter and to Chapters 12, 13, and 14. Shadow formation, in itself, is not so important, but it is a good, simple introduction to the use of ray diagrams. Section 2 requires very little class time because students will learn to use ray diagrams easily even if they are not aware of the exact significance of a ray; that crossing rays do not affect each other is a natural assumption that can be simply noted in passing.

DEVELOPMENT Demonstrate and Analyze Shadows.

A. Most students will be helped by having a little supervised practice with very simple "ray diagrams" before they cover the material on image formation. Furthermore, with your aid, many students will be excited to find themselves able to recognize and to understand new details in something as common as shadows.

1. First show students a sharp shadow which most of them understand easily. You can easily form a suitable shadow by using a small light source in a darkened room. A clear glass lamp is a suitable source. The light source for the ripple tank is excellent. (Familiar classroom examples occur when someone walks into a movie projector beam, or if an object such as a hand, interrupts a small stray light leak from a projector.) You might want to use some special cutouts, or your fingers to produce interesting or amusing shadows. Analyze this simple arrangement by sketching on the board,showing some illustrative light rays.

2. Next show a shadow which is fuzzy; the students will be less certain about these. (Shadows cast with light entering the classroom windows or coming from ceiling lights are usually unsuitable because they are completely fuzzy and have no dark core.) Produce a shadow with a large light source in an otherwise darkened room. A long fluorescent light produces an interesting case, a sharp shadow in one direction and a very fuzzy shadow in another. Be sure that students see that there is a dark core and that the surrounding region is not uniformly gray but has gradations of illumination. Draw the appropriate diagram. Use rays to decide, for several points in the shadow, which regions of the source contribute light.

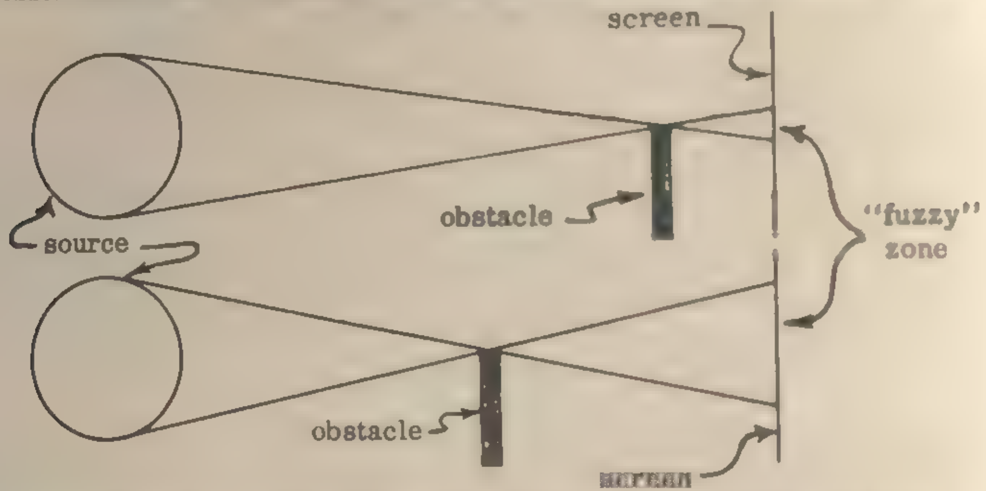
3. Ask the students to predict, and then have them observe, the effects of moving the source, or the obstacle, or the screen. You might also want to use several light sources first alternately, and then together.

Before you demonstrate these effects you could ask the class to describe the successive shadows formed by automobile headlights as the auto approaches a pedestrian at night. When the car is far away, the shadow is quite sharp. However, as the car approaches, two separate shadows are formed by the two headlights. At the same time, each of these shadows begins to have the structure of a dark core surrounded by regions in which there are gradations of illumination.

B. In order to summarize the observations and to be sure the class understands shadow formation, you can discuss HDL Problem 1 in class. Students should be able to suggest that a sharper shadow can be produced by increasing the distance from the light source or by reducing either the size of the light source or the distance between the obstacle and screen. If some students seem uncertain, go over the appropriate ray dia-

grams, first to show the rays which determine the size of the fuzzy region, and then to show, again, the portions of the source which can contribute illumination to several of the points in this fuzzy region.

Here is a diagram which shows what happens when the source and screen remain fixed and the obstacle moves. A similar, but slightly more complicated pattern will be seen if



students look at shadows of their hands while moving them closer to the desk.

Note that with a single light source, such as in the diagram above, the sharpening results from the decrease of the fuzzy zones. However, often there are several light sources (or sources of stray light) which contribute stray light even though there appears to be a single major source. As an obstacle approaches a screen the stray light is decreased tending to produce more contrast between the shadow and its surroundings.

Non-interaction of Light Beams A. Ask students what Figure 12-4 indicates and ask about the relevance of this to ray diagrams. Most students realize that light beams cross one another without interacting, but some may not appreciate that this lack of interaction is continually assumed when ray diagrams are drawn.

B. Ask students what would happen if the two beams of white light in Figure 12-4 were replaced by a blue beam and a yellow beam. You will probably find that some students expect an interaction; a few expect the beams to become greenish. It may be worthwhile to demonstrate crossing colored beams to help convince them. However, you should also ask for other possible experiments. If no student suggests merely looking at two colored objects side by side, ask the class to consider a yellow marble and a blue marble that are next to each other. Some students should be able to point out that the differently colored light beams maintain their color despite the fact that they cross both on the way to, and within, the eye.

COMMENTS Some Terms Are Used Qualitatively. Some "technical" words are used qualitatively and therefore no simple rigorous definition applies. The terms "beam" and "pencil" are good examples. The term "pencil" is not restricted to some definite small divergence, but is used to describe light traveling in a narrow beam whose divergence is not significant for the particular problem being studied. The two terms are sometimes used interchangeably. There is no need to have students distinguish between these terms.

A somewhat similar situation exists with the terms umbra and penumbra. Umbra and penumbra are used when there is an obvious darker region in a shadow surrounded by a less darkened region. However, if one analyzes a shadow carefully, and includes stray light sources as well as the dominant source, the terms are not very useful.

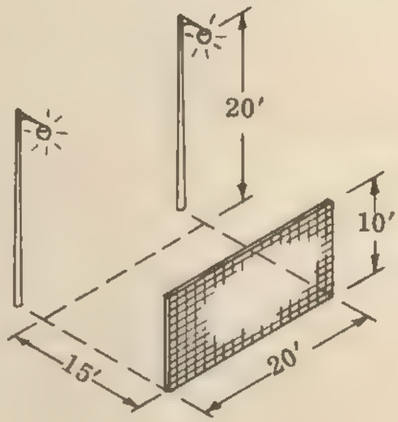
Shadows Do Not Involve Diffraction. From the qualitative description of diffraction in Chapter 11, or from previous knowledge, some students may be inclined to think of dif-

fraction in connection with fuzzy shadows. They may need to be reminded that, on the scale with which we are dealing, diffraction is a small-scale effect and can be neglected. Unless students raise the issue, do not mention diffraction here.

Note: Problem 21 contains, in addition to the suggested project, two simple applications of ray diagrams. You may want to use these parts to see whether the students can generalize their knowledge gained from shadows, or from Problem 2, to a slightly different geometry.

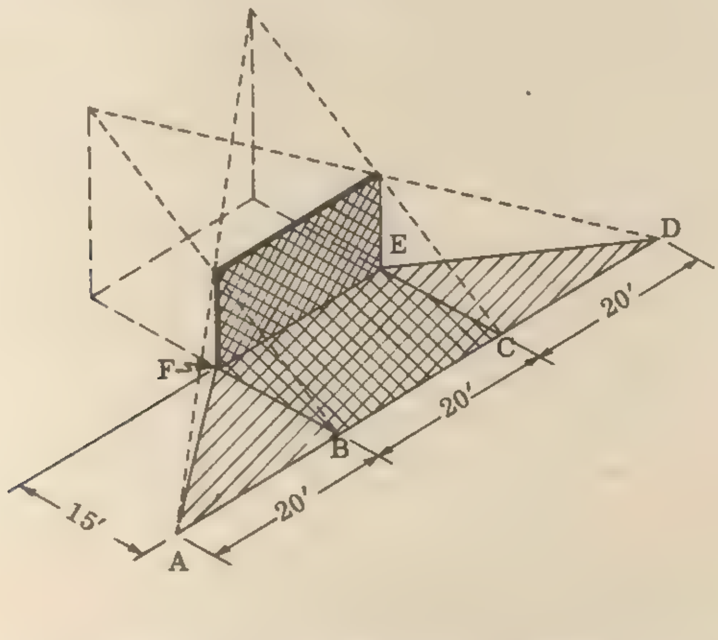
QUIZ PROBLEM Variations of Problem 2 or Problem 21 would make suitable quiz problems. Here is another suggestion:

1) Two street lamps, 20 feet above the ground, are placed 15 feet in front of the ends of a brick wall, which is 20 feet long and 10 feet high, as shown. Show the regions of dense shadow and partial shadow.



Solution: BCEF is dense shadow.

ECD and ABF are partial shadows.



Section 3 - How We Locate Objects

PURPOSE We locate an object visually by noting the point from which light rays come. This almost obvious idea is emphasized in this section, because in Section 5, and in later chapters, this method will be used to locate optical images.

EMPHASIS This idea is very important, not for teaching how we locate objects, but for preparing the way for locating optical images by ray diagrams.

DEVELOPMENT a. Give students simple examples of pairs of diverging rays and ask them to locate the points from which the rays come.

b. Ask them to fill in additional rays that would come from the same point.

c. Are two rays enough to locate a point? They are, but a third ray makes a good check. A surveyor locates an object by two sightings, but often uses a third for a check.

d. An interesting variant which prepares the way for extended images in mirrors might be: The dotted rays come from a man's head, the solid ones from his feet. Is he standing up or lying down?

Extending the rays to this point of intersection shows he is lying down!

Do not dwell too long on this material. It can seem somewhat unmotivated unless you proceed rapidly to the use of ray tracing in locating an image in a mirror. The natural next step is to introduce the location of an image in a plane mirror (Experiment II-1) as an interesting extension of these ideas for locating an object, and then proceed to a classroom discussion of Sections 4 and 5.

The terms "focusing" and "converging" may not be familiar to some students. "Converging" may be explained at this stage, but it is best to defer a detailed discussion of focusing. The students have read (in Chapter 1) about the adjustment of the eye that is necessary to make objects appear "sharp" or "clear". You can merely add that this adjustment is called "focusing".

Although there are other ways of judging distance (and hence position) in addition to extending light rays back, it is the most important for later work. There is no need to discuss in class other important clues to distance, such as stereo vision and apparent size. Stero vision is based on the fact that the two eyes get two slightly different views of a nearby object. These two views, coupled with a prior knowledge of what the object looks like, give a good indication of where the object is. The apparent size of a known object is another important clue, as most students will realize quickly if you remind them of how they get an impression of depth in a picture or painting. Gradients of texture (the "nubbiness" of a rug disappears in the distance), and gradients of color (green foliage at a distance is not as "green" because of filtering and scattering), as well as other factors also contribute to distance perception. You need not discuss angular size now, because it will come up again in Chapter 14 in connection with magnification. An interesting illusion in connection with judging distance is discussed in HDL Problem 3.

Section 4 – The Laws of Reflection

PURPOSE To develop the two laws of reflection.

CONTENT a. When light is reflected, the reflected ray lies in the plane defined by the incident ray and the normal (perpendicular) to the reflecting surface at the point of reflection.

b. The angle of reflection is equal to the angle of incidence.

EMPHASIS The laws of reflection are important. Students should understand them

thoroughly. However, they will require only a moderate amount of class time if the class is facile in the earlier work on tracing rays back to their source, and since the required practice will recur throughout the chapter. It will speed progress in Chapters 12, 13, and 14 if you devote some time to the significance of the normal, and to the definitions of angle of incidence and angle of reflection.

DEVELOPMENT If your students have already done Experiment II-1 on images in plane mirrors, this work can be introduced as an explanation of the formation of such images. You can also get them to formulate the second law of reflection (equal angles) by tracing one of their experimental light rays from the object pin to the mirror and back to the sighting pins, and noting the equal angles of incidence and reflection.

Initially, you can define specular reflection as the type you get from a shiny surface, such as a mirror. Surfaces having varying degrees of smoothness give varying amounts of specular reflection. No real surface gives entirely specular reflection because no surface is ideally smooth.

Once the discussion is confined to specular, or regular, reflection, students can follow the development of this section easily, but there is one detail that needs attention. Students should distinguish carefully between the angle of incidence (the angle between the incident ray and the normal) and the complement of the angle of incidence (the angle between the incident ray and the reflecting surface). Because of the two-dimensional drawings they use, some students may feel that it is more natural to measure the angle from the incident ray to the surface of the mirror. You can show why this is not desirable by using a three-dimensional exercise:

1. Assume the blackboard to be a mirror.

2. Use a meter stick or pointer to represent an incident ray. Let it touch the blackboard at an angle to the normal.

3. Put a chalk mark on the board at the point of incidence.

4. Have a student hold another stick to represent the reflected ray.

5. Using the same point of incidence, change both the angle of incidence and the plane of incidence by changing the direction of the "incident ray". Ask the student for the corresponding "reflected ray".

After you use several "incident rays" ask students what one line will locate both the plane of the reflected ray and its exact direction for all rays incident at one point. This convenient line is the perpendicular to the reflecting surface at the point where the incident ray strikes; it is the "normal". Despite your three-dimensional arguments, some students might privately continue to use the complement of the angle of incidence, because they can locate it more easily on their two-dimensional drawing, and because they realize that it does not change the result. Discourage this because these individualists will be quite confused later by the laws of refraction.

QUIZ PROBLEM See Section 5.

Section 5 - Images in Plane Mirrors

PURPOSE To indicate what an image is, and to analyze geometrically image formation by plane mirrors.

CONTENT The image of a point in a plane mirror is located at that position from which the light rays seem to come.

The image of a point seen by reflection in a plane mirror can be located as follows: (a) Construct the perpendicular from the point source to the mirror, extending it behind the mirror. (b) Mark the point on this extended perpendicular which is as far behind

the mirror as the source is in front of the mirror. This point is the image of the point source.

The position of the image is determined uniquely by the laws of reflection.

EMPHASIS Stress the idea of an image in a plane mirror. Do not dwell on systems which involve several mirrors at angles, except as optional laboratory exercises for students who are not otherwise challenged.

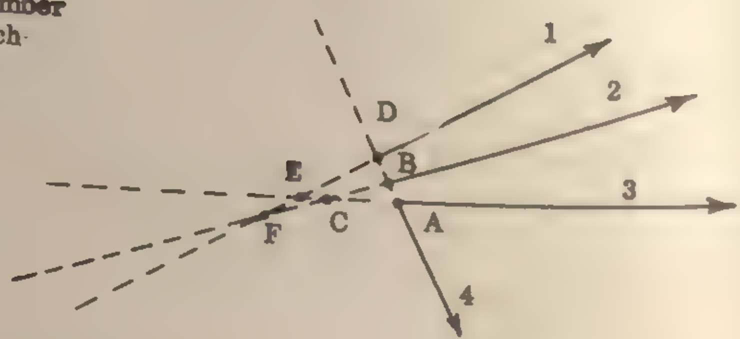
DEVELOPMENT Some teachers have found that it is helpful to conduct the discussion of Sections 4 through 10 in the laboratory, building the discussion around Experiments II-1 and II-2. The sequence of ideas, from the location of objects by rays (this can be done by lining up pins on an object pin) through the laws of reflection, to image formation in a mirror, lends itself naturally to laboratory development.

Naturally, it is also possible to formally separate discussion and laboratory. In this case try to schedule the laboratory before discussion. In either case, as a useful detail of technique, it will be helpful to make sure students understand the method of parallax in locating images because the method is used in later experiments.

If students are shaky about the backward extension of diverging rays to their point sources (which was stressed in Section 3), they may need a review before you discuss image formation in more detail. Here are some suggestions.

(You may foster interest by presenting this as a puzzle.) Draw a few diverging rays similar to these:

Ask the class for the minimum number of point sources of light from which these rays could originate.



(Unless the students bring it up, do not bother to discuss the ambiguity that many pairs of two points might explain the four rays. For example, you can accept any of the following three combinations without discussing the other possibilities.

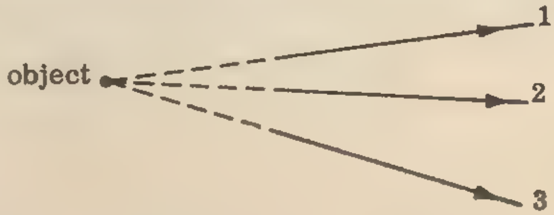
Rays 1 and 2 could come from F while Rays 3 and 4 come from A, or

Rays 1 and 3 could come from E while Rays 2 and 4 come from B, or

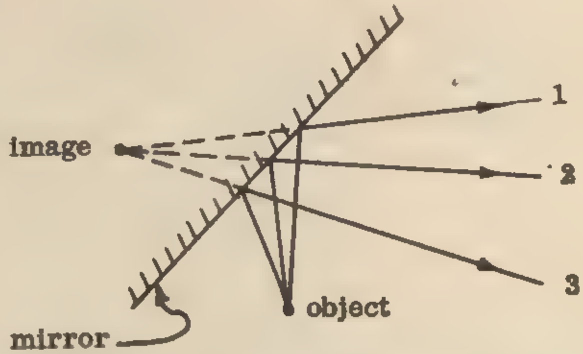
Rays 1 and 4 could come from D while Rays 2 and 3 come from C.

In an actual case each light source at any location would produce many more than two rays and no ambiguity would remain.)

A final step which can be used effectively would be to put on the blackboard a key diagram like this:



Now ask whether the object had to be at that location or whether rays 1, 2, and 3 could have come from a mirror. Draw in a mirror, and ask a student to use the law of reflection to draw in the incident ray which must have produced ray 1. Do the same for ray 2 and ray 3.

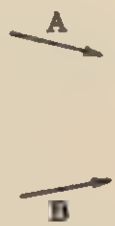


With the mirror present, what one point source might produce these rays? If you don't know the mirror is there, and you look at the rays, where do you think they come from? Stress the concept of an image point which corresponds to some object point. You may want to review, briefly, the proof given in Section 5. Many of the students can follow this easily, but a few need help.

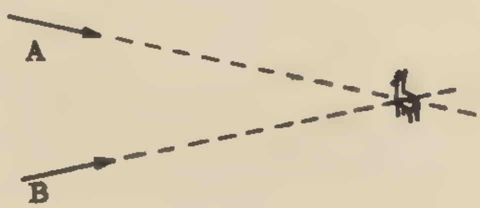
COMMENT While both the terms "virtual image" and "real image" are introduced in this section, it will be better to defer a discussion of real images. It is enough if students know that the image formed by a plane mirror is called "virtual" because the light does not really diverge from the image. There is no light behind the mirror. If a student asks how an image can be other than virtual, you might give him the example of the image on a movie screen and tell him that the subject of real images formed by concave mirrors is the next topic. You can tell him that an image is real if the light rays really do diverge from the image.

QUIZ PROBLEMS

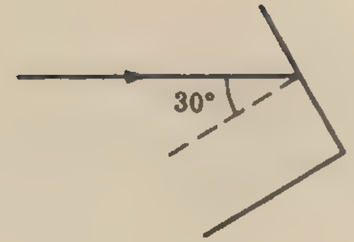
1) (This one is trivial, but may help some students get the point.) Hunters A and B have their rifles pointed as shown. If they are aiming at the same deer, where is the deer?



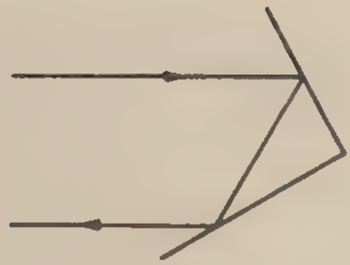
Solution: Extend the lines of sight (rays) to the point where they cross. There is the deer!



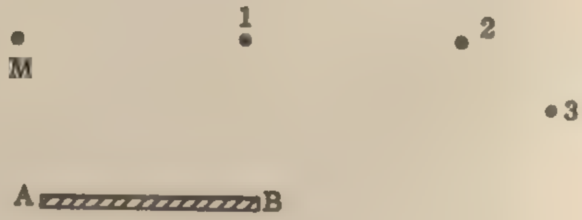
2) Two mirrors meet at right angles. A ray of light is incident on one at an angle of 30°. What will be the subsequent path of the light ray?



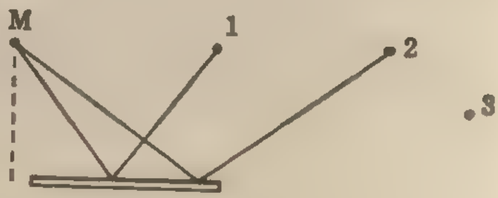
Solution: No matter at what angle a ray hits a pair of mirrors at right angles it will be reflected back parallel to its incoming path.



3) A man stands at M just to one side of a plane mirror AB. Can he see himself in the mirror? Can he see objects (1), (2), and (3)?



Solution: He cannot see himself. He cannot see (3) since no ray can be drawn from 3 to M which obeys the laws of reflection. He can see (1) and (2) by means of the rays shown.



4) a) Find the position of the image of O in the plane mirror AB.

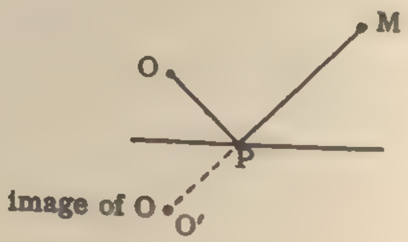
b) Can a man at M see this image?

c) When the man moves from M to N where does the image of O move?

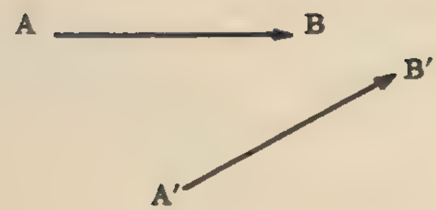
Solution: a) as shown at O

b) Yes, since light can go from O to P to M. Another way of seeing this is that the line MPO′ passes through the mirror.

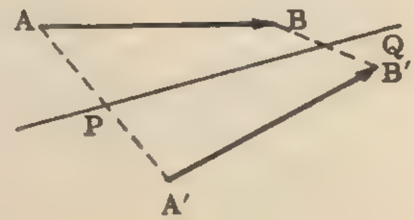
c) This is a "trick" question. Only O and the mirror determine the position of O′. O′ does not move.



5) Can A′B′ be the image of AB in any plane mirror? Prove it.



Solution: If A′ is image of A, the mirror must lie midway between A and A′. Likewise it must lie midway between B and B′. The mirror must thus lie along PQ. But then AA′ is not perpendicular to PQ and hence A′ cannot be the image of A. A′B′ cannot be the image of AB.



Section 6 – Parabolic Mirrors

PURPOSE To build up, from plane mirrors, a "curved" mirror <u>which</u> <u>can</u> <u>focus</u> <u>light</u>. To point out that the laws of reflection apply to curved mirrors as well as to plane mirrors. This section provides the knowledge that the ray parallel to the mirror's axis is reflected through the principal focus. This provides one of the bases for the location of images (Sections 8 and 9) produced by parabolic mirrors.

CONTENT a. A combination of small plane mirrors can be oriented so that each small mirror sends a small beam across the same region.

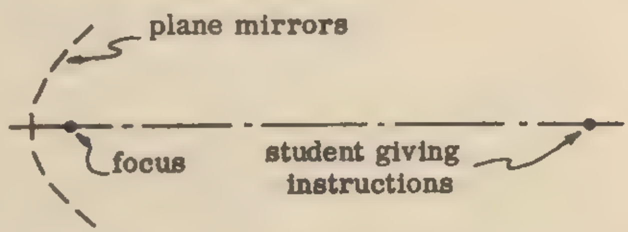
b. This region of concentration can be sharpened by using smaller mirrors which are oriented properly.

c. Continuing and idealizing this process (successive subdivision and reorientation) leads to the ideal surface for the focusing of parallel light. A cross section of this surface is called a parabola.

d. The point at which a parabolic mirror concentrates parallel light is called its principal focus.

DEMONSTRATION AND DEVELOPMENT

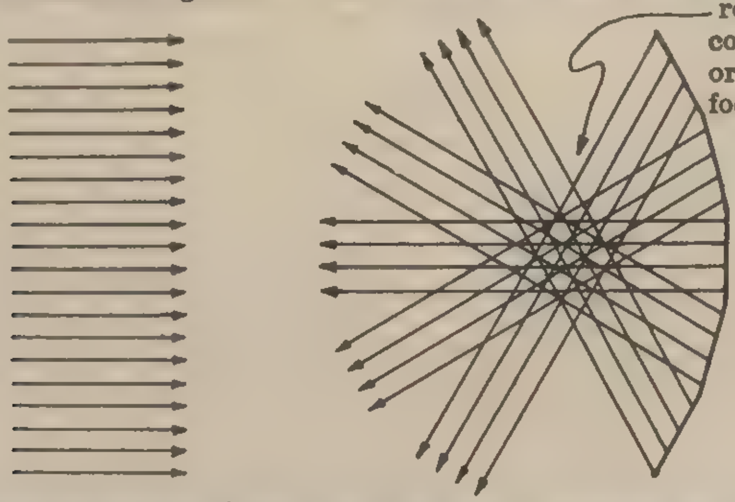
If you have pieces of plane mirror of convenient size, you can reinforce the text discussion by demonstrating the construction of an approximately parabolic mirror "by eye". Arrange the small pieces in the form of a crude parabola about a nearby focus. Attaching the small mirrors with rubber bands to a soft aluminum strip might facilitate this procedure. Put a small, bright source (a flashlight bulb works well) at the intended focus. One student stays at the table with the mirrors to make adjustments and another student places himself, say, twenty feet in front of the mirror and gives instructions. The distant student looks at the small mirrors one at a time and calls instructions until he sees the source in the center of each section. A surprisingly accurate parabola can be constructed.



DEVELOPMENT This section, while simple, is important to an understanding of what a curved mirror does. Studying the "two-dimensional" parabolic cylinder (such as Figure

12-15), before dealing with three-dimensional cases (such as Figure 12-14), will help students understand the geometry of reflection by a parabolic surface.

If students seem unsure of what is illustrated in the figures, you may want to make a blackboard sketch of Figure 12-15 as it would be if seen from above.



Students can examine this figure using the laws of reflection and consider what happens to the region of light concentration as the angles of the side mirrors are changed.

As more, smaller plane mirrors are used, the region into which light is a concentrated can be made smaller until, with a perfect parabolic surface, a point of light is produced. Students should remember that, with a parabolic cylinder, the reflecting surface is curved only in one direction (like the concave surface of a stalk of celery). Hence, parallel light is focused to a line, not a point, in space.

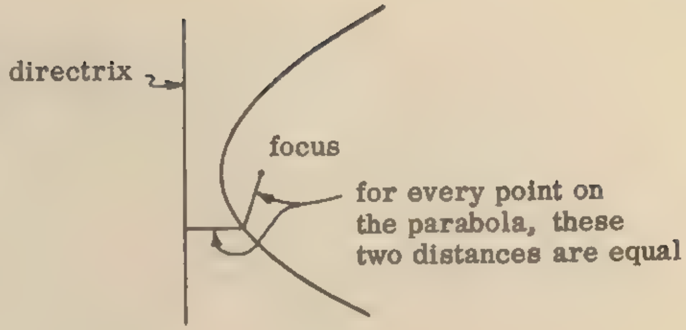
COMMENTS For a curved mirror to have a single point as its principal focus, its surface must be a paraboloid of revolution such as is approximated in Figure 12-14. Students will be helped if you distinguish between cylindrical and "surface-of-revolution" mirrors, even though cylindrical mirrors or lenses rarely enter the subsequent study. The distinction is worth making because, in looking at a cross sectional diagram, some students "see" only a cylindrical surface.

It is of critical importance for entirely inexperienced students to have at least a brief chance, early in their study of the material, to see and handle curved mirrors. They will then better understand the pictures and diagrams in the text. If possible, they should see both spherical and cylindrical mirrors.

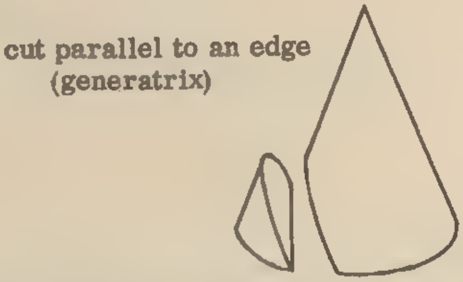
Some students will wonder why the incident light is not visible in Figures 12-14 and 12-15. In Figure 12-14, the smoke particles are illuminated by both the incident and reflected light. The level of illumination was highest where the light is concentrated, and the photographer was clever enough (in exposure and choice of materials) to distinguish between this level and lesser levels where only the incident light hit the smoke. Some of this same argument explains the absence of visible incident light in Figure 12-15. Another explanation for Figure 12-15 might be that the surface on which the light traces are shown does some specular reflecting (more light reflected in the general direction it is traveling than is scattered to the side or back). This would also explain why the reflected light paths that lie close to the line along which the camera is pointed appear to be brighter than the paths which diverge from the camera's line of sight.

CAUTION Probably few of your students will have seen the equation for a parabola. There is no need to introduce it. Most will be willing to accept the focusing property of the parabola as a definition of the curve. Indeed this is a complete and adequate definition.

Some students may know that a parabola can be defined as the curve made of points which are equidistant from a point (the focus) and a line (the directrix).



Other students may know it as a conic section.



The focusing properties of the parabola (defined in either of these ways) can be proved, but the necessary mathematics is too difficult for most students.

QUIZ PROBLEMS See Section 8.

Section 7 - Searchlights

PURPOSE Noting the reversibility of light paths, we observe that a ray which passes through the principal focus before reaching a parabolic mirror, is reflected parallel to the mirror's axis. This is one of the bits of data that will be used in locating images.

CONTENT a. Since a parabolic mirror focuses parallel light, and since light paths are reversible, a light source placed at the principal focus of a parabolic mirror produces a parallel beam.

b. Definition of f, the focal length.

COMMENTS This section and the one that follows (Astronomical Telescopes) are introduced at this point to give the student practice in analyzing simple cases of reflection in parabolic mirrors. Together, these sections illustrate the approach which can be used in the more complex examples of image formation.

The important material of this section is not new to the student. Many students would be able to figure out before reading this section that a light source at the focus of a parabolic mirror would give a parallel beam. However, it is worth stressing the behavior of the rays which enter this problem because they will be key rays in locating images. Be sure that all students realize at this stage that: parallel rays, traveling parallel to the axis, are reflected through the principal focus. Rays traveling through the principal focus will be reflected parallel to the axis of the mirror.

It is enough if students understand the theoretical notions behind the use of a parabolic reflector in a searchlight. Since the source in a practical searchlight is not a point, since it is not economical to build perfect parabolas, and since attention must be given to cooling, sealing against weather, and the like, the actual design of a searchlight is a complicated compromise with which we are not directly concerned.

DEMONSTRATION Probably the best way to demonstrate the principle of the searchlight is with a good quality "focusing" flashlight. When the lamp filament is close to the principal focus, a narrow "parallel" beam results.

You can point out that turning the reflector assembly on its screw thread moves it closer to or farther from the lamp filament. Most such flashlights, when directed to a nearby surface, can be adjusted to produce a "spot" smaller than the diameter of the reflector. You might start with this case, and adjust the flashlight to give various sizes of spots. Ask the students how they can tell when the lamp filament is at the principal focus. Some, using the "sharp focus is always best" principle, will answer incorrectly "When the spot is as small as possible". If you get the "small spot" response, see if there is any argument. If not, ask whether a parallel beam from the mirror ought to produce a spot larger or smaller than the reflector. Someone will come through with "It should be the same size". You are now ready to go quickly to Section 8.

You may be tempted to demonstrate the searchlight principle with a good mirror. If so, be careful not to discuss image formation until the searchlight is understood.

QUIZ PROBLEMS See Section 8.

Section 8 - Astronomical Telescopes

PURPOSE The astronomical telescope is introduced to illustrate the formation of an image of a point source by a parabolic mirror. Consideration of images of point sources which are both on and off the axis of a parabolic mirror helps in analyzing the formation of images of extended objects.

CONTENT a. Parabolic mirrors focus parallel light even though it approaches the mirror at an angle to the axis.

b. The image can be located by tracing any two rays. Two particularly convenient rays are:

1. The ray that travels through the principal focus (which will be reflected parallel to the axis), and

2. The ray that strikes the vertex of the mirror (convenient because the normal is already drawn).

EMPHASIS This section is particularly important because it introduces principal rays.

COMMENTS In this section, students are introduced to real images without being told so. It is probably not worth mentioning this until near the end of your discussion, when you can say that the point at which light from a star is focused is a <u>real image</u> of a distant point. It is "real" because the light rays which form the image really pass through the image. A photosensitive device placed at that point could detect the light. A small piece of tissue paper held there would "show" the image.

In locating the focal point for light that approaches at an angle to the axis, the convenience of tracing the ray that passes through the principal focus is easily seen. Using the ray that strikes the vertex is also convenient. Ask a student to describe how he would trace another ray. You probably have general understanding by students when they see that, after a tangent and normal are drawn at a given point on the mirror surface, the ray reaching that point is as convenient as the ray reaching the vertex.

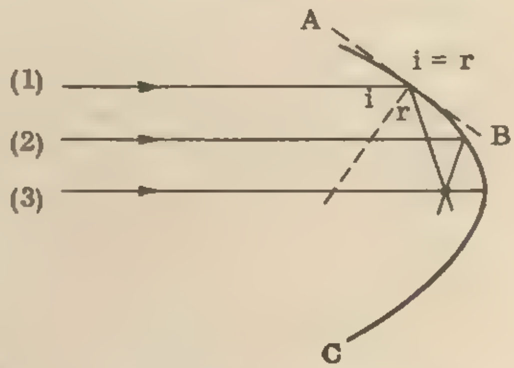
The one new idea in this section is that parallel light reaching a parabolic mirror at an angle to the axis is (almost) focused. If focusing did not occur for both light along the axis and for light at an angle to the axis, two stars could not be photographed simultaneously with an astronomical telescope. If light from one star is parallel to the axis of the mirror, then light from another star would not be. If the incoming light makes a small angle with the axis, the focusing is extremely good, although not quite as good as it would be for axial light.

The mention of telescopes is likely to trigger a good many questions on details of construction, mounting, and the like. These matters are interesting, but you probably can-

not afford the time. There are references listed on page 209 of the text as well as conventional encyclopedia articles. Perhaps you can arrange an out-of-class visit to an amateur astronomer (after you have covered Chapter 14 which includes refracting telescopes).

QUIZ PROBLEMS

1) Plot a parabola from the equation $y = kx^2$, (or trace one), ditto it, and have students trace a few parallel rays by drawing tangents to the curve and applying the laws of reflection directly. If done carefully this can be very instructive.



a) Give them the parabola ABC and the three arrows (1), (2), (3) and ask them to trace the path of these rays--as is done above.

b) Have them continue the rays on through the focus. With a second reflection the rays will go back out parallel. This, of course, illustrates the searchlight principle.

2) It may be instructive to have the students repeat (1a) with a circle instead of a parabola. The rays no longer exactly focus, but they nearly do if they are near the axis.

3) The light from a small flashlight bulb falls on a curved mirror. What is the shape of the curve such that the light is focused back on the bulb itself.

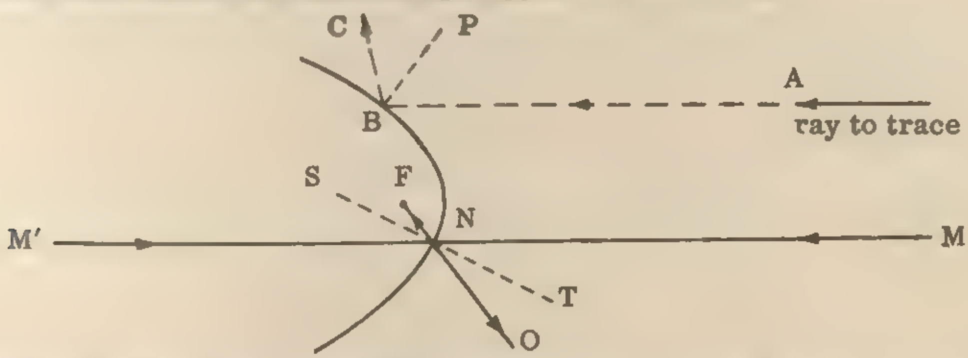
Solution: A sphere centered on the bulb. If the students understand the action of a curved mirror, the answer is rather obvious. There is, however, no simple way to derive it. Once guessed, the answer is easily seen to be correct.

4) Another problem that tests your students' feeling for reflection with curved mirrors is to ditto a parabola as in suggestion (1) and show parallel light incident on the convex surface.

a) Have them trace the rays.

b) From where does the reflected light appear to be coming?

Solution:



a) Extend the ray from A to where it hits the surface at B. Erect the perpendicular to the surface BP. Construct the reflected ray BC so that ∠ CBP = ∠ ABP. Repeat for any other rays called for.

b) The light appears to be coming from the focus of the parabola. Consider the ray MN and its reflection NO. Consider also the ray M'N and, if the parabola were silvered on the concave surface, its reflection NF, F being the focus. By the laws of reflection ∠ M'NS = ∠ FNS, since we know that a parabola focuses parallel light.

∠ MNT = ∠ M'NS. Therefore, since ∠ ONT = ∠ TNM = ∠ SNF, FNO is a straight line and the reflected ray NO appears to come from the focus. Obviously the students are meant to get the answer somewhat more intuitively!

Section 9 - Images and Illusions

PURPOSE To develop a detailed understanding of the images formed by parabolic mirrors (and spherical mirrors).

To use ray diagrams and geometry (not formulas) to find the relationship between object size and location and the image size and location.

CONTENT a. Parabolic mirrors form images.

b. Images can be located by ray tracing:

1. The ray initially parallel to the mirror's axis is reflected through the focus, and

2. The ray passing through the principal focus on the way to mirror is reflected parallel to the axis.

c. Simple similar triangles can be found to relate:

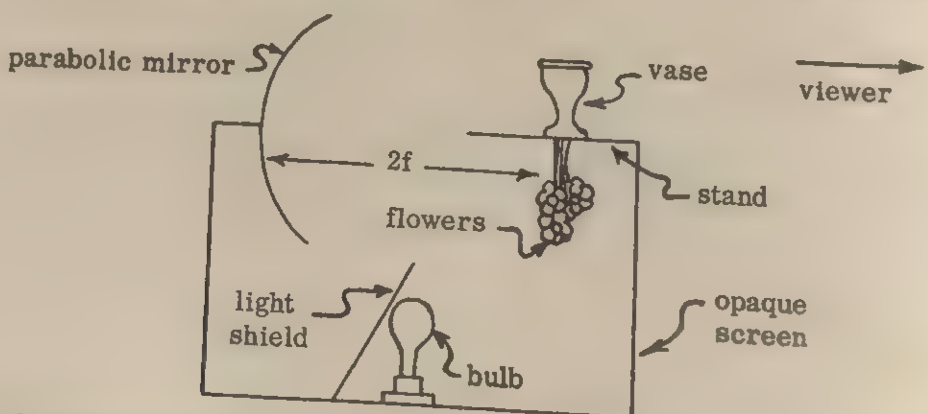
1. Image position to object position: $S_i S_o = f^2$.

2. Image size to object size: $\frac{H_i}{H_o} = \frac{f}{S_o} = \frac{S_i}{f}$.

EMPHASIS This material is basic to the formation of images by lenses as well as by mirrors. It is important for students to draw ray diagrams. If they can draw and interpret ray diagrams they understand image formation. Merely memorizing formulas and treating problems in image formation as trivial algebraic exercises, does not guarantee understanding. It is important to do problems but you can discourage the blind use of formulas by using different notations in your problems. Encouraging students to use the third principal ray (to the vertex of the parabola) whenever it simplifies the geometric construction, will make some diagram problems easier. Even though you discourage quick use of formulas, most of your students will learn the formulas. But, if your emphasis on ray diagrams fulfills its function, students will also learn how images are formed, and how to derive any formulas they need for different arrangements.

DEMONSTRATION It is effective to introduce this section with a class activity designed to let students see a real image formed by a concave spherical or parabolic mirror. Because of this, Experiment II-2 should be done before you discuss this section in any detail. If for some reason you must delay completing the entire experiment, at least do as much of it as will let students see and locate one image.

Following the laboratory, a "spectacular" demonstration may provide an interesting starting point for your class discussion. The following is a familiar, but effective, demonstration. You will need either a concave spherical mirror or a parabolic mirror. The apparatus should be set up as in the diagram.



The bouquet of artificial flowers is suspended upside down. The flowers are shielded from

the viewer and illuminated by the bulb. Both the flowers and the vase above them are at a distance of 2f from the mirror. Since the mirror fo ms an inverted real image, the flowers appear to be right side up in the vase. You can demonstrate that this is a real image by displaying the image on a piece of Kleenex or wax paper held over the vase.

This demonstration can provide effective motivation if you use some showmanship. You can set it up so that it is in the correct line of sight as students enter the room. Flowers in a physics room may be a surprise, and worth a second glance, but the double-takes really come as students walk past the angular field of the image, and the flowers disappear. You can conjure other ways of presenting this demonstration. To keep it a "stunner" try to arrange the presentation so that the first look shows the image. Then you can pick up the vase and leave the flowers, try to pick flowers that aren't there, etc.

DEVELOPMENT You will probably need to show students exactly how to locate an image by using a ray diagram. It may help the class follow if you use as the "object" a diagram of a stick with one tip red and the other tip white. Neither tip should be on the mirror's axis. You can then draw the rays from the ends in different colors so that they can be distinguished easily. First, considering the white tip of the object, draw more than two rays, but emphasize that the two chosen in the textbook are adequate and easiest to construct accurately. Eventually you can erase the rays other than these two. These two rays can be drawn without constructing tangents, normals, and equal angles. (The ray drawn to the vertex which produces a symmetric ray on the other side of the axis is difficult to construct accurately. However, this ray is very useful and it should be used frequently when it simplifies the geometry in problems. For example, it is useful in a problem which asks for relative sizes and gives object and image position but no focal length.) After the image of one tip is located, you can turn to the image of the other tip. Drawing the rays from the second tip in a different color will help to distinguish them. Make it clear that the image point is the point where rays leaving a single point cross after they are reflected. Until you explain this, a few students may think that an image point exists whenever any two rays cross.

Before students become involved in the derivation of formulas on page 204, they should do some ray tracing in scale drawings. Have students trace rays of light from a given object to a cylindrical mirror, then to an image. At the mirror, they should construct normals to the mirror and use the law of reflection as carefully as they can. Occasionally they should be challenged to accurate work. Try challenging them to work to the nearest tenth of a millimeter! They will easily be able to see effects of spherical aberration (and you will be kept on your toes distinguishing between errors in drawing and spherical aberration). This type of exercise helps remind the students of the basic importance of the laws of reflection.

CAUTION The analytical formula, $S_iS_o = f^2$, is derived partially as an illustration of how the geometry can be used to locate an image. The equivalent formula, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, could also be derived simply, but the reciprocals confuse some students and the correct assignment of signs to p, q, and f is more complicated than in the equation $S_iS_o = f^2$ (p is used in many physics texts as the distance from the object to the mirror while q is the distance from the image to the mirror). Each of these formulas has some advantages and some disadvantages. You can easily design problems which make either one seem superior. Every physicist has a slight preference for the form that he knows best; he naturally thinks of problems and situations in which the form he knows is most useful.

If you are most familiar with $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, a little practice will give you all the facility you need with $S_iS_o = f^2$. The inconvenience involved will be slight and it will not last long. Of course, students do not have prior experience with either formula and they find

$S_iS_o = f^2$ somewhat easier to learn and to work with.

Physicists who are adept with optics problems sometimes use both formulas interchangeably. However, when both formulas have been introduced in an early physics course, some students have become confused. They measure distance to the mirror when they should measure them to the focal point, and vice versa.

Inasmuch as you will be encouraging students to draw ray diagrams and to rederive the formula when they need it, this warning against introducing p and q as distances to the mirror may seem unnecessary. However, there will be times when you will want the students to use ray diagrams in which it is convenient for them to label the distance from the image or object to the mirror. In such cases let them call this distance anything they please. Labeling it with the pair of letters corresponding to the terminal points may be best. As soon as you reserve a particular letter for a particular distance, some students will be tempted to memorize and reuse any formulas they may derive.

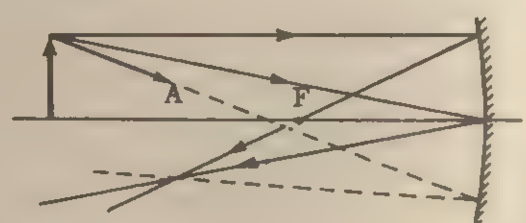
The S_iS_o formulation will also be used to work with lenses in Chapter 14.

QUIZ PROBLEMS Your quizzes should stress ray diagrams rather than the use of formulas. However, some formula problems are not amiss.

1) Given the following diagram and the rays shown (except for F and the dotted ray):

a) Label the position of the principal focus.

b) Continue the ray A.



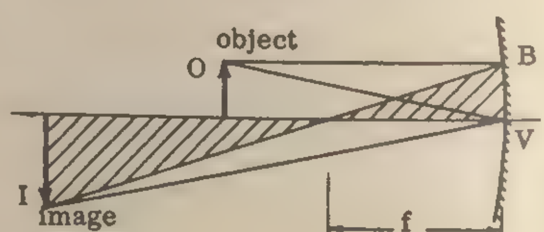
Solution: The principal focus is at F; the continuation of A is shown dotted.

2) An object 3 mm high forms a real image 5 mm high, 25 cm from a concave mirror.

a) How far from the mirror is the object?

b) What is the focal length of the mirror?

Solution:



a) The ray to the vertex reflects at equal angles to the optical axis. The heights of object and image are proportional to their distance from the mirror. The object is therefore 15 cm from the mirror.

b) Drawing OBI we see from the shaded similar triangles that $\frac{BV}{f} = \frac{0.5\text{ cm}}{25 - f}$. BV = 0.3 cm, the object height. $0.3(25 - f) = 0.5f$; $f = 75/8$ cm = 9.4 cm.

Section 10 - Real and Virtual Images

PURPOSE To extend the ray tracing technique to the location of virtual images.

CONTENT a. When an object is between a parabolic mirror and its principal focus, the image of the object is virtual.

b. $S_iS_o = f^2$ may be used to locate the image.

COMMENT Do not bother trying to teach the students sign conventions. If they call S_i negative, they will get a negative answer for S_o and could interpret this as meaning to the mirror side of the focus. However, the sign convention will lead to many errors; it is not worth the trouble. A ray diagram will give the correct qualitative result. The formula will supply the numbers.

A reasonable distinction between a real and virtual image can be made by imagining a translucent screen. If the image can be "captured" on a small screen (tracing paper, for example) it is a real image; if not, it is virtual.

Chapter 12 - Reflection and Images

For Home, Desk and Lab – Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 12-22 to 12-38.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1, 2	1*, 2	3, 21		1*, 2, 3	21
3, 4, 5		5, 6*, 7	4, 8 9, 10	5, 6*, 7, 8	5, 6
6, 7	11			11	
8, 9	13*, 14 15	12*, 16*, 17*, 18*, 19		12*, 16*, 17*, 18*	18*
10		12*	20, 22	12*, 20	

SHORT ANSWERS

1. Decrease obstacle to screen distance.
   Decrease size of light source.
   Increase light to obstacle distance.
2. (a) See diagram and discussion on page 12-22.
   (b) 7.4×10^5 miles.
3. (a) The angle becomes smaller.
   (b) See detailed discussion on page 12-23.
4. (a) 0.12°.
   (b) 4 mm; 3×10^{-4}
   (c) No effect.
5. (a) and (b) Twice the length of the mirror.
   (c) No.
   (d) 2′ 6″.
6. See detailed discussion on page 12-25.
7. See detailed discussion on page 12-26.
8. (a) No.
   (b) Left.
   (c) 1. See detailed discussion on page 12-28.
   2. They are same size but appear different in size.
   3. One image is reversed but the other is not.
   4. 18.4 ft, 23 ft.
9. About 8″.
10. (a) 5.3″.
    (b) 7.5″.
11. Infinite, but strictly speaking is undefined.
12. (a) If object is placed at real image, new real image is formed where object was.
    (b) There would be no image at original position of object.
13. 17 cm.
14. $H_i = f\frac{H_o}{S_o}$.
15. 5.3 cm.
16. 20 cm.
17. (a) 125 cm real (and 75 cm if virtual image is included).
    (b) 4 m real.
18. (a) Sometimes.
    (b) Experiment.
19. See derivation in detailed answers on page 12-35.
20. See detailed discussion on page 12-35.
21. Most of 21 is a project but the numerical answers are:
    (a) 99″
    (b) 10^6 miles.

22. $S_0 = \pm f$ (2 f from mirror, also right at the mirror).

COMMENTS AND SOLUTIONS

PROBLEM 1

Fig. 12-2 shows how a fuzzy shadow is produced. How could you produce a sharper shadow of the same obstacle?

This is a good problem for class discussion; it can be discussed in class without being assigned.

For a given light source, the shadow can be sharpened both by increasing the distance between the source and the obstacle, and by decreasing the distance between the obstacle and the screen. A smaller light source would also sharpen the shadow.

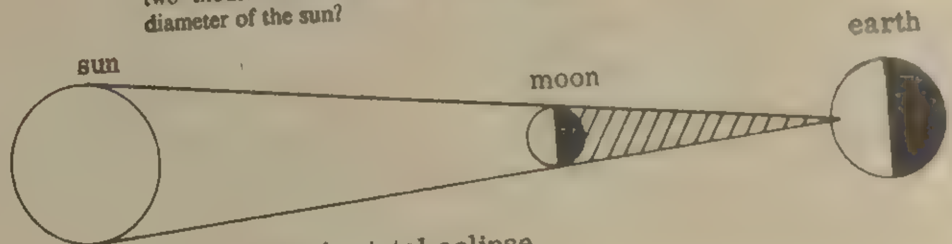
PROBLEM 2

We sometimes see total eclipses of the sun by the moon, and sometimes annular eclipses. In the latter, a ring of light from the sun is seen around the edge of the moon.

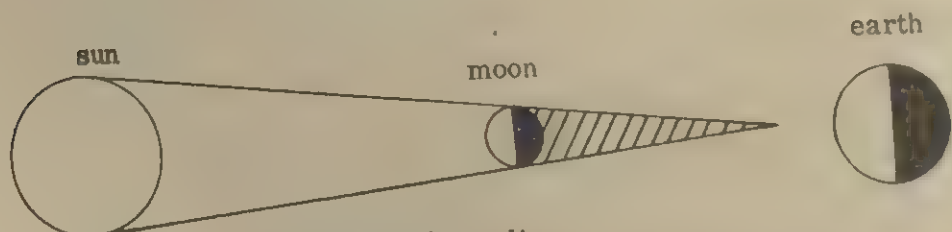
(a) By drawing a diagram of the earth, moon, and sun, explain why two different kinds of solar eclipses occur. Do the distances of the moon from the earth and of the earth from the sun remain the same?

(b) The moon is about a quarter of a million miles from the earth, and the sun is about 93 million miles away. If the moon has a diameter of two thousand miles, what is the approximate diameter of the sun?

a)



case 1 - total eclipse



case 2 - annular eclipse

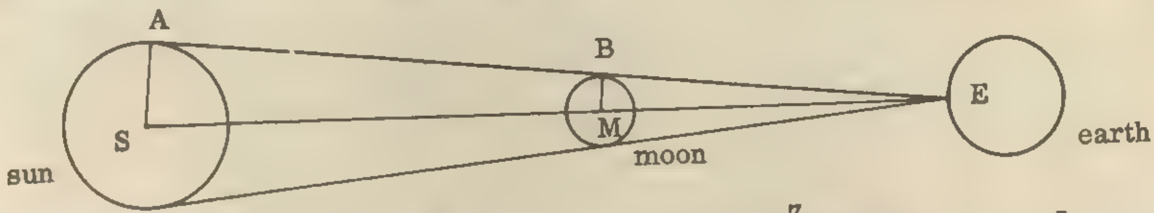
A total eclipse occurs only when a part of the earth's surface is in the umbra, because it is only from points within the umbra that no part of the sun can be seen. The relationship between the sizes and distances of the earth, moon, and sun are such that, in some eclipses, the tip of the umbra just grazes the earth's surface. But in some eclipses, the earth is far enough from the moon (or the moon is close enough to the sun) that the tip of the umbra does not touch the surface of the earth. In this case, an observer sees the outer rim of the sun. Such an eclipse is called annular.

Most eclipses of the sun are partial eclipses, in which not only is no part of the earth's surface in the umbra, but the extension of the line between the centers of the sun and moon (this line also goes through the tip of the umbra) does not touch any part of the earth.

The fact that we have both annular and total eclipses shows that the distance from the

moon to the earth or the distance from the earth to the sun must change. Both do.

b) Since in the total eclipse, the tip of the umbra usually just grazes the earth's surface, we may draw the triangles ASE and BME. If the umbra extended over any large region of the earth's surface, the geometry would become indeterminate.



Then $\frac{AS}{BM} = \frac{SE}{ME}$ and $AS = BM \times \frac{SE}{ME} = 10^3 \text{ miles} \times \frac{9.3 \times 10^7 \text{ miles}}{2.5 \times 10^5 \text{ miles}} = 3.7 \times 10^5$ miles

Thus the diameter of the sun = $\underline{7.4 \times 10^5 \text{ miles}}$. (The accepted value of the diameter is 8.7×10^5 miles.)

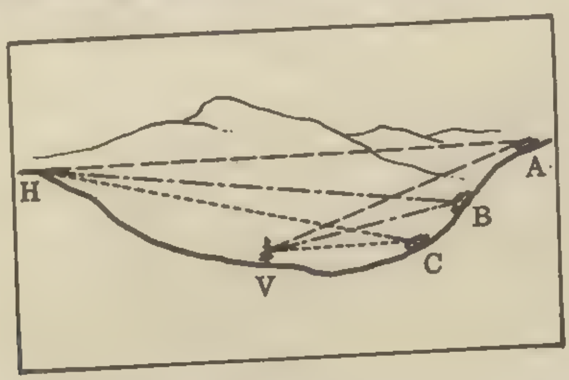
PROBLEM 3

When we drive down a road that runs squarely into a valley, we frequently get the impression that the other side of the valley is getting farther away, instead of closer. Fig. 12-29 shows the light rays that reach the eye from the top and bottom of one side of the valley when the viewer is near the top of the other side. Draw the rays that reach his eyes when he is halfway down; when he nears the bottom.

(a) What happens to the angle between these rays as he gets closer to the bottom of the valley?

(b) How does this help to explain the illusion that the other side of the valley is retreating from him as he gets closer to it?

This problem is concerned with certain psychological aspects of the judging of distance, and gives further practice in the straight-line propagation of light. Very few people seem to be familiar with the illusion dealt with in this problem.



a) As the observer moves down into the valley from A to B to C, the angle the other side of the valley subtends at his eye decreases from ∠HAV to ∠HBV to ∠HCV.

b) Since the same terrain (HV) is progressively seen to subtend a smaller angular field of view, the observer may get the impression that the slope is getting farther away.

If the valley is very deep, the "perpendicular bisector" of one side may intersect the opposite side which the observer is descending. In this case the angle appears to increase for a while and the illusion may be reversed.

PROBLEM 4

In studying Brownian motion a pencil of light was used to amplify the twisting motion of a small mirror bombarded by air molecules (Fig. 9–14). When used in this way, to amplify small motions, a combination of light source and mirror is called an optical lever or amplifier.

(a) If the mirror twists through an angle of .06°, through what angle does the reflected pencil move?

(b) If the distance from mirror to camera is 2 m, about how far does the reflected pencil move across the camera lens? What fraction is it of the circumference of a circle of 2 m radius?

(c) What effect does the position of the light source have on the amplification?

This problem could be done after Section 4 but should be deferred until after students have gone through Section 5 and have a good grasp of the laws of reflection and ray tracing.

This problem illustrates a practical use of the laws of reflection. Part b) requires trigonometry or radian measure of angle. If the students cannot do part b) the problem should not be assigned.

a) If the mirror twists through an angle of 0.06°, the angle of incidence of the ray from the source is changed by 0.06°. Since the angle of reflection must remain equal to the angle of incidence, the angle of reflection must change the same amount. The total change in direction of the reflected pencil is the change in the angle of reflection, plus the change in the direction of the normal. Thus, the reflected pencil moves through an angle of 0.12°

b) The reflected pencil moves 2 tan 0.12° = 0.004 meters = 4 mm. The fraction this is of the circumference of a 2m radius circle is $\frac{4 \times 10^{-3}\ \text{m}}{2\pi \times 2\text{m}} = \underline{3 \times 10^{-4}}$.

c) The position of the light source has no effect on the amplification.

PROBLEM 5

Place a plane mirror (one about 30–40 cm high is convenient) with its center approximately at eye level. Hold a meter stick vertically just in front of your face, with the middle of the stick at eye level; stand in front of and facing the mirror.

(a) Move toward and away from the mirror. Does your motion change the amount of the stick that you can see?

(b) Formulate a general rule connecting the length of the stick that can be seen with the height of the mirror. By making a ray diagram, show that this rule holds for all distances from the mirror.

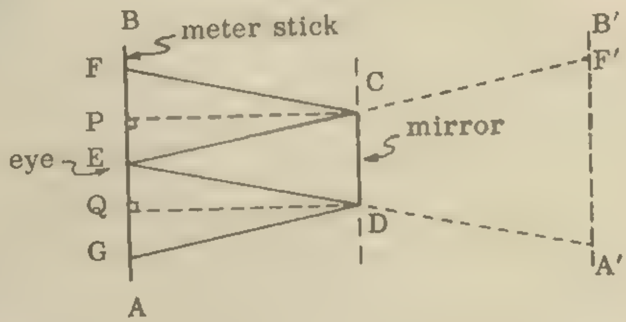
(c) Clothing stores often have mirrors that extend all the way to the floor, designed to allow a customer to see his or her full length. Is it necessary for this purpose for the mirror to be as long as it is?

(d) If the shortest customer has eyes at a height of 5 ft. 0 in., what is the maximum allowable height of the bottom of the mirror from the floor if the customer's feet are to be visible?

a) and b) The length of the stick that can be seen is just twice the height of the mirror, regardless of how close the mirror is.

To prove this, we draw the two rays, FCE and GDE which hit the mirror at its upper and lower edge. From the second law of reflection, ∠ FCP = ∠ ECP and ∠ EDQ = ∠ GDQ, and triangles FPC and EPC and triangles EQD and GQD are identical. Thus FP = PE and EQ = QG. But PQ = CD = 40 cm and since E is centered in front of the mirror, PE equals

EQ; and FG, which is the length of the meter stick which can be seen in the mirror, is twice the height of the mirror, or 80 cm. The same result is found by constructing the image A′B′ of the entire ruler and seeing how much of the image the "window" CD lets us see.



c) No. The rays of light from the feet to the mirror and then to the eyes determine the necessary lower edge of mirror.

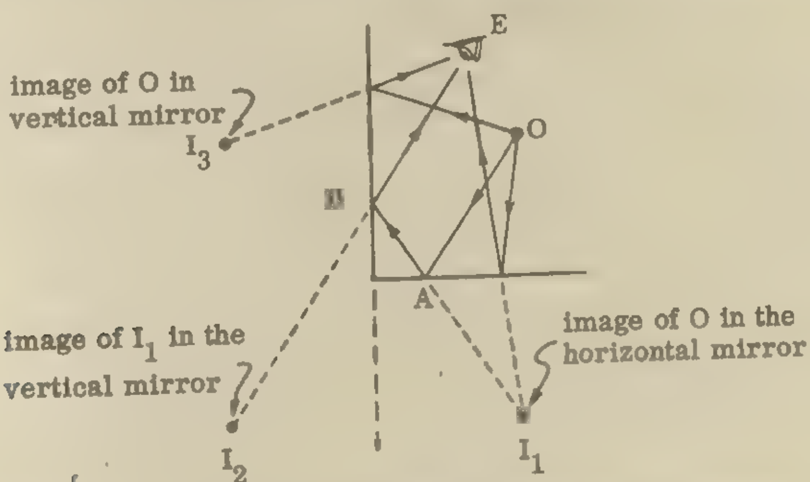
d) The maximum allowable height of the store mirror from the floor is 2′ 6″; half the "eye height" of the shortest viewer.

Since an image is always twice as far from the eye as the mirror is, by proportionality, the height of the image that can be seen is always twice the height of the mirror.

PROBLEM 6

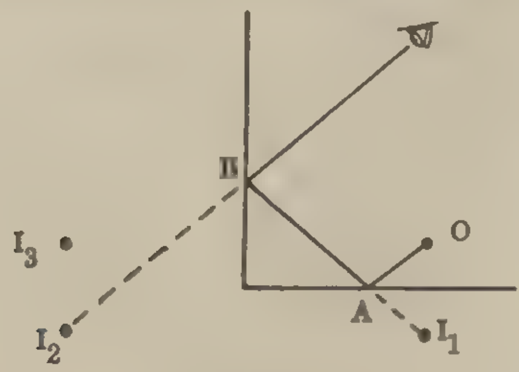
Construct a diagram similar to the one in Fig. 12-13 for two mirrors at 90° to each other.
(a) How does the light reach the eye when it is placed as in Fig. 12-30?
(b) Set up two such mirrors and see if the experimental observation checks your diagram. How can you tell if the light goes the way your diagram indicates? Try first with an object midway between the mirrors, and then with it close to one of the mirrors.

This problem involves the paths of a light ray being reflected from two mirrors and is best done in conjunction with the laboratory experiment on mirrors. Then the students may experiment with blocking certain paths of the light to see which images are affected. In Figure 12-30 the light goes as shown.



If we are to be concerned with light striking both mirrors, we must be concerned with the image of an image. The object O has an image I_1, in the horizontal mirror. I_1 in turn has an image I_2 in the vertical mirror. This may seem mysterious to the students until they construct the actual path of the light OABE as shown. The ray AB hitting the vertical mirror appears to come from I_1, the first image; while the final ray BE appears to come from the final image I_2.

A second case where the object is close to the horizontal mirror is shown:

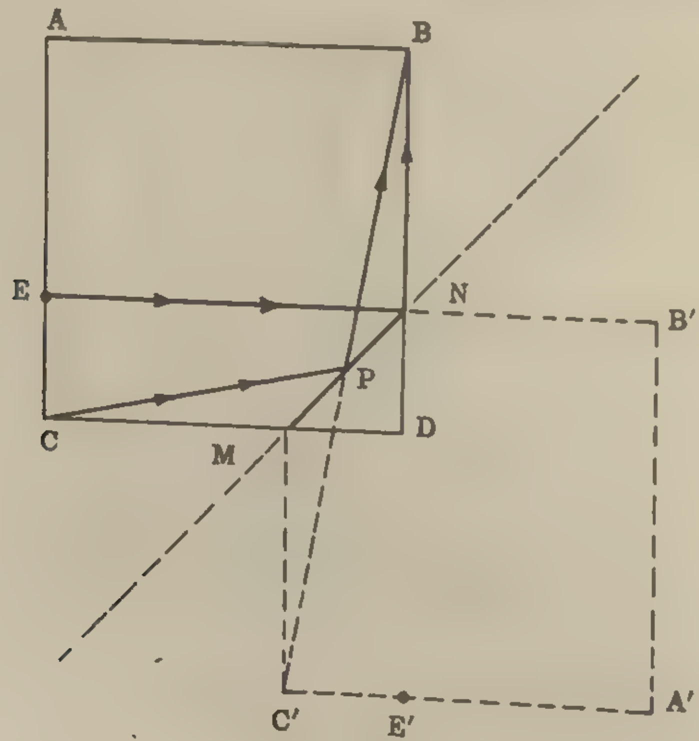


PROBLEM 7

A mirror three ft. wide is placed in the corner of a square room, 20 ft. on a side. Its surface makes an angle of 45° with each wall. You are standing in an adjacent corner. Show by construction what sections of the walls are visible in the mirror and where the images of these sections are located.

This problem and Problem 8 involve the idea that images can be located even though the mirror does not extend far enough to intersect the line between the image and the object.

Find the image with respect to the plane of the mirror, and then consider the mirror itself as a "window" through which we look at the imaginary space behind the plane of the mirror. However, what this means in terms of light paths should always be made clear to students, and these "pictures" should never be used as more than aids to finding the light paths.



In the figure, the size of the mirror, MN, has been exaggerated to make the drawing clearer. The image of corner B is constructed at B′ a distance behind the plane of the mirror MN equal to the distance B is in front of it. Similarly, the image of the corners A and C are constructed at A′ and C′. Notice that D has no image D′. We do not see objects behind mirrors as images in the mirror! Then the dotted image of the room is drawn in.

An observer standing at B looks "through" the mirror at the image of the room. Looking along the wall BD he sees a spot E′, which is an image of E. The actual path of the light ray goes from E to N to B.

The observer also sees C′ through the "window". The actual light ray goes from C to P to B. In fact the observer sees the portion of the walls MCE as an image MC′E′.

It is worthwhile to point out that the observer can walk around in the room and see the whole image MC′E′A′B′N from different points in the room. As he walks, the image remains fixed!

PROBLEM 8

Two persons, *A* and *B*, and two mirrors are located in a room as shown in Fig. 12-31. By construction to scale, investigate the following:

(a) Can *A* see his own image in either mirror?

(b) If *B* raises his right hand, will his image as seen by *A* in M_2 raise the right or the left hand?

(c) *A* can see two different images of *B* in M_1.

(1) Where are they located?

(2) Do they appear to be of the same size? *Are* they of the same size?

(3) Are they both reversed as to left and right?

(4) What is the distance of each image from *A*?

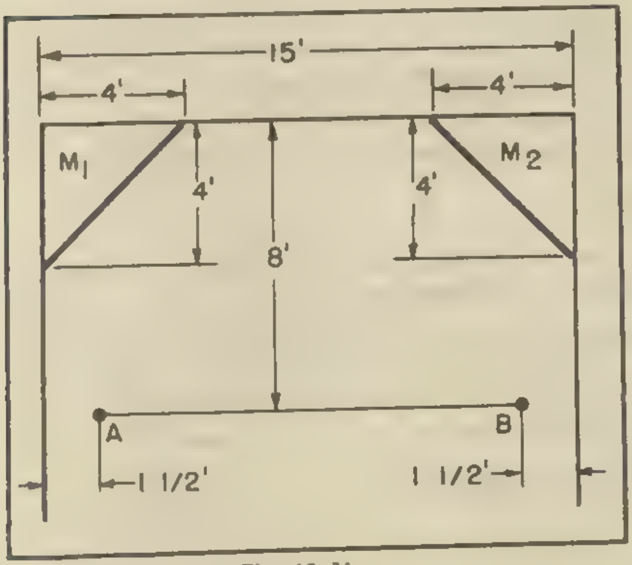


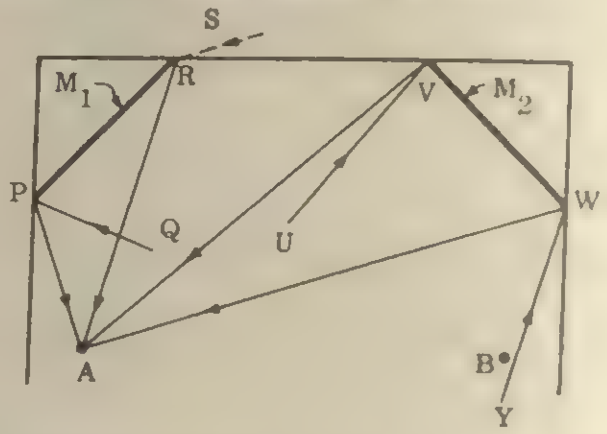
Fig. 12-31.

This is a complicated problem on images in mirrors. It probably is better for class discussion material than homework. It can be assigned after the students have thoroughly absorbed Section 5.

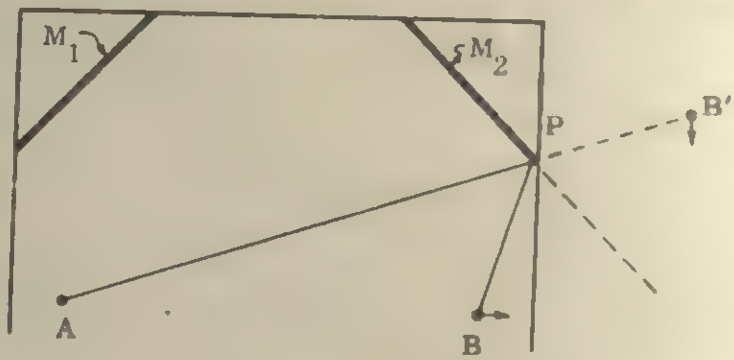
The solutions may be accomplished in several ways. One way is that proposed in Prob-

lem 7 -- considering the mirror as a "window" looking into an image space. A second approach is to draw light rays as they actually travel and see what they indicate.

a) The figure at the right shows Mr. A looking into the mirror M_1 along the directions AP and AR. He then sees things out along PQ and RS. Clearly no such reflection includes Mr. A himself. Similarly, in looking at the mirror M_2 along AV and AW, he sees things in the direction VU and WY. Again, this region does not contain Mr. A. Mr. A cannot see himself in either mirror.



b) In the figure at the right, Mr. A looks at Mr. B in mirror M_2. Light rays go from B to P to A, and Mr. A sees an image of B at B′. From the direction of the arrow it is easy to see that the image raises its left hand if Mr. B raises his right.

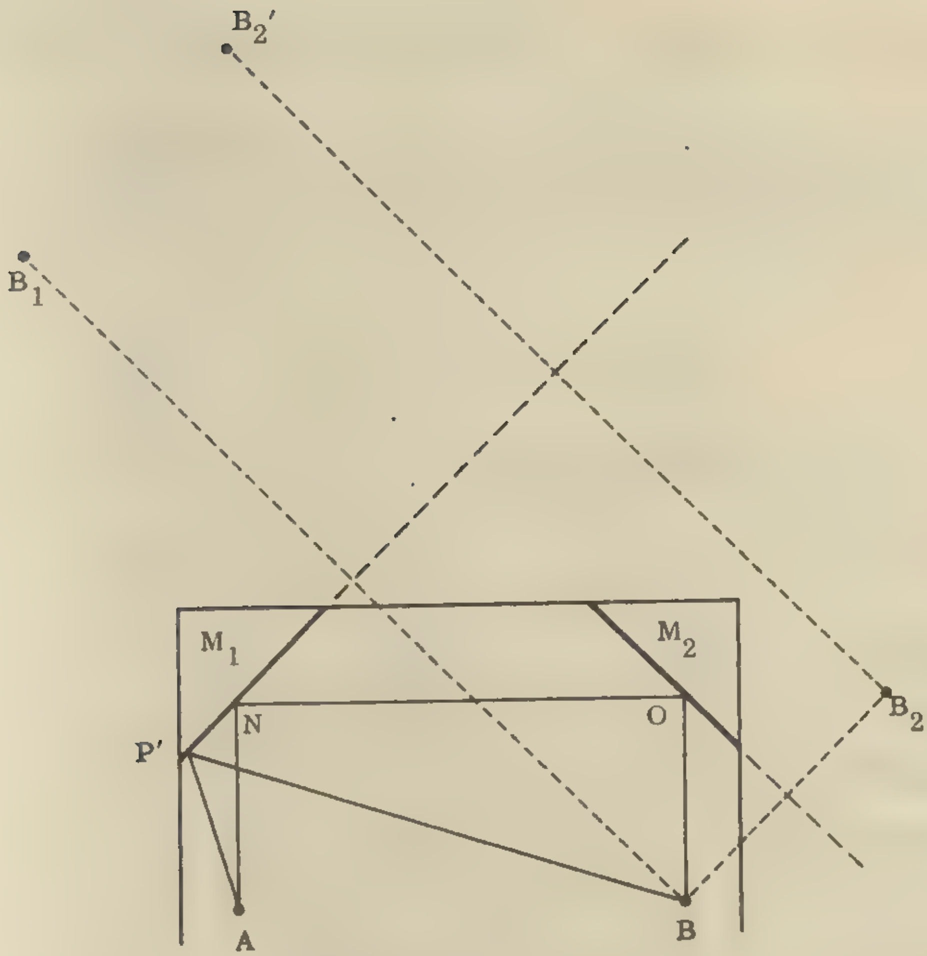


c) In the figure below, light rays travel to A from B off mirror M_1 in two different ways. The rays go from B to P′ to A appearing to Mr. A to come from the image B_1. They also go from B to O and from O to N (appearing to come from the image of B in M_2, namely B_2) and then from N to A appearing to come from the image B_2'. B_2' is the image of B_2 in the mirror M_1.

Since images in a plane mirror are the same size as the object, the two final images B_1 and B_2' are the same size. B_2' is farther away from Mr. A than B_1, and must appear smaller. This is the same as saying the ray BONA is longer than BP′A.

B_1 appears to have left and right reversed. B_2', having been reflected twice, has beer reversed twice, so it is back where it started, unreversed.

B_1 is 18.4 feet from A. B_2' is 23 feet from A.



PROBLEM 9

The rear-view mirror of a car is so placed that its upper and lower edges are horizontal and its center is at the same level as the center of the rear window. The driver's eye is also at this level, and the line of sight from his eye to the center of the mirror makes an angle of 30° with the line joining the centers of the mirror and the window. (See Fig. 12-32.) The distance from his eye to the mirror is 2.0 ft., and that from the mirror to the window is 8.0 ft. What is the least width of the mirror that is needed if the entire width (3.0 ft.) of the rear window is to be seen?

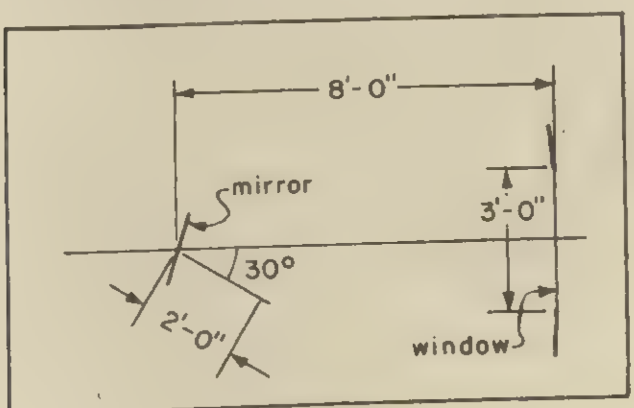
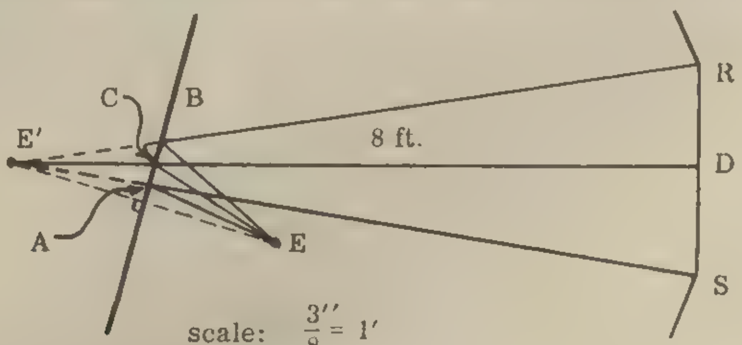


Fig. 12-32.

Although this problem can be solved trigonometrically, it is best done with an accurate scale drawing.

With the plane of the mirror set at 75° to the line between the center of the mirror and the center of the rear window, the ray that travels from the center of the rear window to the driver's eye is reflected from the center of the mirror. If this orientation of the mirror is assumed to give the least necessary mirror width (actually the best orientation is very slightly greater than 75°), then we have the diagram below.



The image E' of E is constructed and lines E'S and E'R drawn to the edges of the rear window. Then if the mirror is as wide as twice the greater of AC and BC, where A and B are the points of intersection of the lines E'S and E'R with the plane of the mirror, the driver will be able to just see the whole rear window in the mirror. By measurement on the scale drawing this width of the mirror is 8''.

Note that in the solution of this problem we used the image of the driver in the mirror, although we are really interested in the image of the rear window in the mirror. That the two give the same results is due to the reversibility of light and the symmetry of the laws of reflection with respect to direction. This is not a completely obvious point and therefore it might be best to use the more cumbersome method of constructing the image of the rear window in the plane mirror.

PROBLEM 10

A *periscope* can be made by mounting two plane mirrors at the ends of a tube. (Fig. 12-33.) The mirrors face each other, are parallel, and each makes an angle of 45° with the axis of the tube. An eye hole is cut in the tube opposite the center of one mirror, and a larger hole is cut in the other end of the tube, opposite the other mirror. Suppose that the distance between the two mirrors is 4 ft. 0 in., and that you are using the periscope to look over a fence at a man 6 ft. 0 in. tall who is 50 ft. away.

(a) What is the smallest possible height of the hole in the top of the periscope?

(b) What is the smallest possible size of the top mirror?

Hint: Draw rays from the man to the eye, and remember the properties of similar triangles.

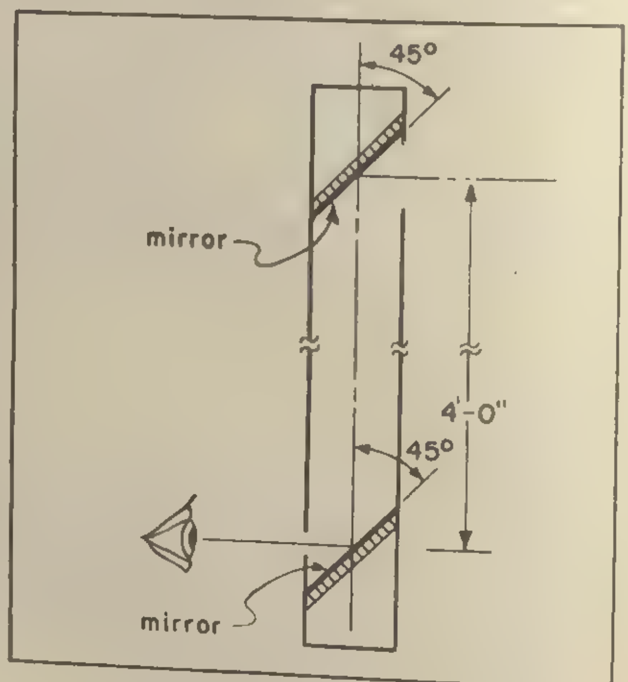
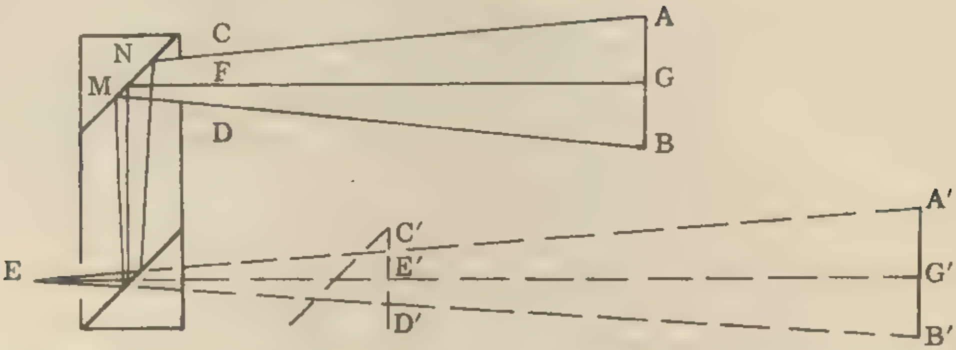


Fig. 12-33.

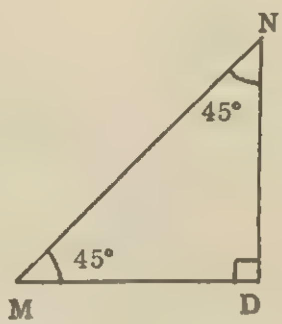
This problem may be done after Section 5.



a) We must construct the image of the man in the lower mirror to get the similar triangles with which we may find the minimum possible height of the hole in the top of the periscope. This height in the diagram is CD, the height at the top of the periscope of the cone of light coming from the man AB to the eye E. In the image, C′D′ equals CD and A′B′ equals AB. Since C′D′ and A′B′ are both vertical and therefore parallel, triangles EC′D′ and EA′B′ are similar and $\frac{C'D'}{A'B'} = \frac{EF'}{EG'}$. We are given that AB equals six feet. EF′ is just about the same as the height of the periscope, which is four feet; and EG′ is the height of the periscope plus the distance to the man, which is 54 feet. Then $CD = C'D' = 6 \times \frac{4}{54} = \frac{4}{9}$ feet $= 5\frac{1}{3}$ inches. The least possible height for the hole in the top of the periscope is $\underline{5\frac{1}{3}}$ **inches.**

b) The minimum size of the top mirror, since the vertical height of the cone of light at the mirror is about the same as CD, may be found from the triangle at the right. We know $ND = 5\frac{1}{3}$ inches $= MD$ since the triangle is isosceles. By the Pythagorean theorem, MN, which is the minimum size of the mirror, may be found from: $\overline{MN}^2 = \overline{ND}^2 + \overline{MD}^2$.

$MN = 5\frac{1}{3} \times \sqrt{2} = \underline{7.5 \text{ inches}}$.



PROBLEM 11 What is the focal length of a plane mirror?

This problem should not be considered until after students understand Section 6. It is best used for class discussion. Students can be asked what happens to the focal length as a parabola is flattened. Eventually, the parabola will be so nearly flat that the difference between a parabolic mirror and a plane mirror will be negligible — the focal length of a plane mirror is infinite. Properly speaking, the focal length of a plane mirror is undefined. It is only in the limit of a nearly flat parabola that we can find a focal length.

PROBLEM 12 What will happen if, in Fig. 12-23, the real light bulb is placed at the position where the real image was previously formed?

(a) Can you state a general rule about moving an object to the position of its real image?

(b) What happens if an object is placed at the

Because of its involving virtual images, it may be better to save this problem until after students have completed Section 10.

a) If a real light bulb is placed at the position of the image in Figure 12-23, the light will travel along the same lines shown in the figure, but in the reverse direction. The second law of reflection is the same regardless of which way the light goes. Therefore, a real image is formed at the original position of the light bulb. In general, if an object is moved to the position of its real image, a real image is formed at the original position of the object.

b) A virtual image is always formed in the region behind a mirror. Therefore, if an object is moved to the position formerly occupied by its virtual image no image is formed at the original object position since light cannot penetrate the mirror.

PROBLEM 13

How large an image of the sun will be formed by the Palomar telescope, whose focal length is 18 m? The sun's diameter is about 1.4×10^9 m, and it is 1.5×10^{11} m away.

$$\frac{H_i}{H_o} = \frac{f}{S_o}; \quad H_i = H_o \frac{f}{S_o}.$$

$$H_i = 1.4 \times 10^9 \text{ m} \times \frac{18 \text{ m}}{1.5 \times 10^{11} \text{ m}} = .17 \text{ m} = \underline{17 \text{ cm}}.$$

PROBLEM 14

Show that the size of the image of the sun formed by a concave mirror is proportional to the focal length of the mirror.

$$\frac{H_i}{H_o} = \frac{f}{S_o}; \quad \underline{H_i = \frac{H_o}{S_o} f}.$$

Since H_o is the diameter of the sun, a constant, and since S_o is the distance of the sun from the earth, essentially a constant, the height of the image H_i is a (constant) × (focal length) as required.

It may also be instructive to draw rays from the edges of the sun to the vertex of the mirror. These extreme rays diverge from the vertex and their distance apart is proportional to their distance from the mirror. The image, however, is formed at a distance from the mirror equal to the focal length (since the sun is very far away). Therefore the image size is proportional to the focal length.

PROBLEM 15

A nail 4.0 cm high stands in front of a concave mirror at a distance of 15 cm from the principal focus. The focal length of the mirror is 20 cm. What is the size of the image?

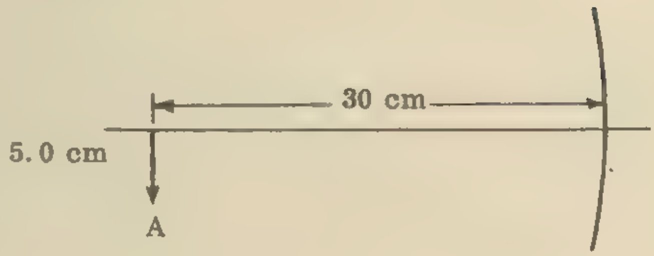
$$\frac{H_i}{H_o} = \frac{f}{S_o}; \quad \frac{H_i}{4} = \frac{20 \text{ cm}}{15 \text{ cm}}; \quad H_i = \frac{16}{3} = \underline{5\tfrac{1}{3} \text{ cm}}.$$

Although the answer to this problem is an unambiguous 5.3 cm, there is some question about the actual physical setup. If the nail is 15 cm from the principal focus away from the mirror the image is real and inverted. If the nail is toward the mirror, i.e. 5 cm from the mirror, the image is virtual and erect.

PROBLEM 16 The image of a candle is 30 cm from the center of a concave mirror. The candle is 10 cm long and its image is 5.0 cm long. What is the focal length of the mirror?

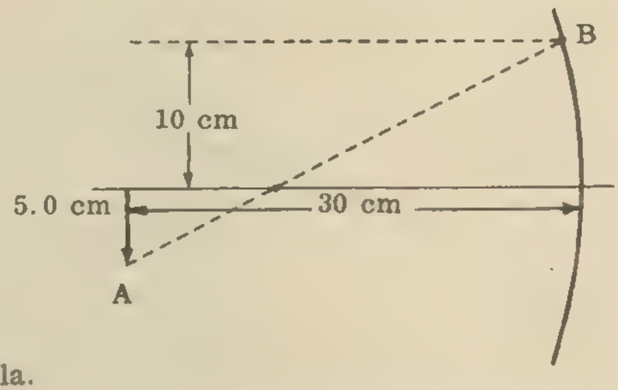
This is a good problem to show how easy it is to use ray diagrams to derive a formula from the geometry of a problem.

We know that there is a 5 cm image 30 cm from the mirror.



We also know that the ray which passed through the focus to reach the tip of the image, approached the mirror parallel to the axis, 10 cm above it (because the object is 10 cm high). Therefore we can locate B.

The line AB crosses the axis at f. From similar triangles f is 20 cm ($\frac{10}{f} = \frac{5}{30 - f}$, or $60 - 2f = f$, and $f = 20$ cm).



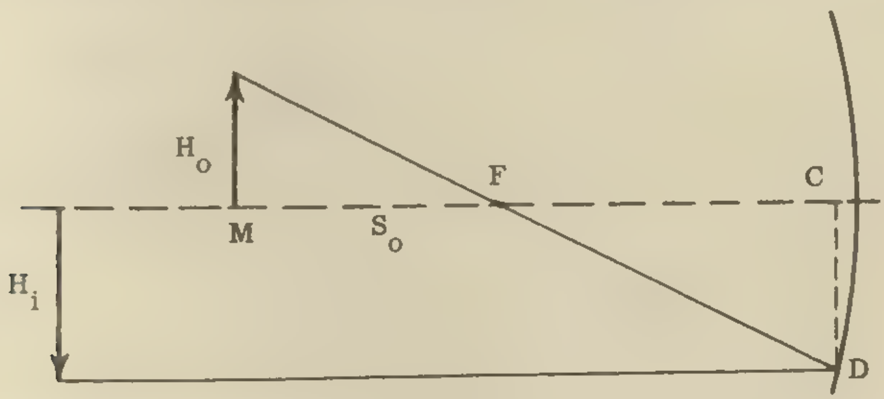
This problem could also be done by formula.

Since $\frac{H_i}{H_o} = \frac{S_i}{f}$, and we know that $S_i + f = 30$, $\frac{H_i}{H_o}f + f = 30$ cm, $f = \frac{30 \text{ cm}}{1 + \frac{H_i}{H_o}} = \frac{30}{1.5} = 20$ cm.

The geometry is more instructive and easier than the algebra. Be sure to show it to your class if you assign this problem.

PROBLEM 17 How far from a parabolic mirror of 1 m focal length must an object be placed to give an image (a) magnified 4 times (b) reduced to $\frac{1}{3}$ its size? Will the images be real or virtual?

a) We can get either a real or a virtual magnified image. If the object is placed farther from the mirror than its focal length, we have the following ray diagram:



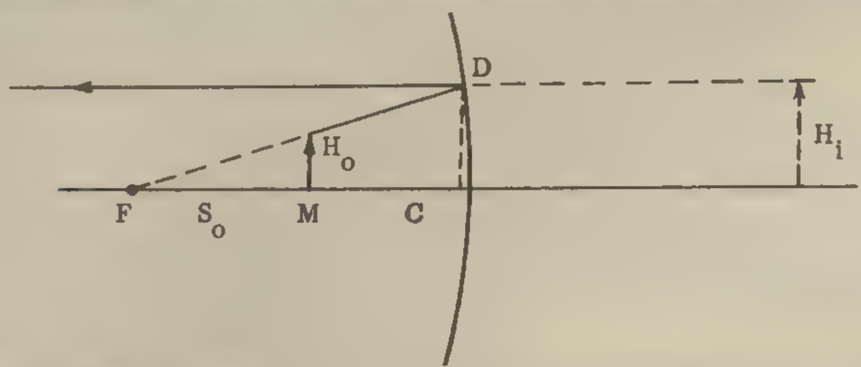
Only one principal ray is drawn for clarity. Now, from similar triangles $H_o / S_o = CD/CF$.

But CD = H_i. Hence $\frac{H_o}{S_o} = \frac{H_i}{CF}$.

But CF is very nearly 1 meter. Hence $\frac{S_o}{1 \text{ meter}} = \frac{H_o}{H_i} = \frac{1}{4}$; $S_o = 25$ cm.

The object should be placed 25 cm beyond the focal point, or <u>125 cm</u> from the mirror.

On the other hand, the object can be placed inside the focal point. In this case



again H_o/S_o = CD/FC = H_i/FC. S_o again is 25 cm. However, this time the object is placed 25 cm closer to the mirror than the focal point, or <u>75 cm</u> from the mirror.

b) For a reduced image, the procedure is the same as in the first part of a):

$\frac{S_o}{1 \text{ meter}} = \frac{3}{1}$; $S_o = 3$ meters.

Hence the object is placed <u>4 meters</u> from the mirror.

With a single concave mirror there is no way to obtain a virtual image reduced in size.

PROBLEM 18

In Section 12-5 we used parallax to locate the virtual image of a candle. We adjusted the position of an object near the image until we could see no parallax (no relative motion).

(a) Can you locate a real image this way?

(b) Use a parabolic mirror to form a real image. Then locate this image by catching it on a piece of paper and by parallax.

This is an interesting exercise which illustrates some of the differences between real and virtual images. Part b) is a laboratory suggestion which is essentially Experiment II-2. This problem might be assigned to prepare the students for this laboratory.

a) Yes, a real image can be located by parallax. Care must be exercised since in using a concave mirror it is necessary to work near the optic axis. Placing a small pointer near the real image in order to observe parallax may interupt the light coming from the object and hence destroy the image. As a practical matter mirrors readily available may not be of high quality. This makes it somewhat difficult to locate the image by parallax.

b) A laboratory exercise.

PROBLEM 19

The distances of an object and its image in a concave mirror are often measured from the center of the mirror, instead of from the principal focus. We call these distances D_o and D_i respectively. We have $S_o = D_o - f$ and $S_i = D_i - f$ where f is the focal length. Using these relations, show that from $S_oS_i = f^2$ follows $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}$.

This problem shows that the Gaussian form of the "lens" equation follows algebraically from the Newtonian form employed in this book.

$$S_o S_i = f^2$$

$$(D_o - f)(D_i - f) = f^2$$

$$D_o D_i - f D_i - f D_o = 0.$$

Divide by $D_o D_i f$ to get:

$$\frac{1}{f} - \frac{1}{D_o} - \frac{1}{D_i} = 0; \quad \frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}.$$

PROBLEM 20

Our discussion of curved mirrors has been limited to the inner, or *concave*, surface. The outer, or convex, surfaces of these curves will also produce images. Such mirrors are called *convex mirrors* and are often used as side mirrors on a car or as ornamental mirrors in a room. Using the laws of reflection and assuming the surface to be parabolic, demonstrate the following facts by suitable constructions.

(a) The area reflected to the eye by a convex circular mirror is larger than that reflected by a plane mirror of the same diameter in the same position.

(b) Light rays parallel to the axis reflect as though they were coming from a point *behind* the mirror. This is the principal focus of the mirror. It is called a *virtual* focus. Why?

(c) Rays starting from a fixed point on the axis are reflected in such a way that they seem to come from a point on the axis behind the mirror. The image is therefore virtual.

(d) The image formed is smaller than the object and is not inverted.

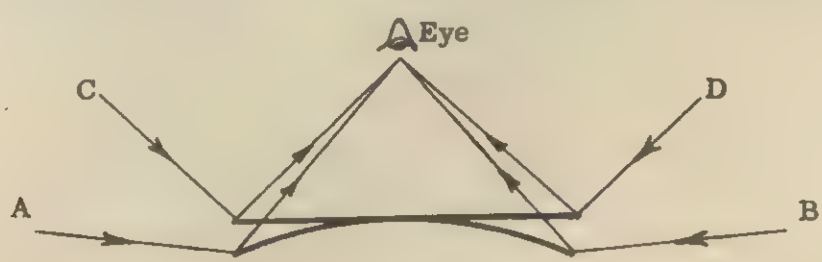
(e) As the object moves in from a great distance, the image moves toward the mirror.

(f) There is a limit to the distance of the image from the mirror — that is, this distance can never be greater than a certain value. What is this value? Try drawing ray diagrams.

This problem requires the student to reproduce, for a convex mirror, a large part of the development which the text does for a concave mirror.

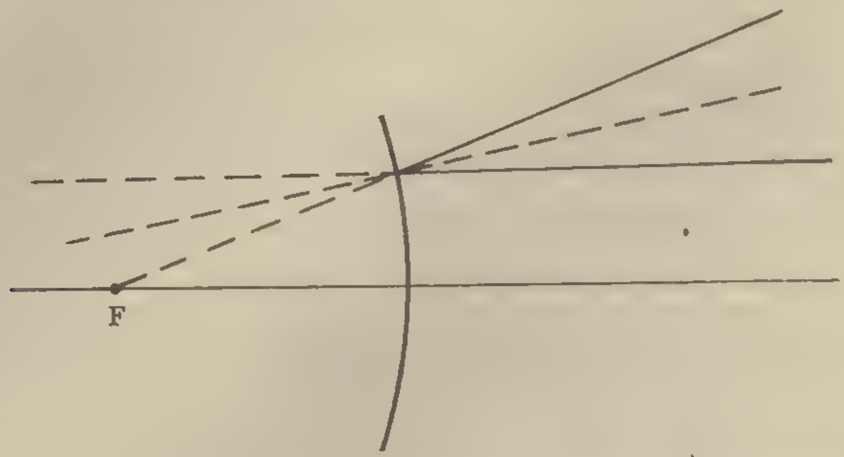
This exercise may be assigned after students have gotten a thorough understanding of the entire chapter. Convex mirrors are not considered again in the course, and it is not at all necessary to assign this problem if you are short of time. It does, however, serve as a high level check on understanding. If a student can do this one, you can be pretty sure he understands the essentials of the chapter. Under no circumstances should sign conventions be introduced here.

a)



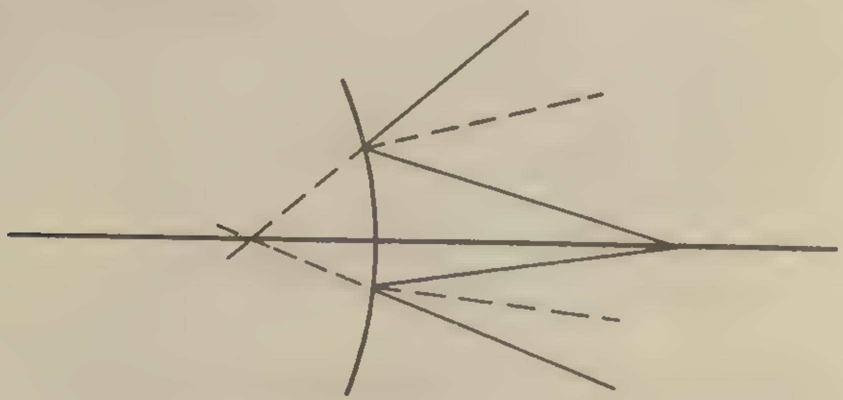
As is evident from the diagram, light in the arc from A to B can reach the eye from the convex mirror, while only light in the arc from C to D will reach the eye from the plane mirror. Thus the convex mirror reflects a larger area to the eye than the plane mirror.

b)



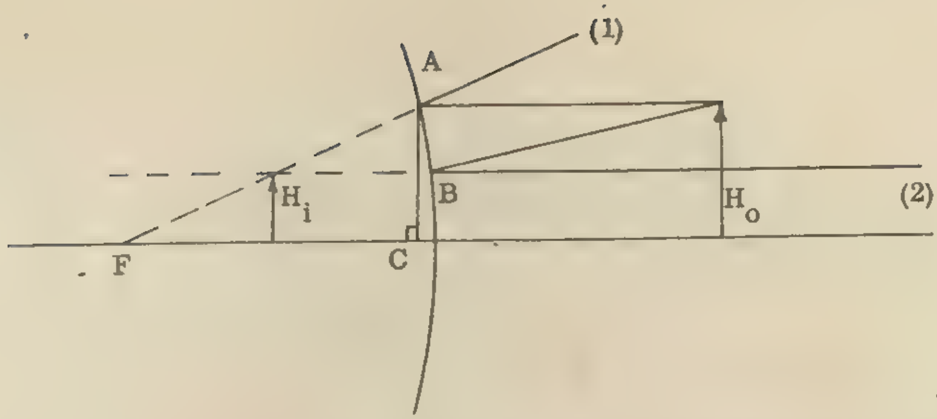
The only practical way to carry out the above construction is to consider parallel rays striking the concave side of the mirror. On the concave side, the angles to the normal of the incident and reflected rays are the same as on the convex side. Using the focal point of the concave side we can then construct the rays reflected from the convex side. The point they seem to come from is called a virtual focus, because light rays never really pass through the focus.

c)



A rough construction will suffice here, and it is obvious that the rays will diverge so as to appear to come from a virtual image on the axis extended behind the mirror.

d)



By drawing principal rays, we see that the top of the image must always lie on the line FA, so that H_i is always less than AC. But since AC = H_o, the image is always smaller than the object. It is also upright since the principal ray (2) (parallel to the axis) is always above the axis if the object is above the axis.

e) In the diagram for part d) we see that as the object moves in closer to the mirror, the point B where principal ray (2) hits the mirror moves nearer A. Since the distance from B to the axis is the same as the height of the image, the image becomes larger. But to do this, it must move in toward the mirror, because its top must always be along the line AF.

f) Since, as the object comes closer to the mirror, the image also comes closer to the mirror, the distance of the image from the mirror must be greatest when the object is very far away. When the object is very distant, the rays reaching the mirror are almost parallel, and the image is just about at the principal focus. The image can never be farther from the mirror than the principal focus.

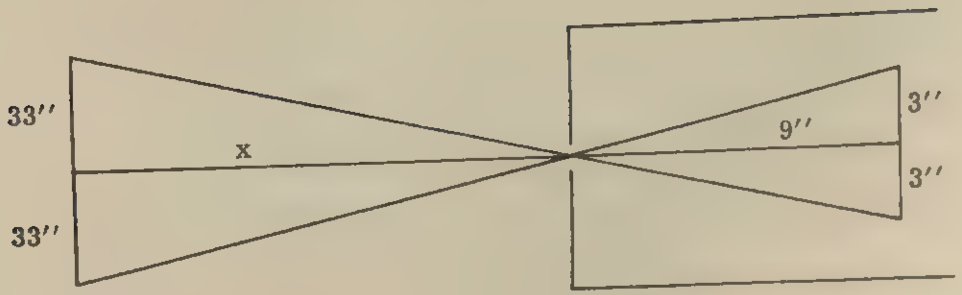
PROBLEM 21

As a project you can make a pinhole camera that is excellent for photographing the sun. Paint the inside of a cardboard mailing tube flat black (to prevent reflection of light). Cover one end with a piece of heavy black paper, in the center of which a hole has been punched with a fine needle. To the other end of the tube fit a piece of wood or metal that allows the tube to be mounted onto the front of a camera, in place of the lens. The connection, of course, must be light-tight. Insert a photographic film in the camera in the usual way, and point the tube toward the sun with the pinhole covered. When the tube is correctly placed, take a time exposure of a few seconds.

(a) If the film in a pinhole camera has a vertical height of 6 in. and is 9 in. from the pinhole, how far must the pinhole be from a man 5 ft. 6 in. tall if his image is to extend the full height of the film?

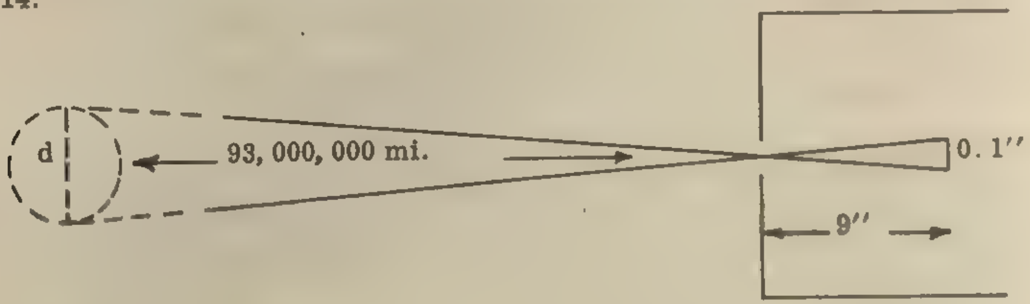
(b) In the same pinhole camera, if the image of the sun is $\frac{1}{10}$ in. in diameter, what is the diameter of the sun? The sun is 93 million miles away.

a)



By similar triangles, $\frac{3}{33} = \frac{9}{x}$, $x = 99''$

b) Some students will find it interesting to consider this problem in relation to Problems 13 and 14.



By similar triangles, $\frac{d}{93{,}000{,}000} = \frac{0.1}{9}$, $d = \underline{10^6 \text{ mi}}$.

PROBLEM 22

In Section 12–9 the question is asked: At what distance from the principal focus must the object be placed so that the image will be at the same place? Show that this question has two answers and explain the second answer, which was not discussed in the chapter.

This problem on virtual images forces the students to think about the properties of a concave mirror.

We can write down in a straightforward way $S_oS_i = f^2$. If object and image are to be in the same place then surely $S_o = S_i$ and it follows $S_i = S_o = f$. Students on their algebraic toes will see that there is another solution to the equation $S_oS_i = f^2$ i.e., $S_i = S_o = -f$. Do not introduce sign conventions in discussing this problem, but merely point out that the object could be placed one focal length toward the mirror from the principal focus, i.e., at the mirror itself. The image will be virtual and at the mirror itself. You should probably have a concave mirror (a shaving mirror will do) handy to defend this answer experimentally!

Chapter 13 - Refraction

CHAPTER SUMMARY

An experimental search is made to determine whether the behavior of a light ray as it bends in passing from air into another medium can be reduced to simple laws. The first law found is identical to the first law of reflection – the refracted ray lies in the plane defined by the incident ray and the normal to the surface. The second law is a remarkable synthesis of what might otherwise require volumes of graphs to describe – the ratio of the sines of the incident and refracted rays is a constant that has a particular value for each material, the index of refraction for that material. The data were available for 1000 years before this law was discovered! If the rays are reversed, the rays follow the same paths backwards; this behavior suggests that light rays are reversible. If light rays pass from one material to another, neither being air, the refraction is governed by the ratio of the indices of refraction for the two materials. The chapter closes with a discussion of total internal reflection, dispersion of white light into its component colors and an explanation of the rainbow.

COMMENT

The discovery by Snell of the second law of refraction is an excellent example of how the behavior of something (light) can be summarized by a law that fits the behavior empirically. Note that no model was invoked to get Snell's law; no one asked, "What is light like?" The question was merely, "What law describes the behavior of light?" Later in Part II, two different models will be tried to see if they can serve as models of what light might be. Both of these models predict Snell's law after the fact of Snell's discovery; in fact one test of a model for light is to see if it predicts Snell's law.

As an example of what engineers and scientists get involved in when a mass of data on the behavior of something cannot be reduced to a simple law, locate, if you can, a vacuum tube or transistor handbook and show it to the class. RCA puts out a condensed edition for $1.00; the complete edition is more impressive. Explain that Snell's law, with a few pages of the tables giving the indices of refraction for different materials, is equivalent to the thick tube handbook.

EMPHASIS

Snell's law, total internal reflection, and dispersion are key points in subsequent developments of the course. Work with problems, the laboratory, and the film will help to pin these down.

SCHEDULING CHAPTER 13

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part II.

	14-week schedule for Part II			9-week schedule for Part II		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2, 3	2	2	II-3	1	1	II-3
Secs. 4, 5, 6	2	-	-	1	-	-
Secs. 7, 8	1	-	-	1	-	-

RELATED MATERIALS FOR CHAPTER 13

Laboratory. Experiment II-3 - Refraction, should be done before discussing Section 3; on the 9-week schedule, it should be done before the chapter is started. See the yellow pages for suggestions.

Home, Desk and Lab. Many of the problems which may appear, at first glance, to require trigonometry, can be solved graphically. Graphical solutions are instructive, but since they take time, you may need to be careful about giving too many in an assignment. Problem 15 is a rather interesting laboratory-type exercise which can be done at home and which will be enjoyed by many students.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 13-15; detailed comments and solutions on page 13-16 to 13-28.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1		1*		1*	
3	2	3*		3*	
5		4, 5*, 7	6, 8, 9, 10*	4, 5*, 7, 8, 9, 10	
6	11, 12			11	
7	16*, 17	18, 19, 20	13, 14, 21*	14, 16 17, 21	17 (variation) 15
8	22	23		23	

Films. None specifically for this chapter, but there are some good sections on refraction in "Introduction to Optics" which you can ask students to recall or which might be re-run if you still have the film.

Section 1 - Refraction

PURPOSE To introduce the general nature of refraction and the first law of refraction.

CONTENT The incident ray, the refracted ray, and the normal to the surface are all in the same plane.

EMPHASIS The first law of refraction is important, but it will not take much class time. It is worth spending a little time pointing out that the approach to refraction will be similar to that for reflection.

COMMENTS Before getting into the first law of refraction, make it clear that we will try to summarize refraction phenomena with a few laws just as we summarized all reflection phenomena with the two laws of reflection. You should re-emphasize that the two laws of reflection describe completely what light does when it is reflected. The formation, location, and size of an image produced by any mirror can all be understood simply through the use of the two reflection laws. Remind students that no matter how complex a mirror problem becomes, there is never any question about where an individual ray will go when it is reflected. Similarly, if we can find laws which describe how a single light ray refracts, we will have tools that will enable reducing to simple terms the action of complicated refracting devices (such as lenses and prisms) in which the refraction of many rays must be considered.

Make sure at the outset that students are measuring the angle of incidence and the angle of refraction from the normal. You can test this by putting on the board a diagram such as that at the right, and asking students to estimate the angle of

incidence. If you get any "about 80 degrees" responses, you have some students who must change their ways.

CAUTION Do not bring up the exceptions to the first law of refraction. If a student asks a question about the "peculiar" materials, you can explain that only a few substances violate the first law; we want to begin with a study of refraction as it occurs in almost all substances; the exceptions are too rare and too complex to serve as examples for beginning the study of refraction.

At this point, even for bright students who ask you after class, it is enough to point out that the exceptional crystals have a complex structure which includes a special "optic" axis. When light passes through these crystals, both the normal to the surface and the direction of the optic axis contribute to the determination of the refraction.

Students who have picked up information on polarization may have learned that the incident ray sometimes breaks up into two refracted rays. True, but again, we start with the typical case.

Section 2 - Experiments on the Angles in Refraction

PURPOSE To show the nature and extent of experimental data which must be collected to predict the refraction of rays if a law of refraction cannot be found. To lay the groundwork for an appreciation of the intellectual achievement involved in finding a "law" from such data.

CONTENT a. Our knowledge of refraction comes from experiment.

b. A collection of data on refraction has clear regularities, but they do not conform to any simple algebraic function.

EMPHASIS The material in this section should not be hurried. It is easier for students to get the necessary qualitative understanding of refraction at this stage before they know Snell's law. They should study the data until they can summarize it with a few simple facts which must be explained by the laws of refraction. This will give them valuable experience in treating data with the help of graphs and tables.

The amount of time you spend on this section should depend upon the amount of practice your students need in data handling, data summarizing, graphing, extrapolation, etc. This is a good point for such work. A similar opportunity will not occur later because after students have used Snell's law, optical instruments (not straightforward refraction) capture their interest.

DEVELOPMENT Early laboratory work on refraction will help. Students need to observe refraction, measure angles, and plot data in order to read this section with understanding. Experiment II-3 should be performed before class discussion of Section 3.

Next, be sure that students understand the general aspects of refraction before they become involved with details. The text makes three descriptive statements (beginning at the end of the first column on page 212) about Figure 13-2 on page 211. You might go over these statements, then supplement them with questions. For example:

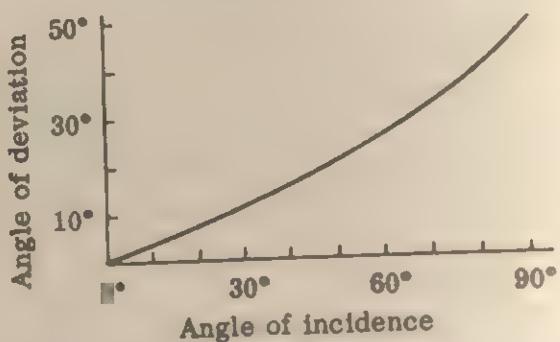
What would happen if the angle of incidence were less than any of those shown? If it were 0°?

Suppose we think of the four pictures in Figure 13-2 as coming from different frames of a motion picture made while the incident ray was moving at a uniform speed from an incidence angle of zero degrees to an incidence angle of 90°. Which moves faster, the incident ray or the refracted ray?

When light goes from air (or vacuum) into glass, which is larger, the incident angle or the refracted angle?

Another major step in the development is to study (using tables and graphs) how the angle of refraction varies when the incident angle is changed for light entering glass from air. You can use the actual experimental data of Experiment II-3 to give the students as much practice as you think they need in interpolation and extrapolation. You might ask students whether it would make sense to extrapolate the curve beyond an angle of incidence of 90°?

If you have time to give the students more insight into refraction and more practice with graphs, ask them to plot, as a function of the incident angle, the angle of deviation (the smallest angle through which the extended incident ray would have to turn in order to coincide with the refracted ray).



The final point to be made in this section is that each time you change either one of a pair of substances, you need a completely new table (or graph). In fact, initially, one might anticipate needing two tables for each pair, depending on which medium contained the incident ray. (Of course, reversibility could be established quickly, and eliminate the need for a second table.)

CAUTION Avoid discussing the dependence of refraction on color. If a student raises the question, tell him that different colors behave similarly, but that there are slight variations due to color which will be discussed later.

COMMENT Do not expect to use the apparent brightness of the reflected rays shown in Figure 13-2 to give even a semi-quantitative idea of reflection coefficients. Since these different pictures were probably developed differently to make them clear, the apparent intensities cannot be determined by comparing the pictures. The coefficient of reflection (i.e., the fraction of the incident ray intensity which is reflected) is small up to fairly large angles of incidence (about 50°), and reaches 1 at 90°. At an incidence angle of 0° (i.e., normal incidence), the reflection coefficient is simply $\left(\frac{n-1}{n+1}\right)^2$. For glass this is 0.04, for water it is 0.02.

DEMONSTRATION Penny in bottom of an evaporating dish on lecture table cannot be seen by students at their seats. But if water is added to the dish the coin becomes visible.

Section 3 – The Index of Refraction: Snell's Law
Section 4 – The Absolute Index of Refraction

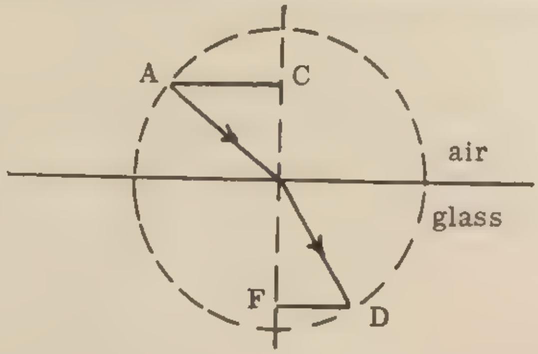
PURPOSE To introduce Snell's law of refraction.

CONTENT The angle of refraction, r, does not depend linearly on the angle of incidence, i, but there is a simple relation between i and r for light going from air into a given substance: ratio of the subtended semi-chords is a constant. In trigonometric language, this is the same as saying that the ratio sin i/sin r is a constant for light going from air into a given substance. This constant is called the index of refraction for the particular substance, relative to air. The absolute index of refraction is the ratio of sin i/sin r for

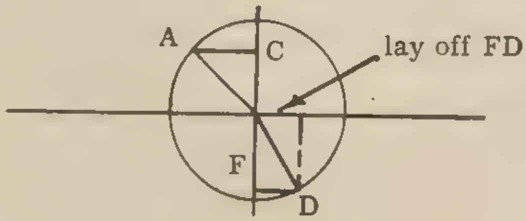
light going from a vacuum into a given substance.

EMPHASIS and DEVELOPMENT This material is important mainly because it introduces Snell's law. But it is also important because it can be used to show students how scientists look for regularity to summarize experimental data. If Experiment II-3 is done before the reading of this section is assigned, including the graphical treatment of the data, the students will come about as close to discovering for themselves Snell's law as can be expected in the time available. The study of this section will then serve to organize their thinking and to point out relations which they may have overlooked in doing the experiment.

COMMENT Even if your students are familiar with sines, it is important to consider Snell's law first as dealing with the ratio of two semi-chords. The fact that AC is always one and a half times as long as FD tells students much more than merely remembering that for air and glass $\frac{\sin i}{\sin r} = 1.5$.



A scale drawing can be used to predict the path of any refracted ray if the refractive indices are known. Protractors and tables of sines are not needed. Some students may not see how to construct FD easily, even if they know $FD = \frac{AC}{1.5}$. Be sure they see that a length equal to FD should be marked on the circle diameter which is the interface (between glass and air) and that a perpendicular to the diameter from this point will locate D.



Be sure at the beginning (and check this when it comes up later) that students know exactly which semi-chord ratio is involved. You will avoid confusion if you take time at the outset to prove (using similar triangles) that the semi-chord ratio is independent of the size of the circle. (Graphical accuracy will improve as the circle radius is increased.) After understanding that the semi-chord ratio is equal to the index of refraction, some students will err in bending the refracted light ray the wrong way (i.e., they will multiply the "incident semi-chord" by the index of refraction instead of dividing the semi-chord by the index). Remind the students to decide first which way the light will bend.

Give the students some problems requiring graphical solutions for work at home, at their desks, and on quizzes. State the problems in such a way that the student can work without sine tables or a protractor. (Provide compasses or special paper on which the necessary circle has been drawn.)

Although the formulation of Snell's law in terms of the ratio of semi-chords should be stressed, you should also discuss Snell's law in terms of sines. Go over the definition of the sine of an angle to be sure that all students understand the first paragraph on page 215 and Figure 13-6 (page 215). You do not have to drill students on this definition or on the use of the sine table (pages 636-637); they will use the sine of an angle many more times and will get further practice.

Section 5 - The Passage of Light from Glass (or Water) to Air: Reversibility

PURPOSE To show that the path of light in passing in either direction through the boundary between air and glass is predicted by the general form of the equation for Snell's law:

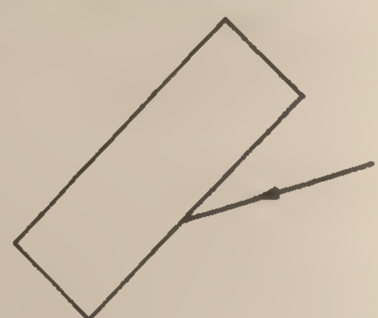
$$\sin \theta_{air} = n_{glass} \sin \theta_{glass}$$

DEVELOPMENT The basis of this conclusion is an experiment such as that shown in Figure 13-7. Your class discussion can begin with this figure, with the questions relating to reversibility asked in Experiment II-3, or with the scene from the film "Introduction to Optics" in which a mirror is used to reflect a refracted ray back along its own path. If you have the equipment you might demonstrate this. In any case the development of the general form of the equation should follow from the experimental observation of the reversibility of light in passing through the boundary between two materials. Students should understand the two main applications of reversibility: that a reversed light ray retraces the path over which it came, and that a light ray passing through a parallel-sided piece of glass emerges parallel to the incident ray.

A point that students are almost sure to raise is: If you take just the final refracted ray in Figure 13-7 and run it backwards, you will get the same refracted rays, but the reflected rays will differ. You can tell them that when we speak of reversibility of light, we do not mean that all the reversed light will necessarily return. We mean simply that if the transmitted ray were reversed, some light would go back over the path of the incident ray. There would be no other ray near the incident ray; or at a slight angle to it.

A good way to make reversibility clear is to draw a diagram as at the right below, showing the same block of glass as in Figure 13-7. Start this time with an incident ray from the right of the glass and moving back over the path of the transmitted ray. Ask students to sketch in the resulting rays. They should know that there will be four rays.

Figure 13-7



In addition to the refracted ray and the transmitted ray, there are the two reflected rays, one from each surface. (Notice that the diagram at the right looks like an upside down view of Figure 13-7.) You should then make sure that students realize that what reversibility has to say about this new situation is simply that the refracted and transmitted rays in the new diagram coincide with the refracted and incident rays in Figure 13-7. The limited form of reversibility discussed here says nothing about either the relative intensities or the reflected rays.

Section 6 - The Passage of Light from Water to Glass: The Relative Index

PURPOSE To show how to predict the refraction that occurs when light passes from one substance to another when neither substance is air.

CONTENT a. If light is incident from medium 1 at an angle, θ_1, and is refracted in medium 2 so that its angle of refraction with the normal) is θ_2, then

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

b. The ratio $\frac{n_2}{n_1}$ is called the relative index of refraction, n_{12}.

EMPHASIS This section is important, but it should require little class discussion except as you discuss problems based on these ideas, e.g., HDL Problems 11 and 12.

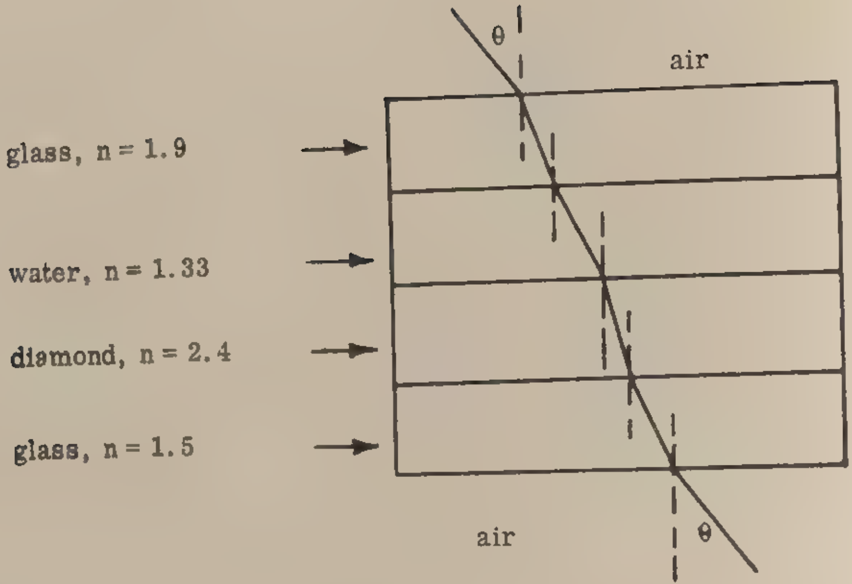
COMMENTS The argument given for the validity of the relative index based on an imaginary layer of air separating the two substances (Figure 13-10) is a "plausibility" argument. It is not intended to be rigorous. (However, this kind of "plausible inference" is a characteristic of physics. It is a common way to arrive at an assumption which then must be tested.) For all we know, until we check it experimentally, there may be a marked change of some kind when the air layer of Figure 13-10 is removed. For example, we know of cases in which two adjacent substances interact completely (alcohol and plastic); it is conceivable that a minute surface layer might form which would make the relative index concept unusable. Of course, no marked change does occur, but students should realizé that the argument given suggests the fact; it does not establish it.

Often students do not really appreciate the great simplification introduced by the relative index even though they know what the index is and how to use it. Be sure that they realize that if it were not for this simplification, they would need an extremely long table which gave the index of refraction for each different pair of materials.

By convention we deal with values of the relative index greater than unity. If the index of refraction of medium 2 relative to medium 1, n_{12}, were less than 1, we would usually speak of n_{21}, the index of refraction of medium 1 relative to medium 2. Also: $n_{21} = n_1/n_2 = 1/n_{12}$. Do not try to drill students in the notation n_{12} or n_{21}. It is enough if they remember that the light ray is always closer to the normal in the medium which has a higher index of refraction.

Students who try to get the relative index of refraction so that they can write $\sin r = n_{12} \sin i$, nearly always get confused. Emphasize the symmetric form $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Try to have students start with this form before they try to decide whether n_1/n_2 or n_2/n_1 is what they need for a particular problem.

Your class may be interested in dealing with a problem involving light passing through several layers of material, each with a different index of refraction:



Notice that:

The incident and emerging rays are still parallel.

Given a particular angle of incidence, θ, the angle at which light travels in, say, the diamond does not depend upon the previous materials. If the incident light went directly from air to diamond, the ray within the diamond would be parallel to the one indicated above.

If we "re-stack" the 4 blocks in a different order, the angle of travel in each medium remains the same as indicated above.

If you think of many more layers with only slightly changing refractive indices, you have a good model of the changing refractive index of the earth's atmosphere (see Problem 9, page 224).

Since for any two media, $n_1 \sin \theta_1 = n_2 \sin \theta_2$; then for three media, $n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3$; etc. for any number of media. The quantity "$n \sin \theta$" is constant (invariant) throughout all the layers. If you know the angle and the refractive index in one layer (air for the problem given), then you know the value of "$n \sin \theta$" in all layers. If you know the refractive index n for a given layer, you can find $\sin \theta$ and therefore θ in that layer. (Remember that all boundaries are parallel planes.)

Section 7 - Total Internal Reflection

PURPOSE To show when light is completely reflected.

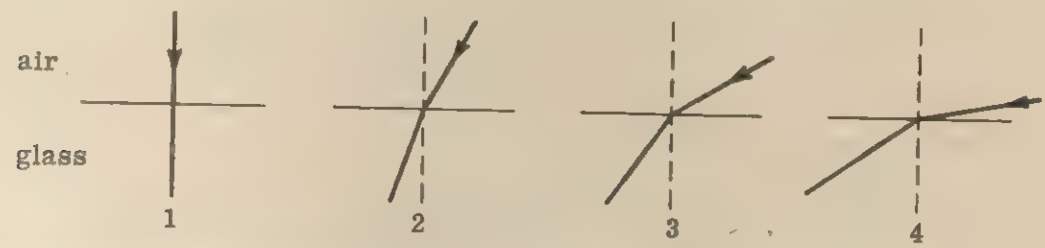
CONTENT Light traveling from a medium with a higher index of refraction to a medium with a lower index will be completely reflected when the angle of incidence exceeds a specific value.

Total internal reflection occurs at <u>all</u> angles larger than the critical angle, θ_c, which is defined by

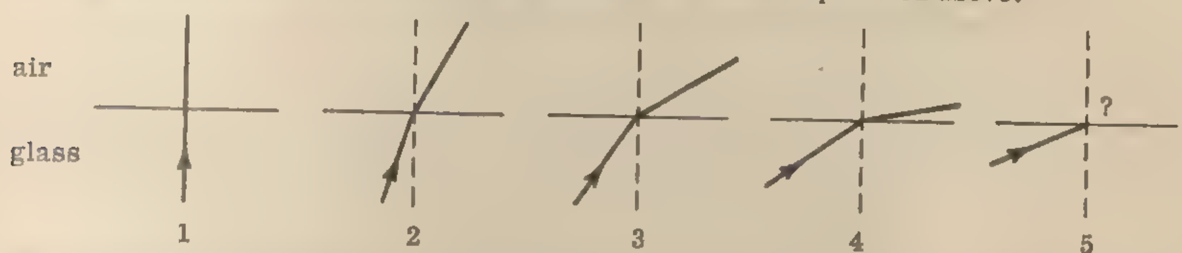
$$\sin \theta_c = \frac{n_2}{n_1}.$$

EMPHASIS This is an important subject which rounds out the discussion of what happens to light at the boundary between two substances. Students probably will have raised questions earlier about internal reflection. The development suggested below is a "full" treatment, which you may want to use or abbreviate depending upon how far your class has gone into this subject in other contexts.

DEVELOPMENT As you look at larger and larger angles of incidence for, say, air to glass

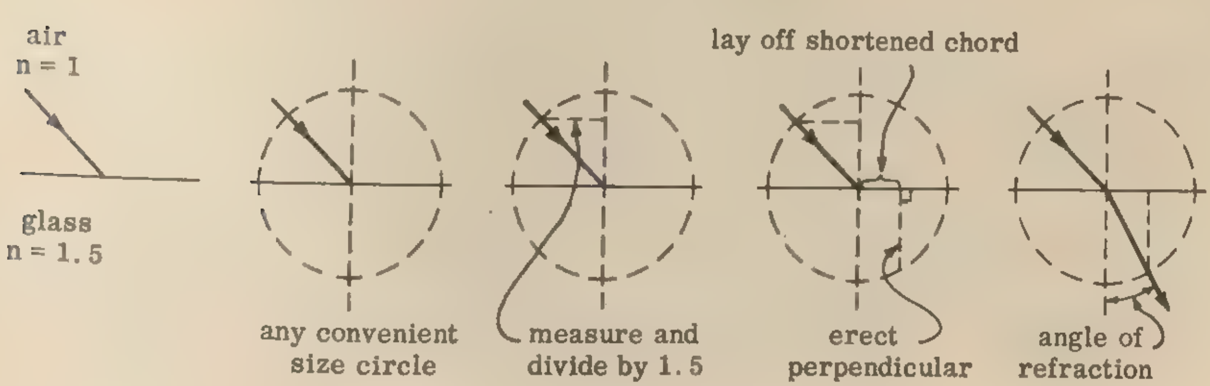


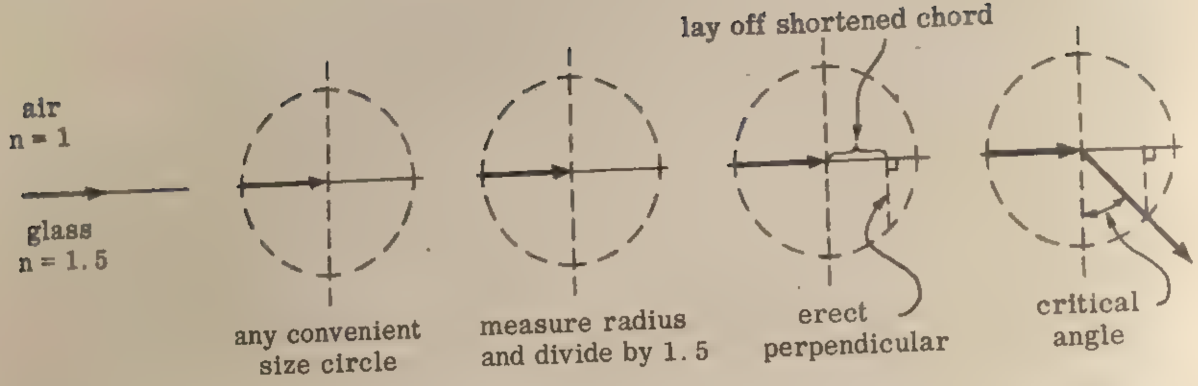
it is natural to apply reversibility and ask about a case not pictured above:



When presented with diagrams of this kind, students usually infer most of the facts about total internal reflection. In particular they should see that if a ray did refract into the air in case 5 above, Snell's law would be violated. There is nowhere in the air for it to go! Students should see from this development that the possibility of total reflection occurs only when light "attempts to leave" a material of higher refractive index to go into a material of lower refractive index.

Students who make scale drawings to find an angle of refraction may have a moment of difficulty when they try to find a critical angle graphically. Steps in the two procedures are:





If any students are still puzzled, or if you think they will be, have them find graphically the refracted angle in glass for an incident angle in air of, say, 88°.

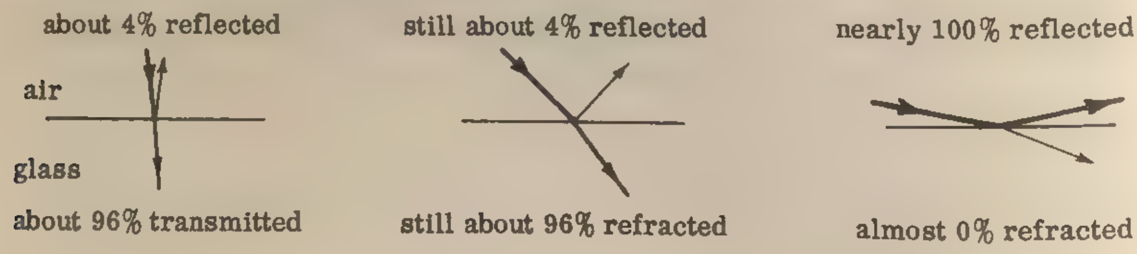
* * *

You can use the idea of reversibility to describe the critical angle. From the previous development you will have shown (or be able to show) that, no matter what the angle of incidence for a ray traveling from air to glass (n = 1.5), the ray cannot be refracted more than 41.8° from the normal. Thus, for any given point on a glass surface, any incident light will be refracted within a cone (in the glass) whose vertex angle is 83.6°. Using reversibility, the student can see that, to escape from glass to air through the given point of incidence, a ray would have to travel within the same cone.

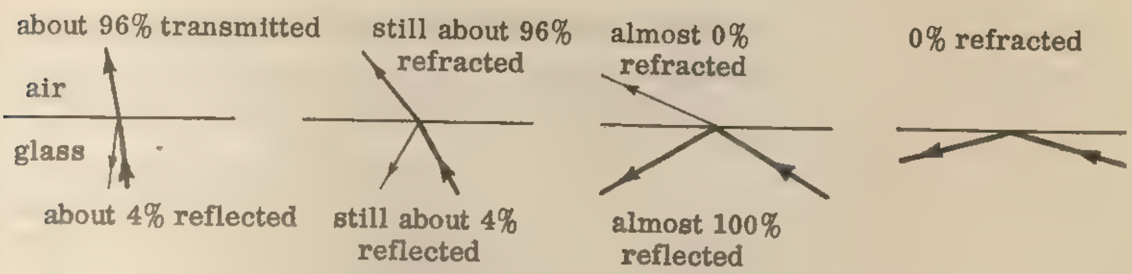
* * *

As a "fooler" you might give a quick problem in class: find the angle of refraction in air. If very many students work for more than a minute, they don't yet see the light!

COMMENTS Students should not get the idea that all of the incident light is refracted until the critical angle is reached, then suddenly all of it is reflected. If a ray is incident on glass from air, the percentage of reflection depends on the angle in air, θ_a. At $\theta_a = 0$, the percentage of light reflected is $\left(\frac{n-1}{n+1}\right)^2$. As θ_a increases this percentage remains constant through about 50° and then increases rather rapidly to 100%.



In considering rays which start in the glass and make an angle of θ_g with the normal to the air surface, you find that the percentage reflection for θ_g is the same as the reflection for the corresponding θ_a. As $\sin\theta_g$ approaches $\frac{1}{n}$ (and $\sin\theta_a$ approaches 1), the reflection approaches 100%.



It will be helpful if you can set up a demonstration such as that pictured in Figure 13-11. In your demonstration, the gradual change of reflected and transmitted intensities will be clearer than is shown in the figure. The student should not be left with the impression that, at $\theta = \theta_c$, an intense light beam skims along the surface.

* * *

After students have a qualitative familiarity with total internal reflection it will take very little class time to establish the equation for the critical angle:

$$\theta_c \text{ is the angle where } \sin\theta_c = \frac{n_2}{n_1}.$$

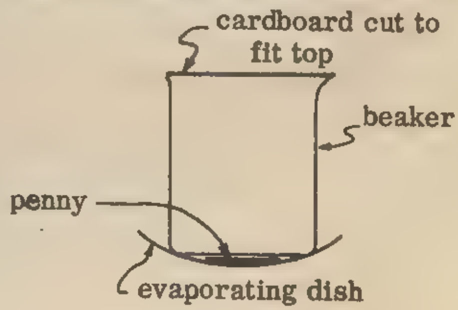
* * *

You may find it convenient to use the terms "optically more dense" and "optically less dense" for describing substances with a higher or lower index of refraction, respectively. These terms are not in the text, but students accept them quickly as a simple method of describing the relative value of the index of refraction of a substance. "Optical density" has nothing to do with ordinary density. Many substances with smaller indices of refraction are much more dense than diamond.

* * *

Some students with an aptitude for hairsplitting may worry about the concept of "grazing incidence". If students seem troubled, suggest that they think of the sequence 89°, 89.9°, 89.99°, etc. In this way they do not need to picture what happens when the light "grazes" the surface in a mathematical sense.

DEMONSTRATION With a coin below small empty beaker in an evaporating dish, the coin is visible through the sides of the beaker. But when the beaker is filled with water and the top of the beaker covered with cardboard, the coin disappears.



Section 8 - Refraction by Prisms; Dispersion

PURPOSE To indicate that the index of refraction depends (very slightly) on color, and that light is bent by a prism.

CONTENT a. White light is composed of different colors.

b. For almost all substances the index of refraction is larger for blue light than for red (i.e., blue light is bent more).

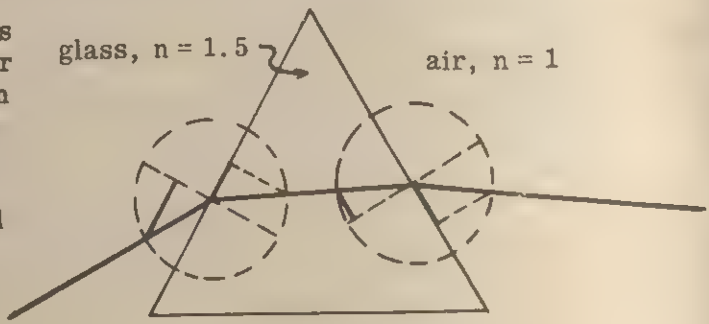
c. We can predict the path of light rays through a prism using Snell's law.

EMPHASIS Dispersion is important both as a tool for examining the colors of light, and later (in Chapter 17) as a clue to the relation between color and frequency. Dispersion can be covered at this stage quite rapidly.

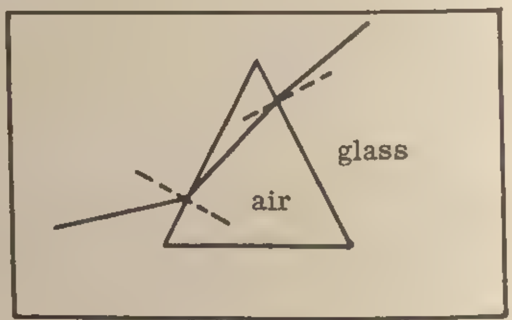
COMMENTS At this stage, dispersion is defined simply as the spreading of white light into its colors, due to slight differences between the refractive indices for the various colors. Later in the text (in Chapter 17), after the students have learned that refractive index is related to wave speed, dispersion will be used in its more general, technical sense to signify that the speed depends on frequency. You should not introduce either speed or frequency now.

Students should now realize that Snell's law (with constant n) is only an approximation for white light. But remind them that if only a single color is present, Snell's law is exact.

You will want to at least briefly discuss the path followed by a light ray of one color through a prism. Students should see such ray tracing as an extension of the work on parallel plates to non-parallel faces. At each surface they have a refraction problem to solve. A careful drawing is a good exercise that guarantees student understanding.

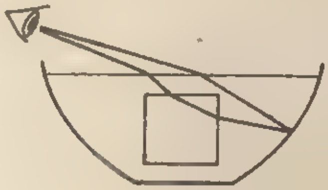


Also students may enjoy the notion of an "air prism" embedded in water or glass.

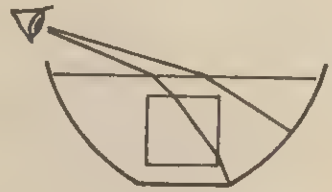


Here is a challenge for your class: Without looking it up, find whether the refractive index of ice is more than, less than, or the same as that of water. If you are lucky, someone will point out that since you can see ice submerged in water, it must not have the same refractive index. A student who knows that water expands a little as it freezes may guess correctly (without logical foundation) that the refractive index for ice is less than for water.

The nice problem is to show experimentally that ice has the smaller refractive index. Tell students that a coffee cup, water, and an ice cube (a clear one) are all they need. Hold the ice cube beneath the water and look just over an edge of the cube at a spot on the cup. If $n_i < n_w$, it is possible to see two spots simultaneously; otherwise not.



$n_i < n_w$
two spots visible

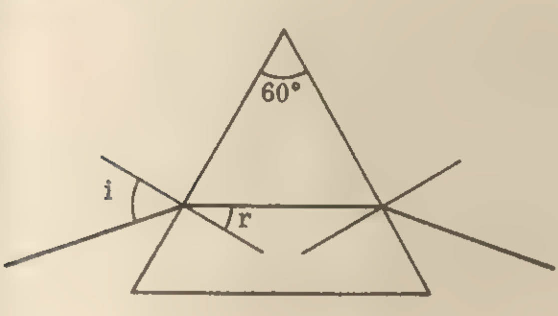


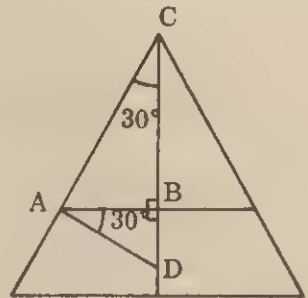
$n_i > n_w$
only one spot visible

ADDITIONAL QUESTIONS for Section 8. (The following problems may be varied in a number of ways for class discussion or quizzes. They might also be used as "extra" problems for good students.)

1) An equilateral glass prism has an index of refraction n = 1.5. For the ray of light that travels through the prism parallel to the base, find the incident angle.

Solution:

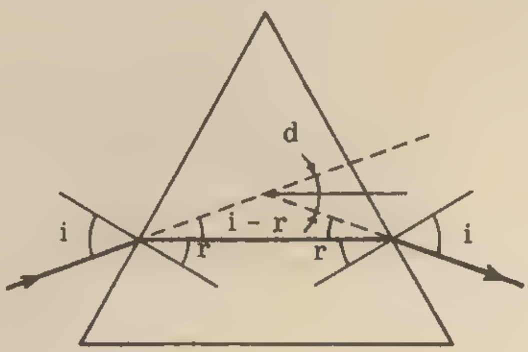




If we can determine the angle r of refraction, Snell's law will give the angle of incidence i immediately. Since the ray in the prism is parallel to the base, it is perpendicular to the bisector of the apex angle. The triangles ABC and ABD are similar, and r = 30°. Thus, sin i = 1.5; sin 30° = 0.75 from Snell's law, and i = 49°.

2) Find the angle of deviation between the incident and emergent rays in problem 1 above.

Solution: From the symmetry, the ray in the glass makes an angle with the normal to the second surface equal to r. Thus the angle between the emergent ray and the normal is equal to i. The angle of deviation d is the angle between the dotted line extensions of the incident and emergent rays. A simple way to find its value is to draw the line parallel to the ray in the glass as shown, and to note that it divides the angle d into two angles, each of which is equal to i - r by geometry. Hence:
$d = 2(i - r) = 2(49° - 30°) = 38°$.



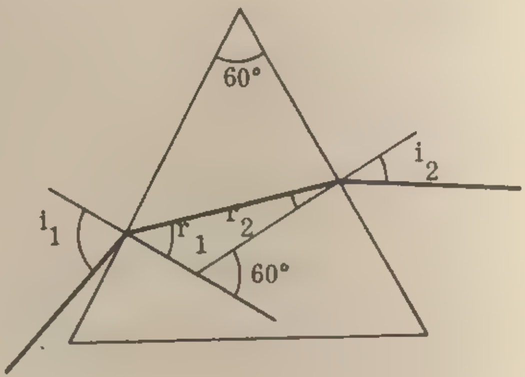
3) Choose another angle of incidence and show that the deviation is greater than the result of Problèm 2.

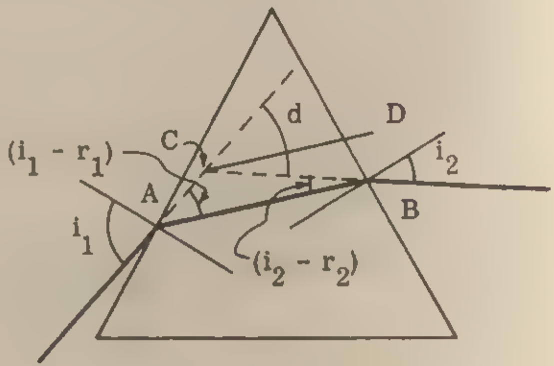
Solution: Students who need exercise may try incident angles slightly larger or smaller than in Problem 1 and follow through to find the deviation. It will be greater than in Problem 2.

The following observations may be helpful. It is easy to show generally that the two angles r_1 and r_2 in the accompanying figure obey $r_1 + r_2 = 60°$. This result follows from the fact that the two normals to the sides of the prism must make a 60° angle with each other. Thus the problem can be carried out analytically. Given a chosen value for i_1, $\sin i_1 = n \sin r_1$ gives r_1.

$r_2 = 60° - r_1 \quad \sin i_2 = n \sin r_2$

To find the angle of deviation d, again draw the line CD parallel to line AB and note that: $d = i_1 - r_1 + i_2 - r_2$.





The Rainbow

(Boxed material on pages 222-223.)

While the rainbow is an interesting phenomenon that can be explained with the facts of reflection, refraction, and dispersion, you may want to avoid a class discussion of rainbows since later parts of the course do not require an understanding of this material. You can treat the boxed information on the rainbow as a reading assignment with no class discussion.

For your convenience, Appendix 2 to this volume of the Guide answers questions which students frequently ask. In most cases, you will want to answer these questions after class.

Chapter 13 - Refraction

For Home, Desk and Lab — Answers to Problems

Many of the problems which may appear, at first glance, to require trigonometry, can be solved graphically. Graphical solutions are instructive, but since they take time, you may need to be careful about giving too many in an assignment. Problem 15 is a rather interesting laboratory-type exercise which can be done at home and which will be enjoyed by many students.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 13-16 to 13-28.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1		1*		1*	
3	2	3*		3*	
5		4, 5*, 7	6, 8, 9, 10*	4, 5*, 7, 8, 9, 10	
6	11, 12			11	
7	16*, 17	18, 19, 20	13, 14, 21*	14, 16 17, 21	17 (variation) 15
8	22	23		23	

SHORT ANSWERS

1. a) Usually the refracting material has no way of "telling" the ray which way to bend out of the plane of incidence.
   b) There are substances, e.g., calcite, for which there is a natural asymmetry that does this.
2. See detailed discussion on page 13-16.
3. a) 0.3511, 20.6°.
   b) 0.4670, 27.8°.
   c) 1.47.
4. a) 80°.
   b) 11 cm.
5. a) 9.0 ± 0.3 cm.
   b) 1.33 ± 0.05, comparable to index of refraction.
6. 32.1°, 5.8 cm.
7. Scale drawing.
8. See detailed discussion on page 13-22.
9. See detailed discussion on page 13-22.
10. See detailed discussion on page 13-23.
11. 2.42.
12. 1.46.
13. See detailed discussion on page 13-23.
14. a) 189 cm^2.
    b) 12.7 cm.
15. Home project (experiment).
16. a) 1.34.
    b) 40.5°.
    c) 60.4°.
17. The quartz will be "invisible."
18. Increases. 54.7°.
19. Endlessly around inside of cylinder.
20. See detailed discussion on page 13-27.
21. a) 8.2°.
    b) Yes.
    c) Smaller.

22. 18.6°.

23. 60°.

COMMENTS AND SOLUTIONS

PROBLEM 1 — Can you give a reason why the incident ray, refracted ray, and the normal should all be in the same plane? Can you imagine a material in which this would not be true?

This problem can be interpreted as a deep question or as a playful question. Viewed lightly, it asks students to give "common-sense" reasons for a fact they already know about the incident ray, the refracted ray, and the normal ray.

In order to prove that in ordinary optical media the two rays and the normal lie in a plane one must assume Maxwell's equations or their equivalent. Students will certainly not give an answer such as this! Just be sure that they do not think they have given a proof. Students' responses to this kind of question are unpredictable. Do not let them take themselves too seriously. The best and most frequently-given reason is that if the three rays did not lie in one plane, the refracted ray would have to lie on one side or the other of the plane determined by the incident ray and the normal. How would the refracted ray decide which side of the plane to travel in? It could not decide! Therefore, it must stay in the original plane.

Giving such arguments as the one above is a good intellectual exercise for students. In the case of the argument above, however, they should realize that the reason given is based tacitly on the idea that a transparent material behaves in the same way regardless of the direction from which the light ray enters it. This is a reasonable assumption for most transparent materials. However, doubly refracting materials such as a calcite crystal furnish a counter example. Consequently, it is impossible for students to "give a reason" which is good enough to cover all cases. (Do not get involved in the details of double refraction here.)

Students may answer the second question either way since some students are notoriously poor imaginers when it comes to school work. Regardless of whether they can imagine it, there are transparent materials in which a refracted ray does not stay in the expected plane. Such materials are rare and we will not be concerned with them in this course.

In a material such as a crystal of calcite there is a lack of symmetry in the structure of the crystal which causes one of the refracted rays to turn in a specific way.

This exercise can be a pleasant activity if you do not push it (or let the class push it) too far. Avoid any of the details concerning double refraction. Conclude with the idea that the reason for belief that, in ordinary materials, the incident ray, the normal, and the refracted ray lie in the same plane is that this is what we observe when we test such materials in the laboratory.

PROBLEM 2 — (a) What are the sines of the following angles: 4°, 30°, 45°, 60°, 73°, 17.8°, 37.3°, 90°?
(b) What are the angles that have the following sines: 0.1045, 0.0000, 0.3090, 0.8660, 1.000, 0.5000, 0.5225, 0.9636?
(c) Plot sin i versus i from 0° to 90°.

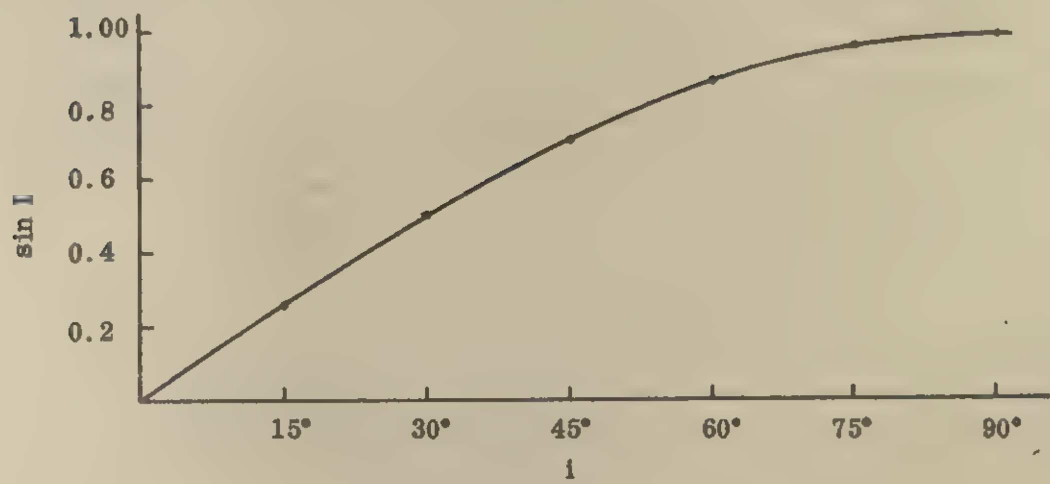
a)

Angle	Sine	Angle	Sine
4°	0.0698	73°	0.9563
30°	0.5000	17.8°	0.3057
45°	0.7071	37.3°	0.6060
60°	0.8660	90°	1.0000

b)

Sine	Angle	Sine	Angle
0.1045	6°	1.0000	90°
0.0000	0°	0.5000	30°
0.3090	18°	0.5225	31.5°
0.8660	60°	0.9636	74.5°

c)



There are occasional weird interpretations of this problem. Some students who are experts in trigonometry think that in part b) they are to give all angles, or rather formulas for all angles, having these values as sines. The problem intends that only angles between 0° and 90° be considered. Also, an occasional student assumes that there is something special about the particular angles and sines of angles which are given and thinks he should memorize them. Be sure that students recognize that there is nothing "special" about the angles or the values given except that if one is dealing frequently with angles and their sines it is convenient to remember values for such angles as 30°, 45°, 60°, etc. For this course such memorization is not necessary.

PROBLEM 3

A rectangular tank 8 cm deep is filled with water. A light ray enters the top surface of the water at a point just touching the side of the tank. After refraction it falls on a point on the bottom of the tank 3 cm from the same side of the tank.

(a) What is the sine of the angle of refraction? What is the angle of refraction?

(b) What is the sine of the angle of incidence? What is the angle of incidence of the entering ray?

(c) Suppose that the same tank were filled with a liquid other than water and you found that in order to fall on the same point 3 cm from the side the angle of incidence of the entering ray had to be 31 degrees. What is the index of refraction of the liquid?

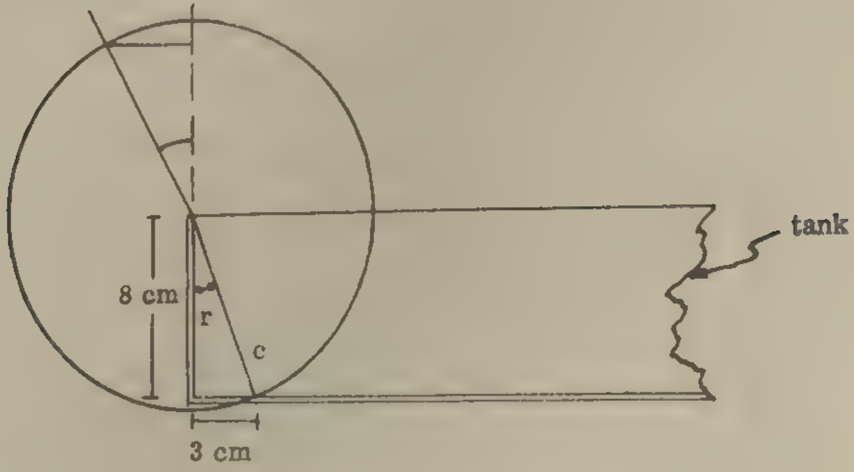
Students can do this by a scale drawing and construction, or they can use the tables to find r. Using the second method,

$c^2 = 8^2 + 3^2$, and $c = \sqrt{73}$.

a) $\sin r = \frac{3}{\sqrt{73}} = \underline{0.3511}$. The angle of refraction $\underline{r = 20.6°}$.

b) $\sin i = 1.33 \sin r = \underline{0.4670}$ from which $\underline{i = 27.8°}$

c) $n = \frac{\sin i}{\sin r} = \frac{\sin 31°}{\frac{3}{\sqrt{73}}} = \underline{1.47.}$

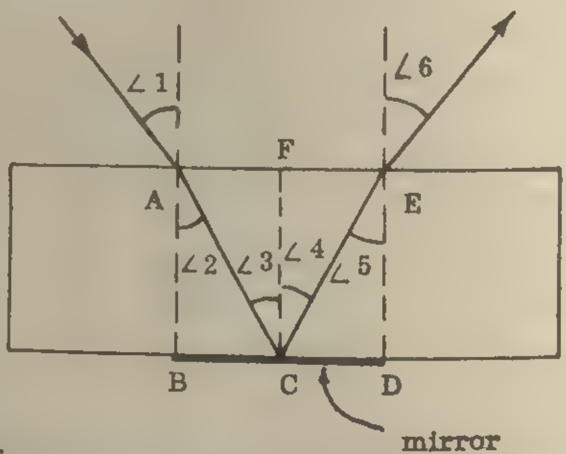


PROBLEM 4

A narrow pencil of light enters the top surface of the water in a rectangular aquarium at an angle of incidence of 40°. The refracted pencil continues to the bottom of the tank, striking a horizontally placed plane mirror which reflects it back again to the surface, and it is again refracted as it emerges into the air.

(a) What is the angle between the incident ray entering the water and the refracted ray emerging from it?

(b) If the water in the tank is 10 cm deep, what is the distance between the points on the water surface where the ray enters and where it emerges?



a) A good student may look at this problem and say, "This is a perfectly symmetrical situation so the angle between the entering and emerging rays is twice the angle of incidence, or 80°." Other students may approach the problem more prosaically by graphical construction or computationally. We know that $\angle 1 = 40°$. By Snell's law $n \sin \angle 2 = \sin \angle 1$. By geometry $\angle 2 = \angle 3$. By the second law of reflection $\angle 3 = \angle 4$. By geometry $\angle 4 = \angle 5$. Finally, by Snell's law, using the reversibility of light rays, $n \sin \angle 5 = \sin \angle 6$. But since $\angle 2 = \angle 5$, $\sin \angle 1 = \sin \angle 6$, and $\angle 1 = \angle 6 = 40°$. The angle between the incident ray and the ray leaving the water is $\angle 1 + \angle 6 = \underline{80°}$.

This problem situation reminds us of a glass mirror silvered on the back where it is still true that the angle of incidence is equal to the angle of reflection.

b) The distance AE on the water surface between the points where the ray enters and leaves the water is, since $\angle 3$ and $\angle 4$ are equal, just 2 AF. We know CF = 10 cm. $\sin \angle 3 = \frac{AF}{AC} = \sin \angle 2 = \frac{\sin 40°}{1.33} = 0.4833$. By the Pythagorean theorem $AC = \sqrt{\overline{AF}^2 + \overline{FC}^2}$,

$$\frac{AF}{\sqrt{\overline{AF}^2 + 10^2}} = 0.4833, \quad AF = 5.52. \quad \text{Thus } AE = \underline{11 \text{ cm}}.$$

We have avoided the use of tangents. Using them would make the problem less involved. Unless the students know trigonometry, it might be best to avoid confusing them with trigonometric functions other than the sine.

A graphical construction should be equally good – and may be faster than the analytical calculation.

PROBLEM 5

Make a drawing (to scale) of the side view of an aquarium in which the water is 12 cm deep. From a single point on the bottom draw two lines upward, one vertical and the other 5° from the vertical. Let these represent two light rays that start from the point. Compute the directions in which the refracted rays will be traveling above the surface of the water, then draw in these rays and continue them backward into the water until they intersect.

(a) At what depth does the bottom of the tank appear to be if you look straight down into the water? Does this help to explain the phenomenon shown in Fig. 11–2 (a)?

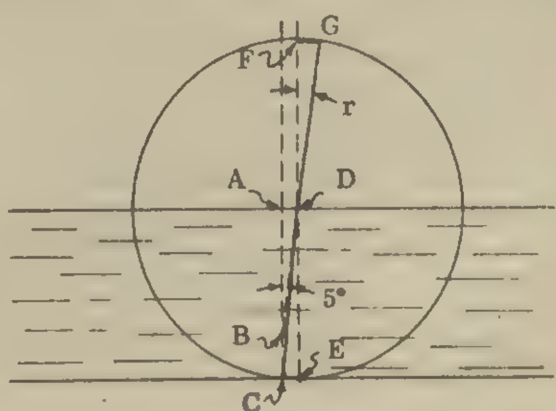
(b) Divide the apparent depth into the true depth and compare with the index of refraction of water.

This is a good graphical construction problem on Snell's law which leads to the idea of "apparent depth".

A student who makes a careful and large drawing (scale 1 cm = 1 cm) should be able to get the following answers:

a) apparent depth = $\underline{9.0 \pm 0.3 \text{ cm}}$.

b) $\dfrac{\text{true depth}}{\text{apparent depth}} = \underline{1.33 \pm 0.05}$. This value agrees within the accuracy of drawing to the index of refraction.



A computational solution is as follows:

a) From Snell's law $r = \angle ABD = 6.66°$
$AD = AC \sin 5° = 1.05$ cm

$$AB = \frac{AD}{\tan 6.66°} = \frac{1.05 \text{ cm}}{0.1167} = 9.0 \text{ cm}$$

b) The ratio of $AC/AB = 1.33$ which is the index of refraction of water. This result should be expected for small angles but not for large angles. Note that by definition $n = FG/CE$ and since $AD = CE$, $n = FG/AD$. But then by similar triangles $FD/AB = FG/AD$. The true depth $AC = DE$. So the true depth/apparent depth = $DE/AB = FD/AB \times DE/FD = n \times DE/FD$. For small angles of incidence, DE is nearly equal to FD, but for large angles FD becomes close to zero and the apparent depth goes to zero. For example, with $i = 5°$. $DC = DG = AC/\cos 5°$, $FD = DG \cos r = 12 \times \cos 6.66°/\cos 5°$, and $DE/FD = 12 \cos 5°/12 \cos 6.66° = 1.003$. For $i = 30°$, $DE/FD = 1.16$.

PROBLEM 6

A person looking into the aquarium described in Problem 5 sees light coming from the bottom. The light that reaches his eye is traveling along a line that makes an angle of 45° with the vertical.

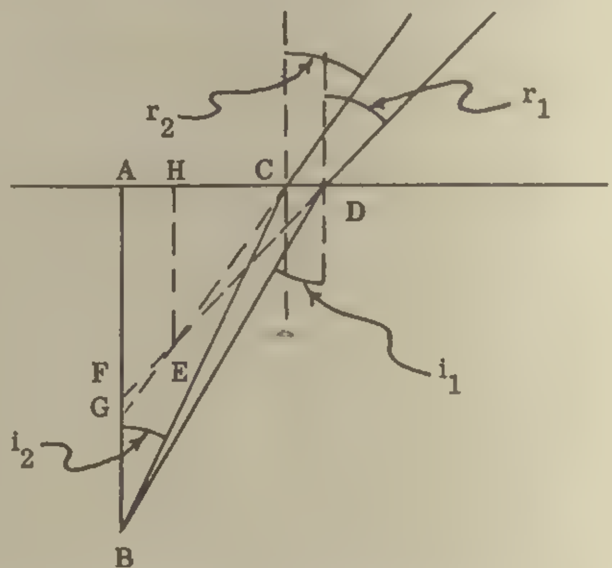
(a) At what angle with the vertical must the light have been traveling in the water?

(b) Make a drawing, like that called for in Problem 5, showing the path of the ray. Then draw another light ray from the point on the bottom at which the first ray started, making its path in the water about 5° closer to the vertical than was the first ray. Compute and draw in the path of this ray above the water.

(c) At what depth does the bottom appear to be when viewed by the person who is looking into the aquarium?

This problem is harder than Problem 5 but not as fundamental. Students should do Problem 5 first.

By application of Snell's law: $n \sin i_1 = \sin 45°$, $\sin i = 0.5317$, $\underline{i_1 = 32.1°}$. Then $i_2 = i_1 - 5° = 27.1°$, and, again by Snell's law, $\sin r_2 = n \sin 27.1° = 0.6058$, $r_2 = 37.3°$.



Finding the apparent depth in this problem is rather tricky. When the refracted rays above the water are extended down into the water, their extensions do not meet directly above the source of light. A calculation of the apparent depth by just finding the intersection of one of the rays with the vertical is not correct. The easiest way to find the point at which the extended rays intersect is to draw an accurate diagram. The value, calculated by analytic geometry, is 5.8 cm. This is the accurate apparent depth. Note that this "apparent depth" is not the same as in Problem 5. When the term "apparent depth" is used without being qualified, it usually means for rays near normal incidence (as in the case of apparent depth for Problem 5).

Given: AB = 12 cm, $n = 1.33$, $r_1 = 45°$.

By calculation: $i_1 = 32.1°$. AD = 7.5 cm.

$i_2 = 27.1°$ AC = 6.1 cm.

$r_2 = 37.3°$ AG = 8.0 cm.

AF = 7.5 cm.

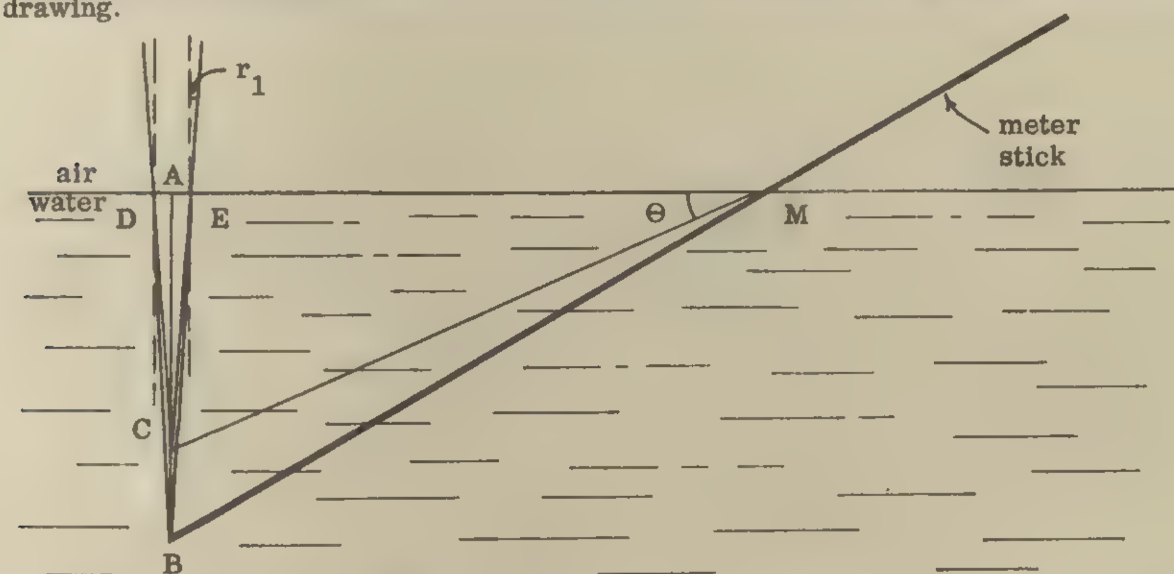
HE = apparent depth = $\underline{5.8 \text{ cm}}$.

One rather careful graphical analysis gave 5.65 cm.

PROBLEM 7

A meter stick is dipping into a tank of water. It makes an angle of 30° with the horizontal surface of the water and its mid-point is just at the surface. Suppose that your eye is located directly above the submerged end of the stick. To find how the stick will appear to you, make a scale drawing showing a side view of the stick and the water. Draw two rays upward from the submerged end of the stick, each being 3° from the vertical. Compute the direction of the refracted rays above the water, draw in these rays, and continue them backward into the water until they intersect. The end of the stick will appear to be at this point. Draw a line connecting the point just determined with the mid-point of the stick and compare your drawing with Fig. 11-1. How do you account for the bent-stick illusion?

Probably the best way to work this problem (as indicated in the instructions) is to compute the angles from Snell's law, but locate the points of intersection using a scale drawing.



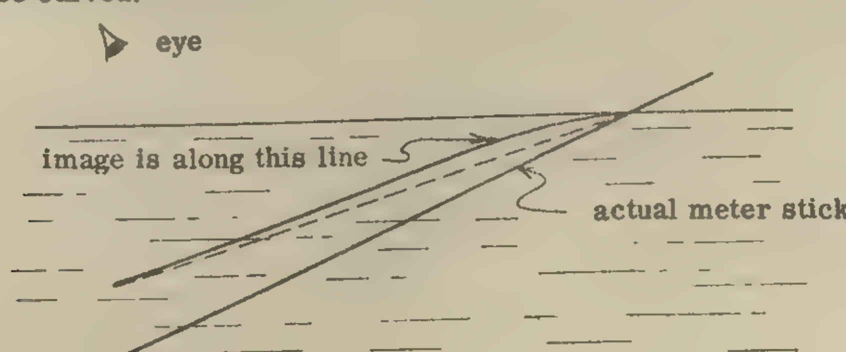
From $n \sin 3° = \sin r_1 = 0.0696$, $r_1 = 4.0°$.

From here on, most students should proceed graphically. However, the analytic solution is:

AB = AM tan 30°
AD = AB tan 3°
AC = AD cot 4°

which gives $\dfrac{AC}{AM} = \dfrac{\tan 30° \ \tan 3°}{\tan 4°} = 0.433 = \tan \theta$, and $\theta = 23.4°$.

However, drawing the straight line between C and M to represent the image of the stick in the water is not quite accurate. If the observer's eye is close to the water, the stick appears to be curved:



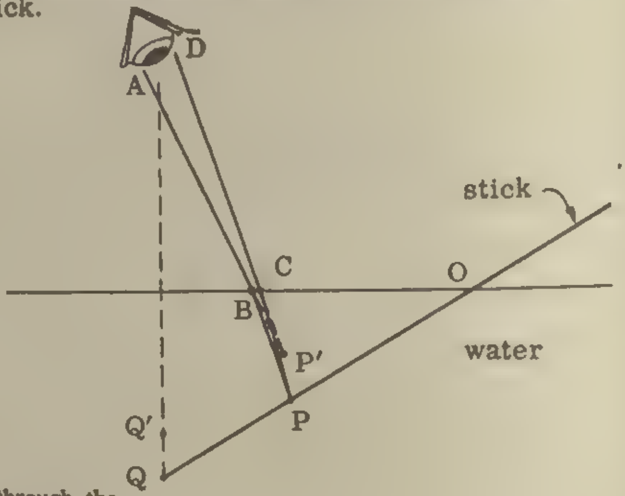
We saw in Problem 5 that the apparent depth is proportional to the actual depth only for small angles of incidence. But if the eye is close to the water, the apparent image is less than 1/n times the actual depth. After students have completed this problem, it is a good subject for class discussion.

PROBLEM 8

In the instructions given in Problem 7 it was assumed that the portion of the stick that was under water would appear to be straight and that the apparent position of the stick could therefore be determined by drawing a straight line connecting the apparent position of the end with the place at which the stick entered the water. Supposing that your eye was 60 cm above the water surface and vertically above the submerged end, can you invent a way of finding out where a point on the stick 20 cm from the submerged end would appear to be? If so, you could check on the accuracy of the assumption. Do not bother to make a scale drawing for this purpose. Instead make a rough sketch and describe the process that you would use.

This exercise on apparent depth combines the concepts developed in Problems 5, 6, and 7. It is probably best used as a class discussion question.

Students are asked how to find where a point 20 cm from the submerged end of the stick would appear to be. This problem is very difficult and can be done only by trial and error, i. e., choose a ray from the 20 cm point going off at a likely angle, and see if it hits the eye. If not, try another angle until two rays very close together both go toward the eye. Extend these rays from the eye back into the water and see where they intersect. This is the apperent position of the 20 cm spot on the stick.



An easier approach to a proof that the stick appears curved is to draw a ray from the eye at some particular angle and follow it into the water and see where it hits the stick. Suppose you draw bent ray ABP. Take another ray from P and extend it back. Call it ray PCD. The apparent intersection of AB and DC is the apparent position, P′, of P.

It will be seen that the three points Q′ (the image of Q), P′, and O do not lie on a straight line.

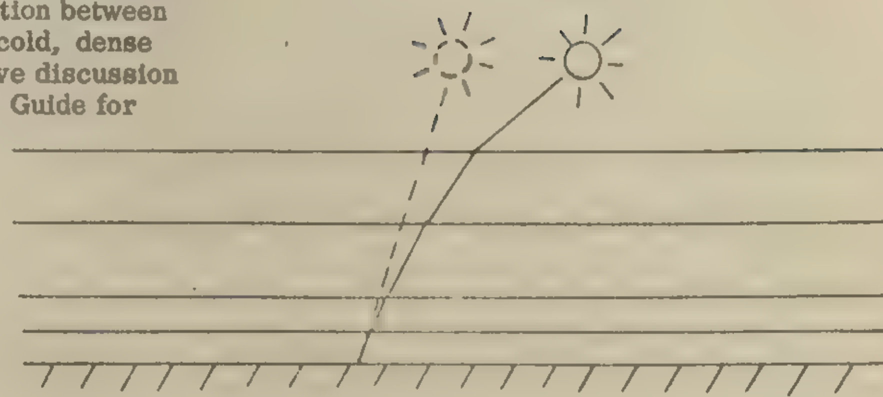
PROBLEM 9

Light from the setting sun comes through the earth's atmosphere along a curved path to your eye, so that the sun looks higher in the sky than it really is. How do you explain this? Illustrate your answer with a diagram.

This problem, involving the bending of a light ray in air of varying index of refraction, may provide an interesting class discussion.

Since the index of refraction for air is different from that for a vacuum, the index for rarefied air is in between the index for sea-level air and the index for a vacuum. The pressure of air drops as the altitude increases, and the index also drops as altitude increases. Assume, for simplicity, that the atmosphere is made of many layers, each higher layer having a slightly smaller index than the layer below, but with each layer having a constant index within that layer. Then we get the following diagram in which the refraction has been greatly exaggerated.

Thus a setting sun is actually already below the horizon; it is only through refraction that we see it. Also, many mirages are caused by refraction between layers of hot, thin air and cold, dense air. (There is a quantitative discussion of "layer problems" in the Guide for Section 6 of this chapter.)



PROBLEM 10 Be prepared to discuss why turning Fig. 13-8 upside down shows you a way to prove the reversibility of light paths.

This exercise can be used as the basis of a class discussion.

Turning Figure 13-8 upside down shows the path of light through the glass in a direction which is the reverse of the path in the figure as normally viewed. The complete symmetry between the two paths argues strongly (a little short of proof) that light is reversible.

PROBLEM 11 If the *relative* index for light going from glass into diamond is 1.61 and the *absolute* index of glass is 1.50, what is the *absolute* index of diamond?

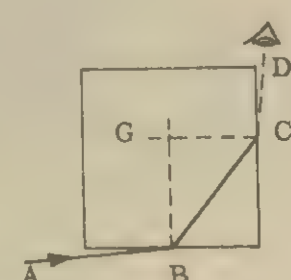
If $\frac{n_d}{n_g} = 1.61$ and $n_g = 1.50$, then $n_d = n_g \times \frac{n_d}{n_g} = 1.50 \times 1.61 = \underline{2.42}$.

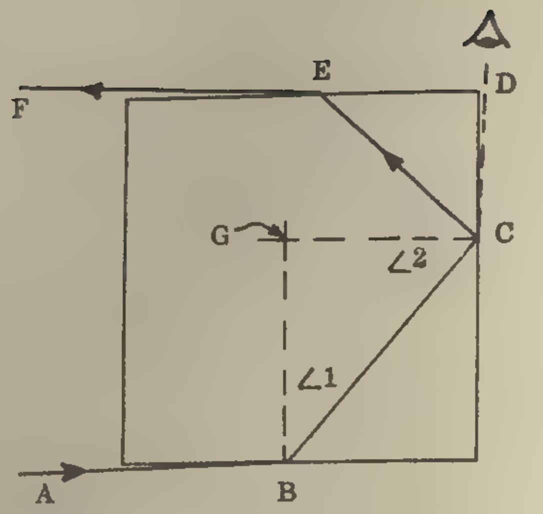
PROBLEM 12 If the relative index for light going from oleic acid into water is 0.91 and the index of water is 1.33, what is that of oleic acid?

If $\frac{n_w}{n_o} = 0.91$, and $n_w = 1.33$, then $n_o = \frac{n_w}{n_{ow}} = \frac{1.33}{0.91} = \underline{1.46}$.

PROBLEM 13 With a square block of glass it is impossible, when looking into a side, to see out of an adjacent side. It appears to be a mirror. Using your knowledge of geometry and the critical angle, prove that this must be true.

If we are to see an object out of an adjacent side of the square block of glass, it must be possible for a ray to come along a path like AB and be refracted out along a path like CD. Light coming in nearly parallel to the face of the cube (as along AB) has the "best chance" of success. Along a path like AB, the angle of incidence is near 90°. Then by Snell's law, $\sin 90° = n \sin r = n \sin \angle GBC$, or $\sin r = \frac{1}{n} = \frac{1}{1.5}$, and $r = \angle GBC = 41.8°$.





But we see that ∠GCB, the least possible incident angle of the ray reaching the block at point C, must be 90° − 41.8° = 48.2°.

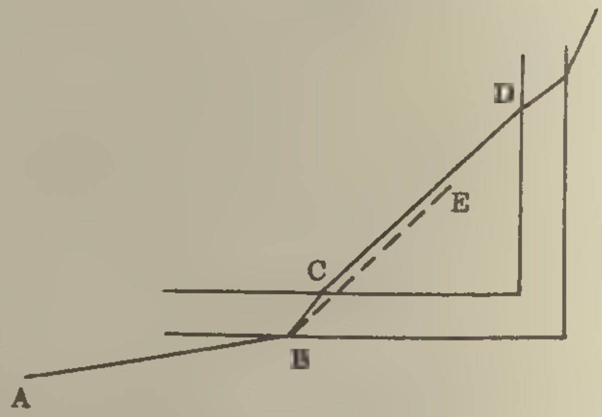
An angle of 48.2° is greater than the critical angle for glass (41.8°). Therefore, all light traveling along the path AB will be reflected back into the block at C, and none will reach the eye along the path CD. If AB enters the cube with a smaller angle of incidence than 90°, then ∠GBC is smaller and ∠GCB is larger, so there is still total internal reflection.

Supplementary Note: The problem could just as well have asked about a rectangular block of glass instead of a square one. In fact, it simply could have asked about the possibility of light getting across a right angle.

You might begin an interesting discussion by substituting a cube of water, n = 1.33, for the cube of glass. Then the calculation shows that the ray can leave the cube.

Next, you might ask what index would just allow a ray to leave the adjacent face. The borderline case is a material for which n = √2. Fancy extensions of this problem can be obtained by considering angles other than right angles.

Finally, a good test of the students' qualitative understanding of refraction can be had by asking about a cube of water enclosed with glass sides. The ray could leave by the path shown here:



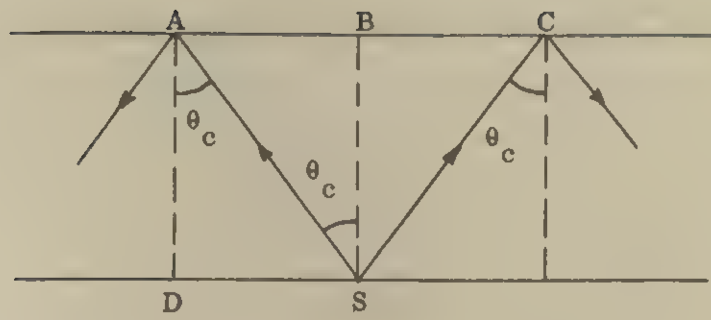
The angle ABE is the same as it would be without the glass. The glass merely displaces the beam to the side.

PROBLEM 14

Carbon disulfide (refractive index 1.63) is poured into a large jar to a depth of 10.0 cm. There is a very small light source at the center of the bottom of the jar.

(a) Calculate the area of the surface of carbon disulfide through which the light passes.

(b) What is the greatest distance in the carbon disulfide traveled by a ray that emerges from the surface?



a) Light will pass through the surface of the CS_2 from below if, and only if, the angle of incidence is less than the critical angle; in other words light within the cone ASC can leave the surface of the CS_2. In part (a) we are asked to find the area of the base of the cone which is $\pi(AB)^2 = \pi(BC)^2$. We know that AD = 10 cm. The critical angle is given by $\sin 90° = n \sin \theta_c$, so $\sin \theta_c = 1/n = 1/1.63 = 0.613$, from which $\theta_c = 37.8°$. Now $\sin \theta_c = DS/AS = AB/AS = 0.613$. If students have not been introduced to tangents, they will have to use the Pythagorean theorem to find AS.

$$\overline{AS}^2 = \overline{AB}^2 + \overline{BS}^2 = \overline{AB}^2 + 100, \text{ and } AS = \sqrt{\overline{AB}^2 + 100}$$

Then, $\dfrac{AB}{\sqrt{\overline{AB}^2 + 100}} = 0.613$

$$\overline{AB}^2 = \frac{100 \times 0.376}{0.624} = 60.3 \text{ cm}^2.$$

And the area is $\pi(AB)^2 = \pi \times 60.3 \text{ cm}^2 = \underline{189 \text{ cm}^2}$.

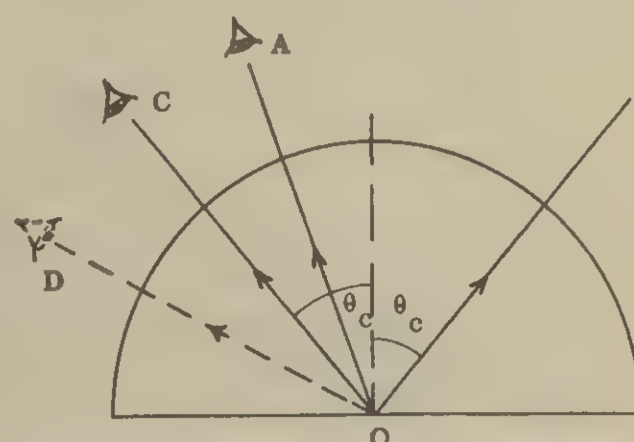
b) In part b) students are asked to find the longest distance travelled by a ray which emerges from the surface. This is the distance AS.

From above, $\overline{AS}^2 = 60.3 + 100$, and $AS = \underline{12.7 \text{ cm}}$.

PROBLEM 15

As a home project use a plastic cheese box (see Experiment II-3 in the Laboratory Guide) to measure the critical angle in water.

Place the semicircular cheese box on a sheet of paper and draw its outline on the paper. Fill the box with water. Stick a pin vertically in the paper next to the box at the mid-point of the straight side (the diameter). Now look at the pin through the curved part. Stick two other pins next to the curved part at the two angles where the image of the first pin disappears. Remove the box, draw the necessary lines, measure the critical angle and calculate the index of refraction.



This is a good problem to be done either at home or in the lab. The students must realize that it is where the rays from the first pin (O) first enter the liquid that the critical bending occurs. That is, an eye placed at A can see the pin. So can one at C (or rather at angles a very little smaller than θ_c).

However, no rays can go from outside the box through O to an eye at D.

PROBLEM 16

The absolute index of refraction of sodium chloride is 1.54 and the relative index of refraction for light going from sodium fluoride to sodium chloride is 1.15. What is
(a) the absolute index of sodium fluoride?
(b) the critical angle of sodium chloride?
(c) the critical angle between the two salts?

a) $\frac{n_{NaCl}}{n_{NaF}} = 1.15$, and $n_{NaCl} = 1.54$. Therefore, $n_{NaF} = \frac{1.54}{1.15} = \underline{1.34}$.

Here the students need to remember that the absolute index must always be greater than one.

b) $\sin \theta_c \text{ (NaCl)} = \frac{1}{n_{NaCl}} = 0.6494$, and $\theta_c \text{ (NaCl)} = \underline{40.5°}$.

c) $\sin \theta_c \text{ (rel)} = \frac{1}{n_{rel}} = 0.8696$, and $\theta_c \text{ (rel)} = \underline{60.4°}$.

The students should be encouraged to go back explicity to Snell's law, and you should be sure they understand that light going from NaCl to NaF may be critically reflected, but not in going from NaF to NaCl.

PROBLEM 17

What will you see when you look at a piece of fused quartz submerged in oleic acid (refer to Table 3)?

Fused quartz and oleic acid both have the same index of refraction, and both are colorless. Since light will pass through the interface without being refracted or reflected, there is no way of "seeing" the quartz at all, unless there are internal imperfections. You might mention that a truly invisible man would have to be colorless and transparent with an index of refraction equal to that of air.

DEMONSTRATION Commonly available materials with nearly the same index of refraction are ordinary mineral oil and lucite or plexiglass. A mixture of benzene and alcohol, in the right proportions, has the same index as pyrex. Some of your students who have studied biology may have seen tissue specimens mounted in Canada balsam which has the same index of refraction as the glass used for microscope slides and cover glasses.

PROBLEM 18

To the jar in Problem 14 we add 5 cm of water. (The water will not mix with the carbon disulfide but will float on top.)
(a) Does this increase or decrease the area of the cone of light as it emerges from the carbon disulfide into the water?
(b) Calculate the critical angle at the surface between the carbon disulfide and the water.

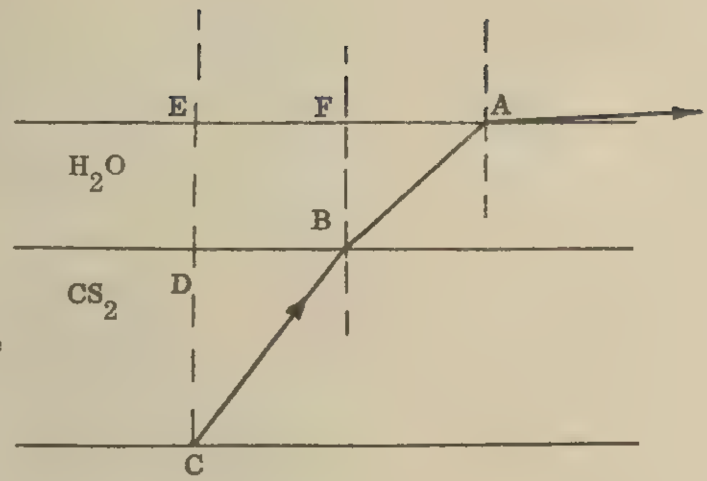
The area of the cone of light at the interface between the carbon disulfide and the water is larger than the cone of Problem 14 going from CS_2 to air because the critical angle is larger in this case. The critical angle at the $CS_2 - H_2O$ surface is found from $n_{H_2O} \sin 90° = n_{CS_2} \sin \theta_c$, and $\theta_c = \underline{54.7°}$.

NOTE It is interesting to note that if we restrict ourselves to the cone of light that will <u>leave</u> the water at the water-air surface, then the cone in the CS_2 is the same as in Problem 14.

To find the maximum surface area of the cone of light leaving the water, we must suppose that light leaving the water at A just grazes the surface. This light must have come along the path BA. The angle FBA is found from:

$$n_{air} \sin 90° = n_{H_2O} \sin \angle FBA$$

$$\sin \angle FBA = (1.33)^{-1} = 0.7519$$
$$\angle FBA = 48.8°.$$



This light must have come from CB, where

$$n_{CS_2} \sin \angle DCB = n_{H_2O} \sin \angle FBA$$

$$1.63 \sin \angle DCB = 1.33 \sin \angle FBA$$
$$= 1.33 \times 0.7519 = 1$$

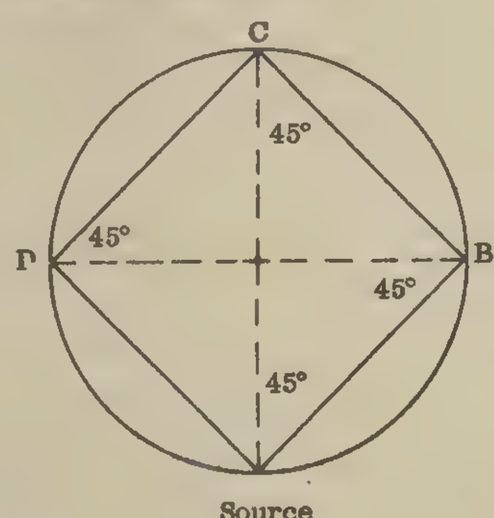
$$\sin \angle DCB = (1.63)^{-1} = 0.613$$
$$\angle DCB = 37.8°.$$

Notice that the $\angle DCB$ is the same as the angle of the ray that escaped in Problem 14. Thus DB is the same as the radius of the cone in the CS_2 of Problem 14.

PROBLEM 19 A light source in a cylindrical glass container of carbon dichloride (C_2Cl_4; $n = 1.50$) sends a pencil of light from a point on the circumference. The pencil is parallel to the bottom and makes an angle of 45° with the radius. (See Fig. 13–21.) What will be the path of the light?

This problem on critical angles shows an example of light "piping".

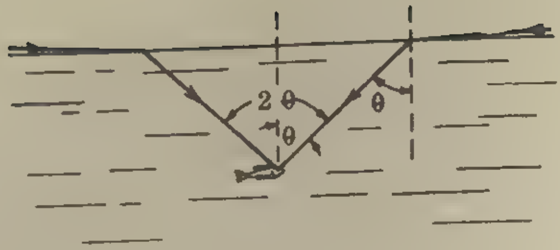
From the geometry we see that the light beam makes an angle of 45° with the normal to the surface at B. Note that the interface between the carbon dichloride and glass is irrelevant. The index of refraction of carbon dichloride is 1.50, and the critical angle is 41.8°. Since the angle of incidence is greater than the critical angle, there will be total reflection. The same situation occurs the next time the light ray hits the wall of the cylinder, and the next, and the next; so, the light beam will retrace itself endlessly around the inside of the cylinder, gradually being absorbed. (Instantly absorbed from our point of view.)



PROBLEM 20 Show that the angle of the cone of rays reaching the fish from above the water is about 98°. (See Fig. 13–12.)

If θ is the critical angle, the angle of the cone of rays reaching the fish is $2\,\theta$.
$\sin \theta = 1/n_w = 1/1.33 = 0.751$, and

$\theta = 48.8°$. $2\,\theta = \underline{97.6°}$.



PROBLEM 21

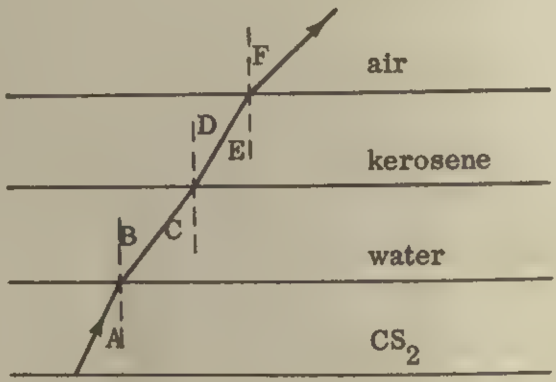
It is possible to place carbon disulfide, water, and kerosene in separate layers in that order, since they do not mix. A container having these three liquids in layers of equal depth has on its bottom a light source which projects a pencil of light upward through the liquids at an initial angle of 5° from the vertical.

(a) Calculate the angle of refraction of the ray as it finally emerges from the kerosene.

(b) Would a pencil of light travel over the same path in the reverse direction from kerosene through water into carbon disulfide?

(c) Suppose the cylinder had been filled with water only. How would the angle of refraction of the emerging ray compare with the angle calculated above?

This problem on relative refractive index encourages algebraic work since one of the refractive indices is not given (but turns out to be unnecessary). If you assign this problem without warning, some students will go to the trouble of looking up the refractive index of kerosene. But if you warn students against this, it will take away the surprise.



a) We do not know the refractive index of kerosene, but we find that the final refractive angle, F, can be determined without it. From Snell's law,

$$n_{CS_2} \sin 5° = n_w \sin \theta_w = n_k \sin \theta_k = \sin \theta_{air}.$$

Therefore, $\sin \theta_{air} = 1.63 \sin 5°$ or $\theta_{air} = \underline{8.2°}$.

b) The answer is yes, but it should be emphasized that reversibility is an empirical law.

c) For water only, the angle of the emerging ray would be smaller than in part (a) and can be computed from Snell's law:

$1.33 \sin 5° = 1 \times \sin \theta_{air}$, $\sin \theta_{air} = 0.1160$, and $\theta_{air} = 6.7°$.

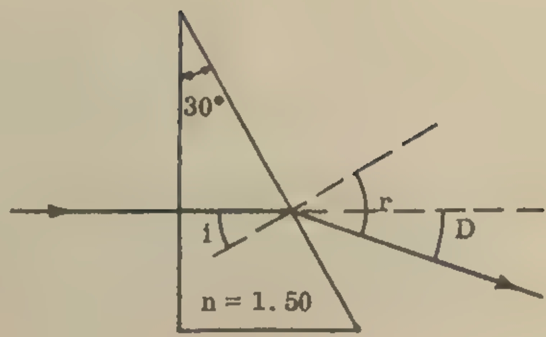
PROBLEM 22

A ray of light enters a triangular prism perpendicular to one face and emerges from the opposite face. The prism faces make an angle of 30° and the prism has an index of refraction of 1.50. Through what angle will the light be deviated in passing through the prism?

Since the ray enters the prism perpendicular to the face, it is refracted only as it leaves the prism. By geometry $i = 30°$, and by Snell's law:

$$1.50 \times \sin 30° = \sin r = 0.75$$
$$r = 48.6°.$$

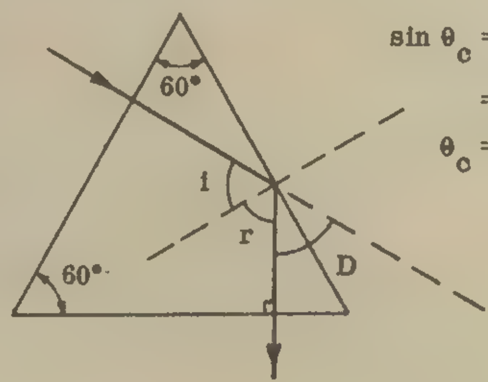
Then the angle of deviation is $r - i = \underline{18.6°}$. It should be pointed out that the angle of deviation is not the angle of refraction.



PROBLEM 23 If the prism in Problem 22 were equilateral, through what angle would the light be deviated?

This problem asks the student to trace a ray through a prism. He must realize that the ray totally reflects rather than refracting at the second face.

The interesting point about this problem is that the student is forced to recognize that deviation, although discussed in the text only with regard to refraction, is also applied to reflection. The angle of incidence, i, is greater than the critical angle. The ray is reflected off the second face of the prism, and by the second law of reflection and geometry there is no further deviation. The angle of deviation is $\underline{60°}$.



$$\sin \theta_c = \frac{1}{1.50} = 0.6667$$
$$\theta_c = 41.8°$$

Chapter 14 – Lenses and Optical Instruments

Just as mirrors can be shaped to converge light by reflection, so most transparent substances can be made to converge light by refraction. Beginning with a set of tapered prisms, as a means to this end, the idea is extended first to a smoothly curved cylindrical lens and then to a spherical lens. The focal length is used as the basis for drawing ray diagrams to find images and to determine magnifications.

The discussion of lenses in cameras, projectors, microscopes and telescopes gives an interlude of "gadgetry" before tackling the hard problem of figuring out why light behaves the way it does now that we have learned how it behaves.

CHAPTER EMPHASIS

These subjects are interesting to students. You should cover image formation thoroughly. However, it should not take long because many of the ideas are similar to those used in analyzing curved mirrors in Chapter 12.

Lenses, image formation, and lens instruments as such are not essential to an understanding of later chapters in this volume. However, when students later work with ripple tanks in studying water waves as analogs of light waves, they will see convergence (focusing), divergence, and other properties of "lenses" for water waves. A reasonable familiarity with the material in this chapter will help students appreciate such analogies.

Laboratory work will reinforce this chapter and vice versa. Experiment II-4 should be performed before the discussion of Section 3.

SCHEDULING CHAPTER 14

	14-week schedule for Part II			9-week schedule for Part II		
Subject	Class Periods	Lab Periods	Exp't	Class Periods	Lab Periods	Exp't
Secs. 1, 2, 3	2	2	II-4	1	1	II-4
Sec. 4	1	-	-	1	-	-
Sec. 5	1	-	-	1	-	-
Sec. 6, 7	1	-	-	1	-	-

RELATED MATERIALS FOR CHAPTER 14

Laboratory. Experiment II-4, Images Formed by a Converging Lens, is a good experiment, but is similar enough to II-2 to be omitted if your students did that experiment and you are short of time.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 14-19, detailed comments and solutions on page 14-20 to 14-36. Problems 18 and 19 are difficult problems that might be used near the end of your work on this chapter.

Section	Easy	Medium	Hard	Class Discussion
1		1*, 2		1
2	4	3, 5*	6	3, 6
3	9*	7*, 8, 10	11	7*, 11
4		7*, 10		7*
5	12			
6	14*	8	13, 15, 18*, 19*	9, 13, 15
7	16, 17			

Films. There are no PSSC films designed for this chapter.

Materials. Students can design their own optical experiments and devices from surplus optical supplies offered by several companies. Two such companies are:

Edmund Scientific Company
101 E. Gloucester Pike
Barrington, New Jersey

A. Jaegers Optical Company
Merrick Road and Horton Avenue
Lynbrook, Long Island, New York

Section 1 - The Convergence of Light by a Set of Prisms

PURPOSE To show how a beam of parallel light can be made to converge along a line.

CONTENT a. A set of prisms can be arranged so as to converge parallel light.

b. The size of the region of convergence can be reduced by replacing each prism with several sections of smaller ones, each having smaller vertex angles.

c. As the number of prisms is increased indefinitely, smoothly curved surfaces are approximated. Lenses with such surfaces converge parallel light to a line.

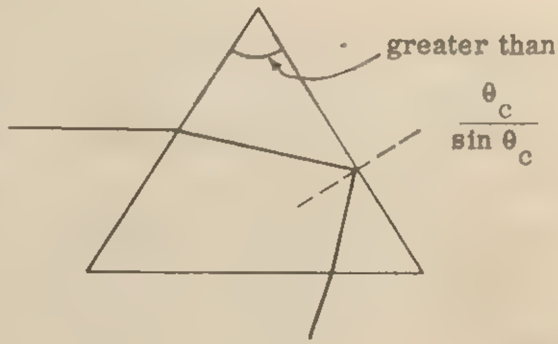
EMPHASIS Students should understand this section. This should not take long because of their previous experience both with refraction and in approximating a parabolic mirror with plane mirrors.

COMMENT By the time students reach this section, most of them will be able to use Snell's law to trace light through a prism. However, some students will not readily have a qualitative feeling for whether the light bends "up" or "down" unless you point out explicitly that a light ray bends toward the thicker part of the glass. You can mention this frequently. However, even after students are familiar with this mnemonic, they should occasionally apply Snell's law at each refracting surface of a lens.

CAUTION It is best not to mention waves or wave fronts at this time. Later in the course, students will see that the bending of light rays toward the thicker part of the glass can be remembered by considering what would happen to a set of parallel wave fronts if they encountered a region in which their speed is changed. Introduction of wave fronts at this time is not necessary, and jumps considerably ahead of the text's development. At this point we are not trying to explain light, but rather to note and describe how light behaves.

DEVELOPMENT Some students may not fully understand how prism combinations produce convergence. You can clarify the idea and start a stimulating class discussion if you ask a series of questions such as the following:

1. How would a change in the vertex angle of a prism affect the bending of light? Does a "fat" prism or a "thin" prism bend light more? (You will need to be careful about making the prism too "fat". For light parallel to the base of the prism, when the vertex angle of the prism exceeds $\frac{\theta_c}{\sin \theta_c}$, total internal reflection occurs.)



2. How would you arrange a selection of glass pieces such as

to concentrate a beam of light most intensely near a line?

(Many students will know immediately that

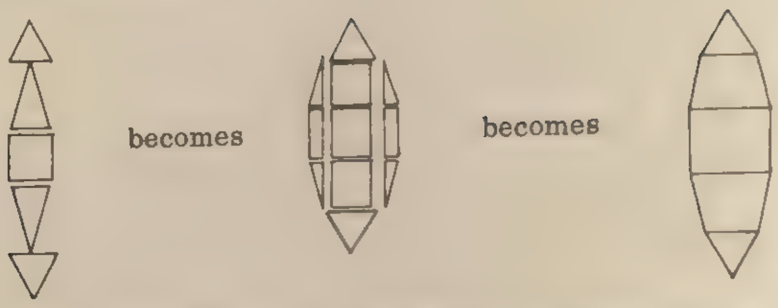
will work the best. Here is a place to call on slower students.)

3. What would happen to the direction of emerging light if  were cut in half and replaced by  ?

4. What would happen if  were replaced by  ?

5. How can you make  have a smoother outer surface?

Students should realize that

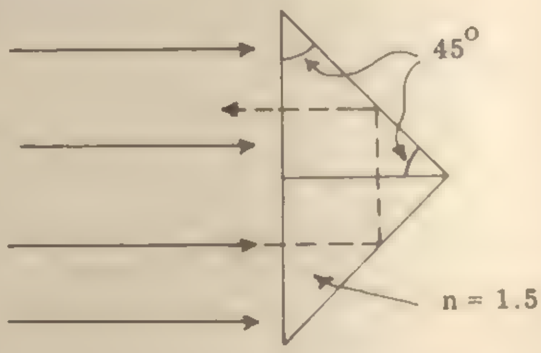


(Note: This process of "building up" a smooth lens is essentially the opposite of the development of the idea of a Fresnel lens. See Problem 3 on page 234.)

COMMENTS If a student brings up the problem of different colors (dispersion), you can answer either that this effect is small (refer to Chapter 11, Section 6), or that he should assume for now that the light is of a single color. In Section 7, the effects of chromatic aberration are mentioned.

The statement that a lens is a device which can redirect a light beam through refraction (lines 3 and 4 of Section 1) is not intended as a technical definition, but as an initial approximation. A more precise definition is not needed at this point.

Problem 2 will provide a surprise and will remind the students that not all prisms always refract light. If you assign this problem, be sure to point out, after students have done it, that total internal reflection will not be involved in the lenses they will study in Chapter 14.



Problem 1 asks students to determine the approximate "focal length" of a "lens" constructed of three prisms. This is a good exercise to accompany this section. Even though the problem asks for the focal length to just one significant figure, you may need to emphasize that precise mathematical treatment of a physical approximation is not warranted.

Section 2 - Lenses

PURPOSE To describe the physical characteristics of lenses, and to indicate some of the related optical properties.

CONTENT a. For precise focusing of light to a line by thin cylindrical lenses, the "ideal" surface is very close to circular.

b. Lenses with two spherical surfaces (thick in the middle, thin around the edges) converge light at a point. If an incident beam is parallel to the lens axis, the light converges at a point on the axis called the principal focus. The distance from the principal focus to the center of the lens is called the focal length of the lens.

c. For thin lenses, the focal length is the same no matter which side of the lens the light enters. Decreasing the radius of either or both of the spherical surfaces of

a lens shortens its focal length. Relationships between the structure of a lens and its focal length are expressed quantitatively in the lensmaker's formula.

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where n is the relative index of refraction of the lens, and R_1 and R_2 are the radii of the two spherical surfaces. (While not mentioned in the text, you may want to note that, for a lens immersed in a medium, $n = \frac{n_{glass}}{n_{medium}}$.)

EMPHASIS Treat the qualitative ideas thoroughly. Do not require the students to memorize the lensmaker's formula. It is enough if they know its qualitative features. Do not introduce a sign convention to deal with diverging lenses.

EARLY LAB Many teachers have found that a brief, informal, and early laboratory exercise is extremely helpful in introducing the student to spherical and cylindrical lenses, focal length, etc. Without this, some students do not properly interpret the shape of lenses from the cross-sectional diagrams. Students should handle lenses and they should actually see both focal points and images. After this type of laboratory exercise many students can read the text more intelligently. This would be a good time to do Experiment II-4. It is somewhat more quantitative and leads to the mathematical results of Section 3.

If you are short of time and do not do this experiment because of its similarity to Experiment II-2, it will be especially important to give students a few minutes to get the "feel" of lenses and prisms either in class or laboratory.

Able students will probably enjoy understanding the $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relationship as well $S_i S_o = f^2$; but even for able students, you will be far safer to stick with $S_i S_o = f^2$ until it is thoroughly understood.

COMMENTS The similarity between the development of the spherical lens from the cylindrical lens, and the development of the spherical mirror from the cylindrical mirror, should be stressed.

Able students may wonder why the difference between the ideal surface and the spherical surface in the case of lenses does not seem to be emphasized to the degree it was for mirrors. This subject is discussed for mirrors in Chapter 12, Section 9, and for lenses in Section 7 of this chapter. You can explain that spherical surfaces are used for convenience of manufacture, and that, for most applications, good lenses are usually made by combining several lens elements of different shapes and different indices of refraction. The types of glass used, and the shapes of the elements in such compound lenses are suitably chosen to minimize the major troubles for a particular application. Non-spherical surfaces are intentionally used in some fine optical systems, and also, surprisingly, in some very crude ones where the lenses are cast rather than ground and polished (e.g., condensing lenses in some projectors and photographic enlargers).

CAUTION Stress the qualitative features of the lensmaker's formula. If the light is bent more, the focal length is shorter; therefore a "bulgier" lens (or one with smaller radii) has smaller values of f. Similarly, a large relative n decreases f. A good way to stress these qualitative features is to ask the class what happens in the limiting cases (e.g., if the surface is plane, i.e., R gets larger and larger; if n approaches 1; etc.).

Do not give students many problems in which they merely substitute numbers in the lensmaker's formula. If you do, they will merely memorize the formula and may lose sight of its qualitative features--which are the important ones for students to use.

DEVELOPMENT You will not need to give students a positive and negative sign convention for dealing with concave surfaces or negative, diverging lenses. Even in advanced courses, the sign conventions often cause confusion. You will not have trouble with the formula as long as you restrict it to converging lenses.

You may want able students to treat negative or diverging lenses as, for instance, in Problem 11. In any case, you will want to defer such discussions until after Sections 3 and 5. The lensmaker's formula can be used if both surfaces of the lens are concave. $\frac{1}{R_1}$ is added to $\frac{1}{R_2}$. The fact that this lens causes light to diverge is known to students from the fact that it is thin in the middle. Of course with careful ray tracing students can handle any lens– and, in some cases, even reason out the application of the formula.

Problem 6 is an example of a case where n in the lensmaker's formula requires not the index of refraction for glass in air, but rather the relative index for glass in water. In the formula you should use $n = \frac{n_{glass}}{n_{water}}$. See Section 13-6.

Lenses in which one surface is concave and the other convex are called <u>meniscus lenses</u>. For such lenses, since the two surfaces give opposite effects, $1/R_2$ is subtracted from $1/R_1$ (or vice versa). If such a lens is thicker in the middle than at the edges, it is converging, or "positive". If the edges are thicker than the middle, it is a diverging lens (and may be called "negative").

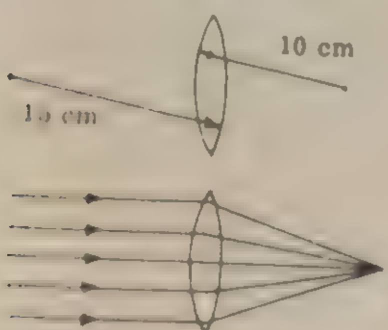
When a student begins a new problem he should decide two things qualitatively before he considers the formula:

a. Is the lens converging or diverging?

b. Do the two surfaces of the lens "help each other"?

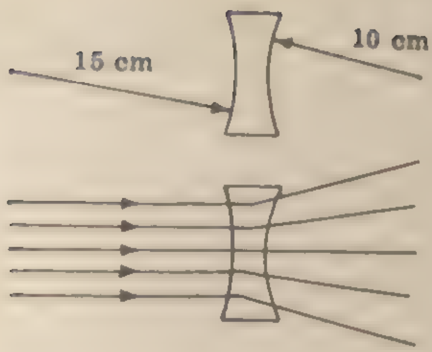
He should then decide whether he needs to multiply (n – 1) by $\frac{1}{R_1} + \frac{1}{R_2}$, or $\frac{1}{R_1} - \frac{1}{R_2}$ (or $\frac{1}{R_2} - \frac{1}{R_1}$).

Here are some examples. $n = 1.5$.



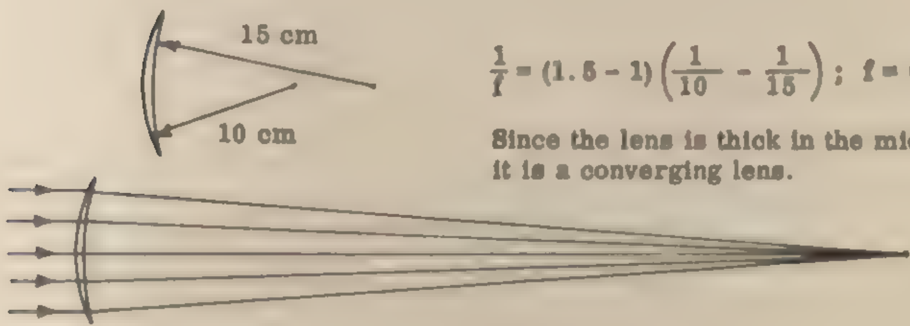
$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10} + \frac{1}{10}\right)$; $f = 12$ cm.

Since the lens is thick in the middle, it is a converging lens.



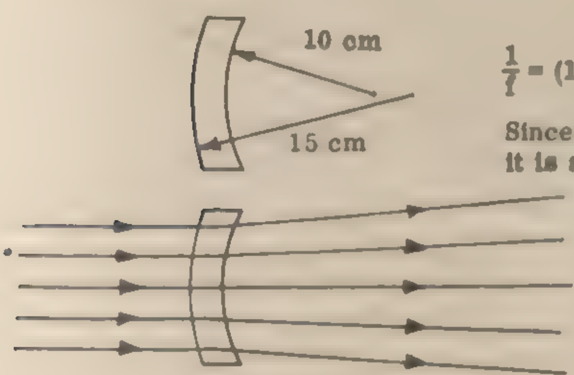
$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10} + \frac{1}{10}\right) \; ; \; f = 12 \text{ cm.}$$

Since the lens is thicker at the edges, it is a diverging lens.



$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10} - \frac{1}{15}\right) ; \; f = 60 \text{ cm.}$$

Since the lens is thick in the middle, it is a converging lens.



$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10} - \frac{1}{15}\right) ; \; f = 60 \text{ cm.}$$

Since the lens is thicker at the edges, it is a diverging lens.

Derivation of the Lensmaker's Formula. At the end of Part II, pages 302-303 of the textbook, a derivation of the lensmaker's formula is given. However, as indicated in the footnote on page 228, since this derivation employs concepts from the study of waves, it probably should not be discussed at this time. If some of your students press for a derivation, you could show them (outside of class) a derivation similar to the one given in Appendix 3 at the back of this volume of the guide. This derivation involves only Snell's law, geometry and radian measure.

Section 3 - Real Images Formed by Lenses

PURPOSE To show the application of ray diagrams to the location of images formed by converging lenses.

CONTENT a. Two principal rays can be used to locate the real image produced by a converging lens.

1. One principal ray leaves the object traveling parallel to the lens axis. At the lens it is bent so as to go through the "far" principal focus.

2. The second principal ray leaves the same point on the object but travels through the "near" principal focus. This ray is bent by the lens so that it travels parallel to the axis after leaving the lens.

b. The image point is the point at which these two principal rays meet. All other rays (going through an ideal thin lens) would also go through this point.

c. The ray diagram and simple geometry can be used to relate image and object position and size. $S_i S_o = f^2$.

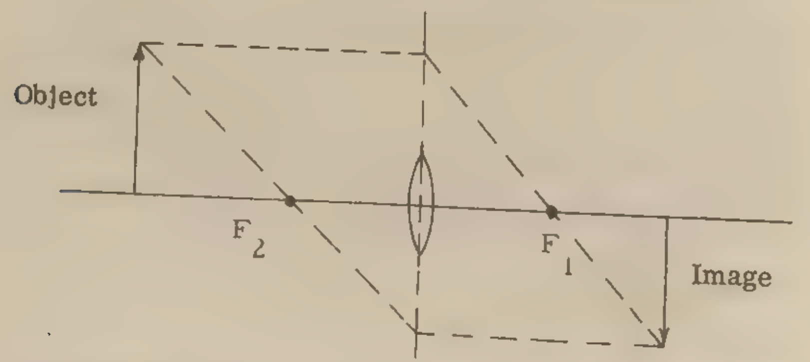
EMPHASIS Work through this section thoroughly until students can handle problems easily. Emphasize the ray diagrams. Discourage routine, unthinking use of the formula $S_i S_o = f^2$. Concentrate on converging lenses--at least until the students understand them well. A firm grasp and consequent retention of this section are more important than the applications which appear in the rest of the chapter.

DEVELOPMENT The central purpose of this section will not be served if you proceed too quickly to the problems. Most students need some class discussion of ray diagrams if they are to work through the problems with understanding. You might want to begin a discussion with questions such as:

1. Why are the ray diagrams drawn as though the light ray is unbent until it reaches the center of the lens? Is a light ray really bent sharply, or does it bend gradually all of the time it is in the glass? If not gradually, where does the bending actually occur?

Although students should know that the light bends only at a face between two materials, and although the thin lens approximation is explained in column 1 on page 229, you will find some students who cannot answer these questions. Some fail to notice the change in drawing from "surface bending" to "center bending" as they go from Figures 14-4, 14-5 and 14-6 (pages 228-229) to Figure 14-7 (page 229).

2. How can you find the image for the object shown below which is larger than the lens?



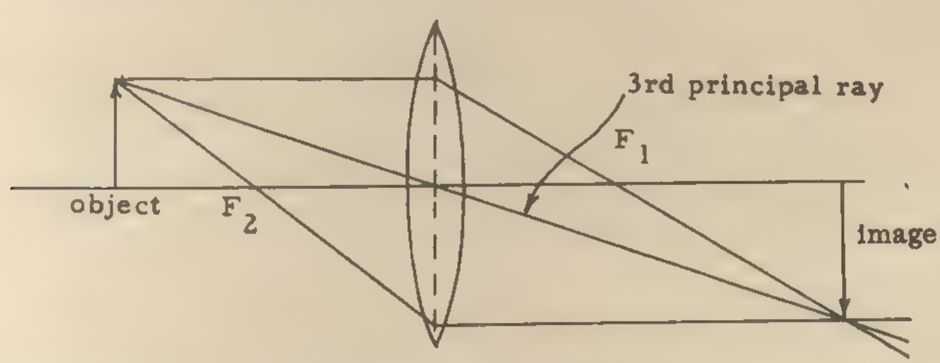
(This is equivalent to asking students to explain and justify the diagram of a camera in Figure 14-8, page 230.) Students may at first be perplexed by the fact that the principal rays can still be used to locate the image even though they do not travel through the lens.

One way to reinforce student understanding here is to point out (perhaps by questioning, see Problem 16 on page 236) that replacing a small diameter lens with a larger lens of the same focal length, does not change the position or size of the image. It only makes the image brighter. Thus if we have a lens which is too small to accept the principal rays, we can imagine for the purpose of image location that the small lens is replaced by a sufficiently larger one of the same focal length. Thus the dotted lines of Figure 14-8 on page 230 allow us to construct geometrically the point at which all the rays in the shaded cone will intersect.

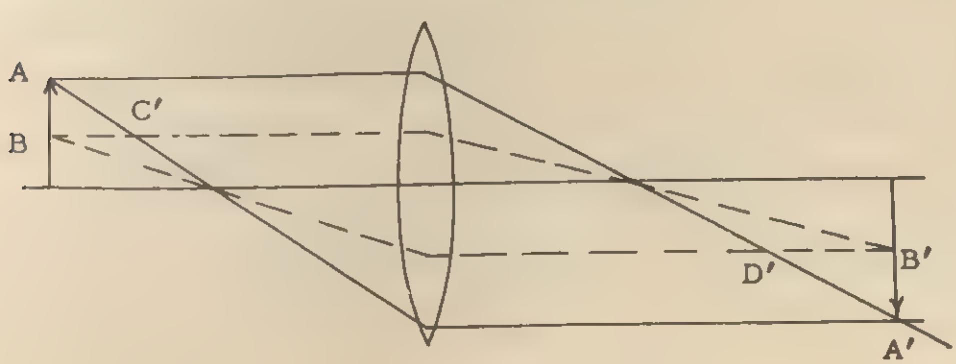
Another approach is to start with a large diameter lens and to locate the image in a conventional way. Then ask the students to draw several other rays closer to the center. Finally, you can point out that if the edges of the lens are blocked off or cut, the central rays still focus at the same point.

A demonstration might be in order in which part or nearly all the lens is covered by a piece of opaque paper or cardboard and yet except for changes in brightness, the image does not change in size or position.

3. Which light ray is not deviated? Your class should realize that since there is a ray that is bent down and another which is bent up, some ray in between should leave the lens in the same direction it entered. Some students may suggest, by symmetry, that this undeviated ray is the one that goes through the center of the lens. See if they also realize that this must be true for the center rays near the axis of a thin lens because the opposite sides of the lens are nearly parallel at this point and therefore act like a plane piece of glass. This undeviated ray through the center of the lens is very useful for constructing ray diagrams of images and is often called the third principal ray:



4. Does the location of a single image point (such as shown in Figure 14-7, page 229) locate the entire image? Some students may not understand this question because they do not realize that at least two points on a straight-line image are needed to locate the image. It would be worthwhile to have students check by drawing principal rays from two points on the object:



You can easily test their understanding of this by having them locate the image of a tilted object.

5. How would you find the ratio of image to object size (i.e., H_i/H_o) if you know the distances from the object and image to the lens? Try to get the students to think about the geometry of a ray diagram. If they know about the principal ray through the center of the lens, they can write the answer directly from similar triangles:

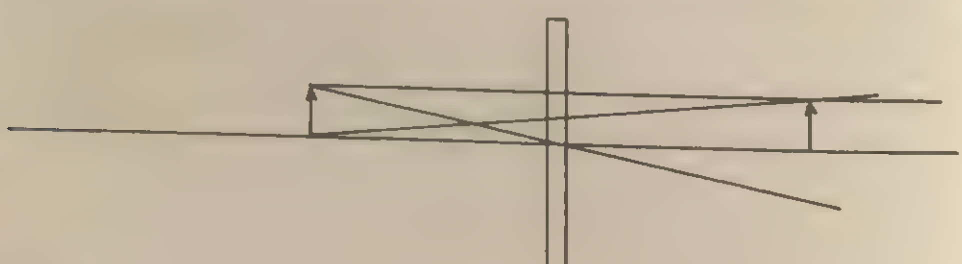
$$\frac{H_i}{H_o} = \frac{\text{image distance to lens}}{\text{object distance to lens}}.$$

Of course, the ratio H_i/H_o can be derived easily by using the formulas they know, $\left(\frac{H_i}{H_o} = \frac{f}{S_o} = \frac{S_i}{f}\right)$. For example, they can get a convenient expression for f/S_o by adding 1 to each side of the equation $\frac{f}{S_o} = \frac{S_i}{f}$. Thus $\frac{f}{S_o} + \frac{S_o}{S_o} = \frac{S_i}{f} + \frac{f}{f}$. This becomes $\frac{f + S_o}{S_o} = \frac{S_i + f}{f}$, or $\frac{f}{H_o} = \frac{S_i + f}{f + S_o} = \frac{\text{image distance to lens}}{\text{object distance to lens}}$.

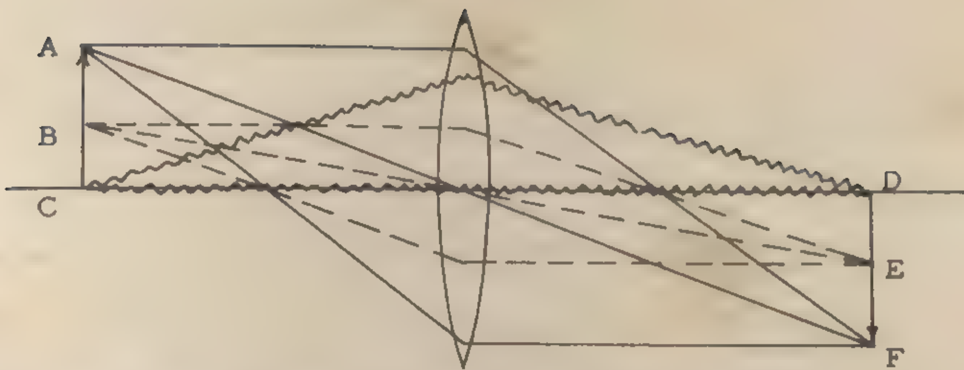
Students often do much more algebra to get this result and are impressed by the simple, geometric derivation involving the central principal ray.

COMMENTS A good test of student understanding of the location of images by tracing rays can be based on Problem 5 on page 234, or introduced separately. Suppose that you have a thin lens made of ordinary window glass (except that the surfaces are more accurately plane) and an object as follows:

What is wrong with the following ray diagram and corresponding location of the "image"?



If your students dispose of this question in short order, fine! If they do not, remind them that only the intersection of rays which <u>leave the same point</u> is useful in locating an image point. You can emphasize this by drawing a figure on the board like the one below. Draw (<u>in colored chalk</u>) all of the rays leaving point A in red, all leaving point B in yellow, and all leaving point C in white. (Since this is not color printing, we are using various kinds of dashing to represent the colors.) Image points such as D, E, and F are the ones to be found when an image is to be located.



Although this drawing looks like a mess in black and white, it will be very clear in a drawing using colored chalk. You will find that some students remember for a long time that the image occurs where "rays" of the same color intersect.

* * *

The second principal focal point (F_2 of Figure 14-7 on page 229) is introduced for the first time in this section. Some students may be a little uncertain as to what F_2 is. F_2, by definition, is the point at which parallel light incident on the right of the lens (moving to the left) will be focused. (The sixth sentence in column 2, on page 228 is the definition of F_2; however, some students think that F_2 has been defined earlier and that they should know from other reasoning that parallel light would be focused at F_2.) The reversibility referred to in the following sentence merely indicates that a ray going from left to right through F_2 emerges parallel to the axis. Reversibility does not necessarily mean that F_1 and F_2 are at equal distances from the lens. These distances are the same for a "very thin lens", if F_1 and F_2 are in the same medium.

Section 4 - Camera, Projector, and Eye

PURPOSE To discuss with the aid of ray diagrams, several devices in which a single lens is used to form real images.

CONTENT a. A converging lens produces a real image of an object when the distance between the lens and the object is greater than the focal length of the lens. Three general situations are considered:

1. For extremely distant objects, the real image is close to the principal focus and smaller than the object.

2. For closer objects, as long as S_0 is longer than f, the image is beyond the principal focus (but S_i is shorter than f) and the image is smaller than the object.

3. For objects close to the principal focus (S_0 shorter than f), the image is larger than the object and at a distance from the lens that is large compared with the focal length (S_i longer than f).

b. Focusing arrangements vary. In the camera and projector, a lens of fixed focal length is moved back and forth. In the eye, focusing is accomplished by changing the curvature of the lens (hence its focal length and converging power).

EMPHASIS The emphasis in this section should be on the action of the simple lens in each of these devices. Review with ray diagrams how a simple lens forms an image and how the position and size of the image can be found. Although students should learn the basic principle of these devices, you will not be able to afford the time for a detailed

excursion into the biophysics and psychology of the eye, or into the details of photographic lenses. Able students with interests in these areas can be encouraged to search out supplementary references.

COMMENTS Work on this section should be principally in informal laboratory and problems. Problems 9 and 10 are appropriate.

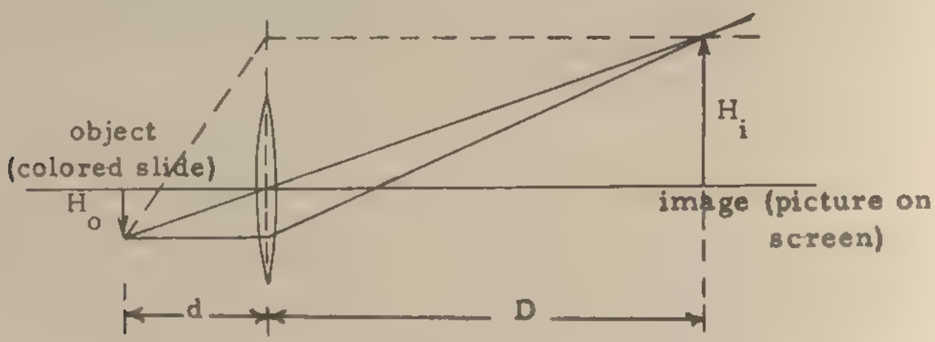
Students will be in fairly good command of image formation when they see beyond the bare formula $S_iS_o = f^2$ to its meaning for the geometry of images: the product S_iS_o is fixed by the constant f^2 for any lens. Thus, if S_i increases, S_o must decrease, and vice versa.

DEVELOPMENT If time permits and you feel a discussion is needed, illustrate with ray diagrams examples which clarify the relationship between the focal length and the distances and sizes of objects and images. Use ray diagrams for objects placed, say, at 510 cm, 110 cm, 35 cm, 20 cm, 14 cm, and 11 cm from a 10 cm focal length lens. Repeat using a 5 cm focal length lens. Which arrangements approximate the usual use of a camera? A projector? The eye?

Here is a problem for a good student:

> What focal length lens would have to be used in a projector if a colored slide 24 mm x 36 mm is to be magnified to 4 feet x 6 feet in a living room which is 25 feet long?

If the central principal ray is understood, the solution is simplified. The diagram shows by similar triangles that $H_i/H_o = D/d$.



H_o = 24 mm, H_i = 4 feet and the distance D may be about 20 feet. Then,

4 feet/24 mm = 20 feet/d; d = 120 mm.

(Note that no units have to be changed to get d in millimeters.)

The distance d is very close to the required focal length, and for all practical purposes a focal length of 120 mm would be sufficient. However, the exact value can be obtained from $d = S_o + f$ and $H_i/H_o = f/S_o$. Substituting, and eliminating S_o gives

$$\frac{H_i}{H_o} = \frac{f}{d-f}; \quad f = \frac{d}{1 + H_o/H_i} = \frac{120 \text{ mm}}{1 + 24/(4 \times 304.8)} = 117.6 \text{ mm}.$$

(The factor 304.8 is the number of millimeters in a foot. Here we must convert units.)

This problem illustrates the statement at the end of paragraph 2, Section 4 that a lens of small focal length is useful for photographing small objects. The length (and bulk) of the photographer's camera is usually limited just as the projectionist is limited by the length of his living room. For a "life-sized" photographic image, a camera with a lens of 25 cm focal length can be half the length of a camera with a 50 cm focal length. For a projector in a room with a limited projection distance, the shorter the focal length of the lens, the larger the image.

Supplementary Information on Camera Lenses. While not intended for presentation to your class, Appendix 4 at the back of this volume describes briefly some of the characteristics and aberrations of photographic lenses.

Supplementary Information on the Eye. Background information on the optical characteristics of the eye will be found in Appendix 5 at the back of this volume.

Section 5 - The Magnifier (or Simple Microscope)

PURPOSE To show how a virtual image is formed by a converging lens when it is used as a magnifying glass.

CONTENT a. If an object is placed between a converging lens and its focal point, the image is on the same side of the lens as the object, enlarged, farther from the lens than the object, and virtual (i.e., the light coming from the object through the lens only appears to have come from the image; it does not actually travel through the image).

b. The apparent size of an object depends on its actual size and its distance from the observer; the apparent size can be conveniently expressed as the angular size. Magnification increases the angular size.

EMPHASIS The formation of a virtual image by a single converging lens should be treated thoroughly. The material on magnifications should be treated only if you have time to do it in some detail.

COMMENTS Students can understand the characteristics of the virtual image if you review the ray diagram shown in Figure 14-10 of the text on page 231. This diagram makes the three characteristics of the image obvious, particularly if you add the third principal ray (from the object through the center of the lens). Note that the light appears to be coming from the image, but that the light going from the object to the eye never actually passes through the image. In this sense the image is virtual. The principal ray "through F_2" does not actually pass through F_2 but rather reaches the lens from a direction as if it had come from F_2. (It might be wise to use a pattern of language such as "The virtual image seems to be located . . ." rather than ". . . is located . . ." when first talking about virtual images.)

The distance of most distinct vision for students (with normal vision) may be as small as 7 cm to 11 cm. This distance will be smaller for nearsighted students and greater for farsighted students. You can reassure most students that they have not made a gross error if they find their distance of distinct vision to be much less than 25 cm.

Notice that the term magnification is not defined precisely, but it is always used consistently to mean angular magnification. The magnification, as used in the text would be

$$M = \frac{\dfrac{\text{Size of Final Image}}{\text{Distance from Final Image to Eye}}}{\dfrac{\text{Size of Object}}{\text{Distance from Eye to Where the Object can be Viewed}}}$$

Some misunderstanding may arise because for the magnifier (last sentence on page 231, column 1, continued in column 2), the two distances are the same, so that the angular magnification reduces to H_i/H_o. In some texts, the ratio H_i/H_o is called "lateral" magnification. There is no need to introduce this new term to your students. Also many texts derive the magnification of a magnifying glass under conditions where the virtual image in infinitely far off. In this case, the magnification is $M = \frac{d}{f}$ instead of $M = \frac{d}{f} + 1$.

DEVELOPMENT If you want your students to understand magnification well, you should give them a few examples which illustrate angular size in cases not involving lenses or

optical instruments. Would a thin rod 5 feet tall and 40 feet away look bigger or smaller than a 7 foot rod 56 feet away? (You must not think about familiar-sized rods seen against backgrounds containing familiar-sized objects. Imagine the rods on a beach, or in the middle of a frozen lake.) If you usually sit 50 feet away from a standard movie screen, how much bigger should the screen of a drive-in movie be to give you your normal view when your car is 200 feet from the screen? How far from a television set should you sit to have the same "view" as in a theater?

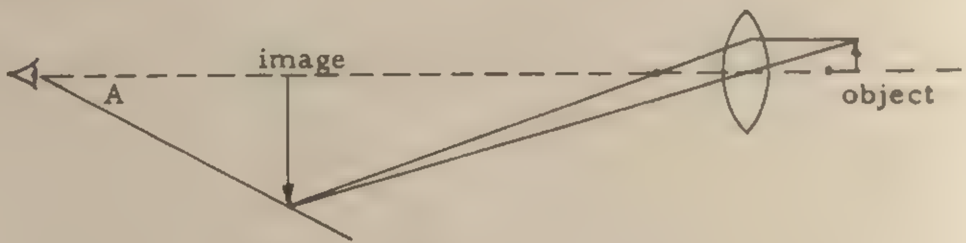
After a few examples such as these, students realize quickly that one of the clues to the apparent size of an object is the angle made by the rays from the eye to the edges of the object. Students will also realize how reasonable this is if they think of these rays as principal rays through the center of a thin eye lens, which (because they are undeviated) will determine the size of the image on the retina.

Once the concept of angular size has been established, you can reduce the purpose of most optical instruments to that of helping the eye produce an image with a larger effective angular size.

* * *

One possible way of stimulating the class to think about angular magnification is to discuss another way--seldom actually used by anyone--of using a high power, short focal length lens as a magnifier. You should not discuss this without actually showing the students the phenomenon.

This second way is to place the lens at a distance from the object slightly greater than the focal length of the lens. A large real image will be formed. You then just look at this image with your eye. Presumably you will place this image roughly 25 cm from your eye. A ray diagram is shown along with an eye simply looking at the object from a distance of 25 cm.



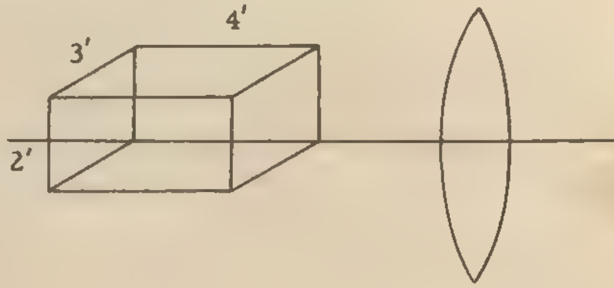
Obviously, A can be made much larger than B if enough distance is allowed between the object and the eye. In fact, A can be made as large as you like. Is this a "better" way of using a magnifying glass? Let your students try it and see.

To answer the question, hold a high power glass (f less than 2") right on the object, which might be a letter on a typed page, and slowly raise it. The enlarged virtual image discussed in this section will be seen at first. Then the image gets blurry as the focal distance is passed. Finally, an inverted image will appear. This is the real enlarged image spoken of above. By playing with the lens a little, the students will convince themselves that the magnification (angular) can be made very large, but that this is not a "good" way to use a lens since they see very little of the object. In technical terms, the field of view is too narrow. (See Figure 14-13, page 233 for an illustration of this use of the magnifier.)

* * *

For the brighter students: All of the work with lenses and image formation has dealt with two-dimensional images. When we project an image on a flat screen or take a picture on flat film, we are concerned only with two-dimensional images. However, when you look through a microscope or telescope, the real image viewed with the eyepiece is a three-dimensional image. (On the retina it becomes a "two-dimensional" curved image.) You will probably not want to discuss this question with your class, but it may come up when you are not expecting it. For example, if you demonstrated a real image of a bouquet of flowers (or any similar real image) with a concave mirror, the three-dimensional aspect of the image was clear to students as they viewed it. In a camera, a three-dimensional image is formed in space. A two-dimensional "slice" of this image is captured on the film or on a ground glass screen.

If you are looking for more involved work for capable students you can use this notion of a three-dimensional image to construct hard problems. For example, find the shape, size, and volume of the image of a box as shown at the right. Imagine the "box" is a frame of wire.



Section 6 - The Compound Microscope; Telescopes

PURPOSE To show that lenses can be used to form images which may then be viewed through other lenses. This is the case in compound microscopes and telescopes.

CONTENT a. The objective lens of a compound microscope forms an enlarged, real image of the object which is then examined through a simple magnifier called the eyepiece.

b. The objective lens of a telescope forms a small but bright, real image of the object which is then examined through a simple magnifier called the eyepiece.

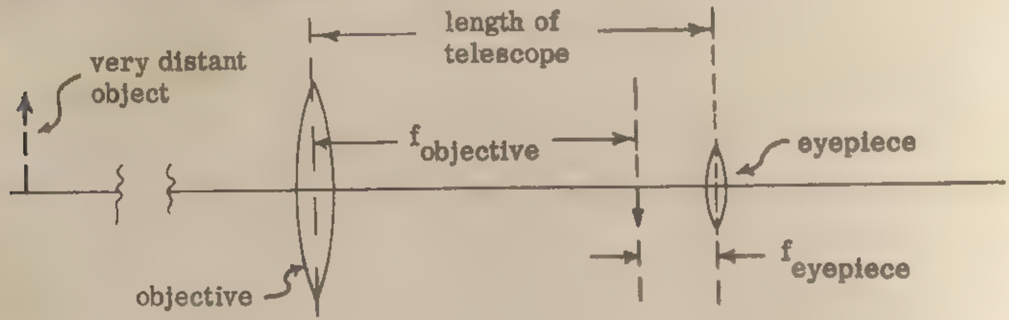
EMPHASIS Treat briefly. This section is simply an application of what has gone before. Aside from the central idea of the microscope and telescope--producing as large a real image as is practical, then viewing an enlarged virtual image of the real image--there are no important facts which students need to learn. Students do not need to learn magnification formulas.

DEVELOPMENT An effective way to demonstrate the whole idea behind compound optical systems, and the telescope in particular, is to announce to your class that you are now going to look at the view from the window by forming an image of it on a piece of ground glass or waxed paper. Do this with a lens of moderate focal length, (25 cm). Become interested in some detail and ask how you could see it better. One way is to produce a larger image by substituting a longer focal length lens. How could you see it even better? Look at it with a magnifying glass. You may want to emphasize the effect of short focal length for the magnifier by substituting ones of higher power. As you examine the image, complain of a spot on the ground glass. So you remove the ground glass. You have just invented the telescope!

The qualitative idea of looking at the image formed by one lens with a second lens is the whole idea of this section. It would be perfectly proper to stop with this or you may wish to carry things a little further by doing a quantitative example or two.

Students will enjoy simulating a telescope in the laboratory if you have extra time. Although some students may want to use cardboard tubes and fancy focusing devices in later work, very simple arrangements illustrate the point adequately. Use, say, a 30 cm lens and a 5 cm lens on an optical bench. Finding the image from the 30 cm lens on tissue paper will help students get the telescope working. Examine the image with the short focal length lens and you have the telescope. If you have a good, short focal length eyepiece, students will enjoy viewing their images with it. If you don't have short focal length eyepieces, perhaps you can borrow some microscope eyepieces from the biology laboratory.

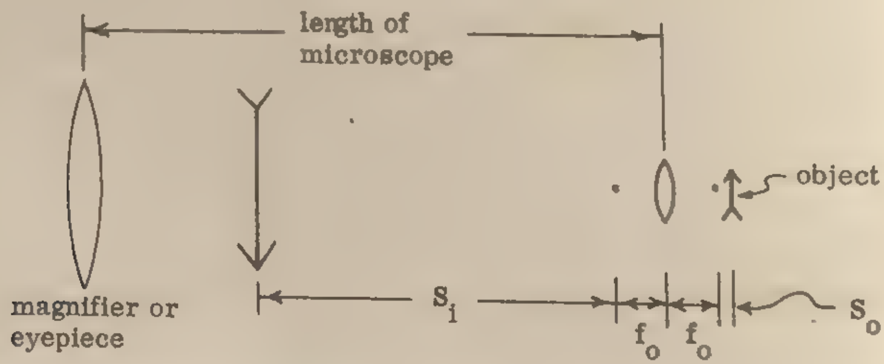
COMMENT From their recent study of magnifiers, students readily see that a small value for f is desirable for the viewing lens of a telescope, but some students have difficulty seeing that a large value of f is desirable for the objective lens. This large f value is helpful because the telescope is used to view objects at large fixed distances. Since S_o cannot be reduced, the largest image is produced by a large f $(H_i/H_o = f/S_o)$. (If a student suggests using an almost plane piece of glass, you can remind him that this would require a telescope a few miles long!) The tube of the 40 inch refracting telescope at the Yerkes observatory is 60 feet long.



$$\text{Angular Magnification} = \frac{f_{objective}}{f_{eyepiece}}$$

In a microscope, the situation is different. Here $S_i + f$ of the objective plus f of the magnifier must be contained within the length of a practical-sized instrument. Of these three values, S_i of the objective lens contributes most to the length of the microscope tube.

S_o of the objective lens can be made very small.



Thus, whereas S_o is the "fixed" distance in the telescope, in the microscope the "fixed" distance is S_i. For any given practical length for $S_i + f$, the largest image can be produced with an objective lens of short f. Even if a "long" microscope were convenient to use, the increased magnification would result in a dimmer image which is a disadvantage because the amount of light that can be put on or through the object being viewed is limited.

The angular magnification of a microscope is given by

$$M = \left(\frac{S_i}{f}\right)_{\text{objective}} \times M_{\text{eyepiece}}, \text{ where } M_{\text{eyepiece}} = \left(\frac{d}{f} + 1\right) \text{cm.}$$

(d = distance of closest distinct vision.)

Section 7 - Limitations of Optical Instruments

PURPOSE To indicate that our treatment of lenses has been approximate and that images formed by simple lenses have many defects.

EMPHASIS Treat very briefly, perhaps as a reading assignment coupled with part of a laboratory exercise.

DEVELOPMENT Do not try to discuss lens defects in detail. Instead you might suggest that students look for these defects in the laboratory. An image formed on ground glass by a good anastigmatic camera lens can be compared with the image formed by a simple lens of about the same focal length. The difference in image quality is not (necessarily) due to more accurate spherical surfaces of the camera lens, but is due to the use of just the right combination of spherical surfaces and different varieties of glass in the good lens. If you have a monochromatic source, it is instructive to compare the two lenses using a single color.

COMMENT Some students try to reproduce the effects shown in Figure 14-13 using their own eyeglasses. This leads to confusion because

a. Many of the lenses are divergent (for nearsighted students) while others are cylindrical (to correct astigmatism). Only spherical convergent lenses will work well.

b. The focal length of most eyeglasses is so long that students' arms are not long enough to get their eyes farther from the lens than the focal point (as they must to see an inverted image).

Chapter 14 - Lenses and Optical Instruments

For Home, Desk and Lab - Answers to Problems

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Problems 18 and 19 are difficult problems that might be used near the end of your work on this chapter.

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 14-20 to 14-36.

Section	Easy	Medium	Hard	Class Discussion
1		1*, 2		1
2	4	3, 5*	6	3, 6
3	9*	7*, 8, 10	11	7, 11
4		7*, 10		7
5	12			
6	14*	8	13, 15, 18*, 19*	9, 13, 15
7	16, 17			

SHORT ANSWERS

1. a) Between 10 cm and 22 cm.
   b) No.
2. Total internal reflection. Emerge parallel to incident rays.
3. (Fresnel lens.) See discussion on page 14-21.
4. 20 cm.
5. a) 10 cm; infinite.
   b) See discussion on page 14-23.
6. a) Greater.
   b) 80 cm.
7. a) 59.
   b) 10.17 cm.
8. See discussion on page 14-25.
9. a) 0.93×10^{-2}.
   b) 9.3 mm.
   c) 10^{-2}
10. 6.6×10^{-6} cm.
11. a) 6.67 cm.
    b) See discussion on page 14-29.
    c) See discussion on page 14-29.
    d) See discussion on page 14-30.
    e) See discussion on page 14-30.
12. a) 1.5.
    b) 2.5.
    c) 16.
    d) 151.
    e) Graph.
13. 680.
14. 0.015 cm.
15. See discussion on page 14-32.
16. See discussion on page 14-33.
17. a) 0.964.
    b) No.
18. 40 cm from the lens.
19. a) No.
    b) Yes.

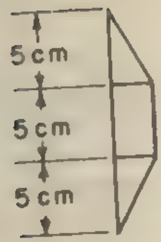
COMMENTS AND SOLUTIONS

PROBLEM 1

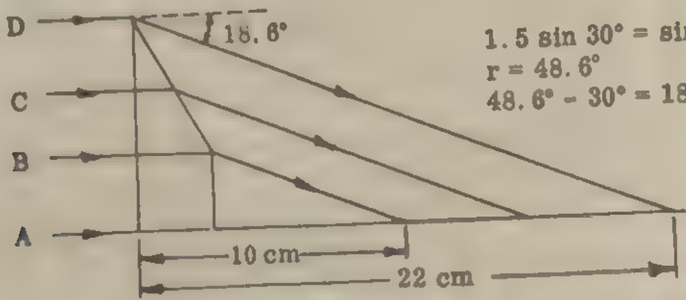
A crude converging lens can be constructed by placing two 30°-60°-90° glass prisms together with a glass block as shown in Fig. 14-15.

(a) What is the focal length of this "lens" to one significant figure?

(b) Would such a lens form a clear image? Explain.



Students can solve this problem by finding the angle of refraction through the application of Snell's law, then either computing the "focal length" using a trigonometric function or measuring the "focal length" from a scale drawing. The results are dependent upon whether the light is incident on the flat face or the opposite faces, but, to one significant figure, this does not matter.



$1.5 \sin 30° = \sin r$
$r = 48.6°$
$48.6° - 30° = 18.6°$

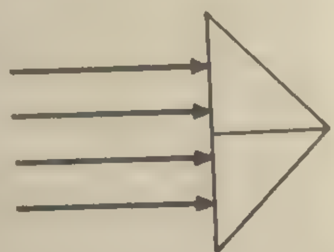
a) The central ray A goes through undeflected. If an index of refraction of 1.5 is used for the glass, the three rays B, C, and D cross A at distances of approximately 10, 16, and 22 cm from the flat face. An answer of 15 cm is pretty good, but, to one significant figure, either 10 cm or 20 cm should be accepted.

If the student has read Section 2 he may try to use the lensmaker's formula, using an approximate radius of curvature. The "value" of the radius of curvature is about 10 cm, giving a focal length of about 20 cm.

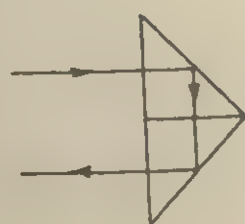
b) This "lens" would not form a clear image since parallel light is not focused to a single point, but is merely somewhat concentrated.

PROBLEM 2

If two 45° prisms of glass (index = 1.50) are arranged as in Fig. 14-16 they will not converge parallel light. Why not? What will happen to the light?



The two prisms shown will not converge light since light entering one will be totally reflected twice and emerge parallel to its initial direction. After this problem has been done, make it clear that total internal reflection is almost never important in lenses. It does prevent the making of some very bulgy types.



PROBLEM 3

Some lighthouses and light buoys mark the positions of dangerous rocks and shoals. The light must be concentrated at a low angle with respect to the horizon (light directed upward is wasted) and must be equally visible from all points of the compass.

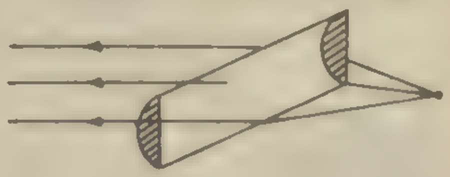
(a) Can you design a "lens" which will do this?

(b) Instead of using a continuous curved surface, such lights often use a lens made of sections of prisms. Can you draw a diagram of such a lens? It is called a *Fresnel lens* after the French physicist who first devised such a lens.

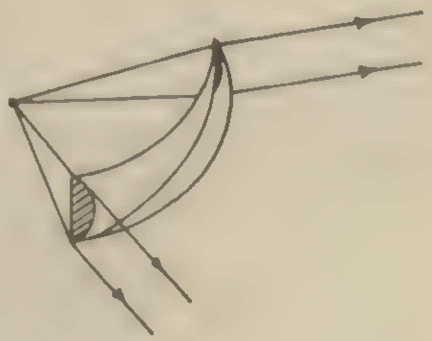
(c) Automobile headlights are constructed to give a wide, flat, horizontal beam. Parabolic reflectors are made to give a narrow beam which passes through a Fresnel lens in the front of the headlight. Examine an automobile headlight and see if you can understand how it gives broad, horizontal beams.

This problem can be used to stimulate a class discussion. It is probably better for class discussion than home assignment.

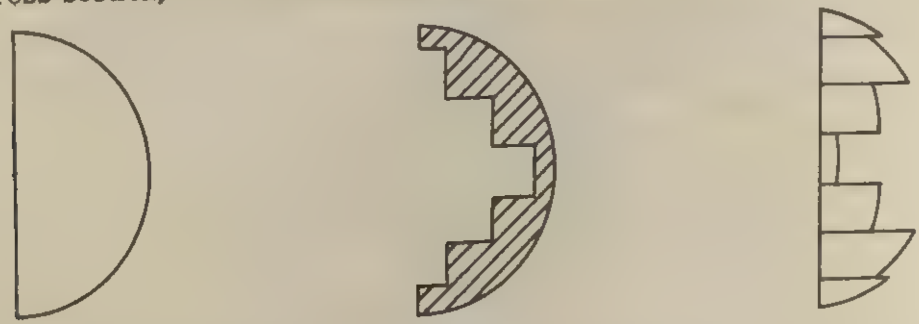
a) Light from a single source can be bent into a parallel band with a cylindrical lens. However, the source cannot be at the correct distance from all vertical sections of the cylindrical lens unless the lens is bent around with the source at its center. If the lens is bent completely around



the source, a band of light will go out in all directions— exactly what is needed in a lighthouse.



b) In cross section, the lens we have just made looks like that at (a).

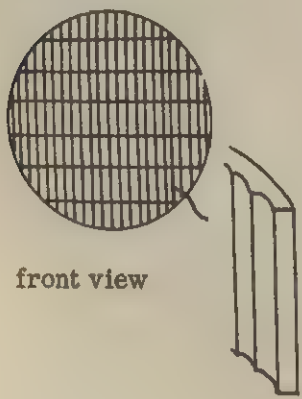


(a) (b) (c)

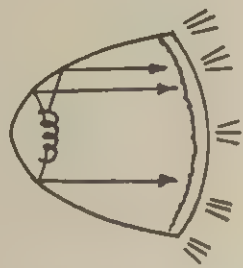
In a large size this would be quite bulky. We could make a lens with similar characteristics by removing glass as in (b). Except for the small amounts of light falling on the horizontal surfaces, each section of the lens at (b) will refract in the same way as its corresponding section at (a). Finally, the lens sections can be shoved together as at (c). A lens like that at (c) would be hard to make with a good optical finish, but a high degree of optical precision is not required for lighthouse lenses, railroad signal lights, and the like. They can be cast to shape directly as at (c).

c) Automobile headlight

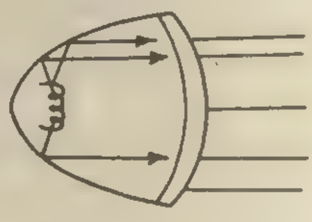
The parabolic reflector produces nearly parallel light. Different makes of lenses differ slightly but, in general, they are made up of segments, each of which is a small cylindrical lens. These segments diverge the incident light to left and right, but not up and down.



front view



view from above



side view

PROBLEM 4

Use the Lens Maker's Formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

to find the focal length of a glass lens ($n = 1.50$) with one flat surface and one with a radius of 10 cm. (Such a lens is called a *plano-convex lens.*)

We have $n = 1.5$; $R_2 = 10$ cm. The flat surface does not contribute since R_1 is very large.

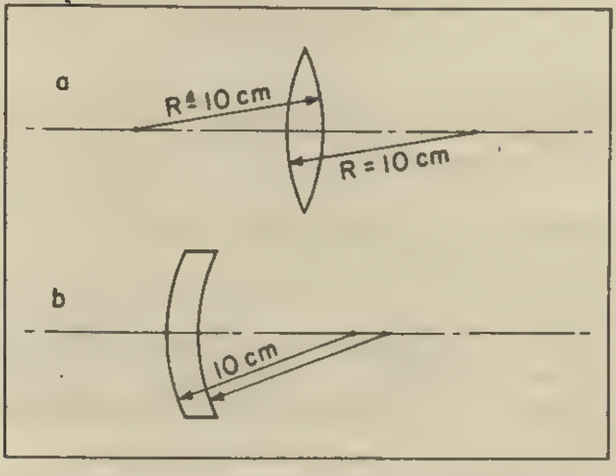
$$\frac{1}{f} = (1.5 - 1.0) \times \left(0 + \frac{1}{10\text{cm}}\right) = \frac{1}{20\text{cm}}$$

$f = \underline{20 \text{ cm}}$.

The lens converges light because it is thicker in the middle than at the edges.

PROBLEM 5

(a) What are the focal lengths of the two lenses shown in Fig. 14-17? (Index of glass = 1.50.)
(b) How does the focal length of (b) compare with a flat block of glass?



a) In the cases of both lenses, we must make a qualitative decision about whether R_1 and R_2 "help" each other. For the first lens, since both curvatures contribute to making the lens thicker in the middle,

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10\text{cm}} + \frac{1}{10\text{cm}}\right) = \frac{1}{10\text{cm}}$$

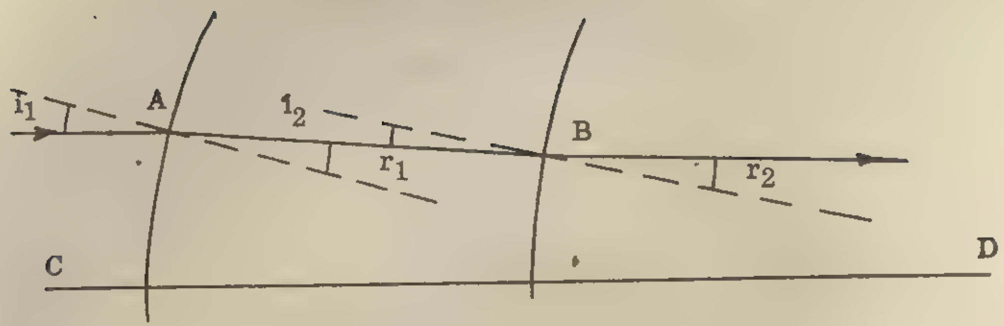
$$f = \underline{10\text{ cm}}.$$

For the second lens the two curvatures "hurt" each other, one converging and one diverging the rays. The lens is uniformly thick and the reciprocal radii should be subtracted.

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10\text{cm}} - \frac{1}{10\text{cm}}\right) = 0$$

f becomes very large (or becomes <u>infinite</u>).

b) Some students will say that the pair of similarly curved surfaces will not be different from a flat plate of glass. Although this is not quite correct for a thick curved plate, it is an adequate solution; there is not much difference. However, incident rays parallel to the axis entering from the left will be slightly convergent when they leave the lens.



For the incident ray and the leaving ray to be parallel, i_2 and r_1 must be equal. When this is the case $\frac{i_2}{r_2} = \frac{r_1}{i_1}$, $r_2 = i_1$. In the case of a thick curved plate, the point of incidence B at the second surface is closer to the axis CD than is the point of incidence A at the first surface. The normals to the incident points are not parallel, and i_2 is smaller than r_1. It follows that r_2 is smaller than i_1, and the leaving ray is directed slightly toward the axis.

Avoid a discussion of whether you call a uniformly thick piece of glass (curved or not) a lens. Physicists would not call something a lens unless it focused a small axial beam.

PROBLEM 6

A lens (index = 1.50) has a focal length in air of 20.0 cm.

(a) Is its focal length in water greater or less than in air?

(b) What is its focal length in water?

Hint: Notice that every individual refraction depends on the relative index of refraction.

The lensmaker s formula is given as $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, in which n is the relative index of the lens material and the medium in which it is immersed. Thus (n-1) should be replaced by $\left(\frac{n_{glass}}{n_{water}} - 1\right)$, when the lens is immersed in water.

a) Since (n-1) for the lens immersed in water is less than the same factor when the lens is immersed in air, the focal length will be greater when the lens is in water.

b) If we use $n_g = 1.500$, $n_w = 1.333$, then

$$\frac{\frac{1}{f_{air}}}{\frac{1}{f_{water}}} = \frac{(n_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{\left(\frac{n_g}{n_w} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \frac{n_g - 1}{\frac{n_g}{n_w} - 1} .$$

The factors containing R_1 and R_2 cancel because the surfaces have the same curvatures in the two cases. Then

$$\frac{f_{water}}{f_{air}} = \frac{n_w (n_g - 1)}{n_g - n_w} = \frac{1.333\,(1.5-1)}{1.5 - 1.333} = 4.$$

Hence the focal length in water is $4 \times 20 = \underline{80 \text{ cm}}$.

PROBLEM 7

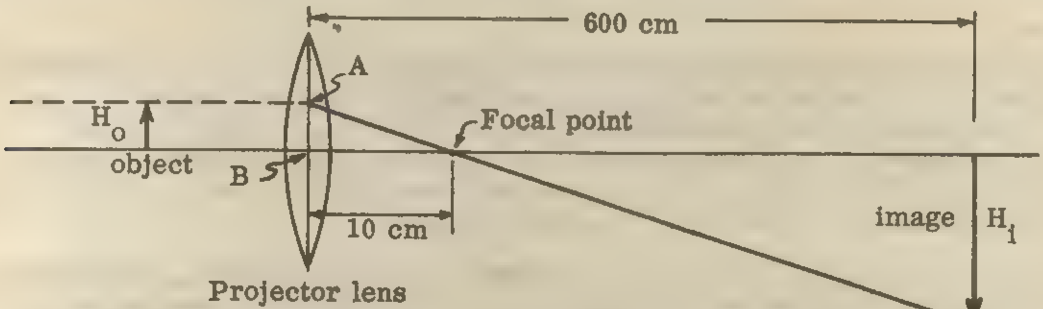
A lens whose focal length is 10 cm is used in a slide projector to give a real image on a screen at a distance of 6.0 meters.

(a) What will be the magnification?

(b) How far is the lens placed from the slide?

If you have not previously done so, you should point out that magnification here means H_i/H_o.

a) Students should draw a ray diagram of a projector. Students who have a little familiarity with cameras and projectors should have no trouble with this. Students who are completely unfamiliar with cameras and projectors may be able to solve this problem easier after having studied Section.4.



Use a principal ray originally parallel to the axis so that $H_o = AB$. Then by similar triangles

$$\frac{AB}{10} = \frac{H_i}{600-10} \text{ or } \frac{H_i}{H_o} = \frac{590}{10} = \underline{59.}$$

If a student puts this on the board and merely uses $\frac{H_i}{H_o} = \frac{S_i}{f}$, ask him to (1) draw a diagram, (2) label S_i, f, H_o, H_i, (3) put numbers on diagram, and (4) ask him to show that the formula is correct from geometry. The point of this is to stress the ray diagram and the geometry.

b) To find how far the lens is from the slide, draw a principal ray from the bottom of the image through the lens center to the top of the object and using similar triangles get

$$\frac{d}{600} = \frac{H_o}{H_i} = \frac{1}{59} \text{ or, } d = \frac{600}{59} = \underline{10.17 \text{ cm}}.$$

Using this construction, the focal point near the object need not be identified.

A similar construction using the focal point nearest the object would have given the same answer.

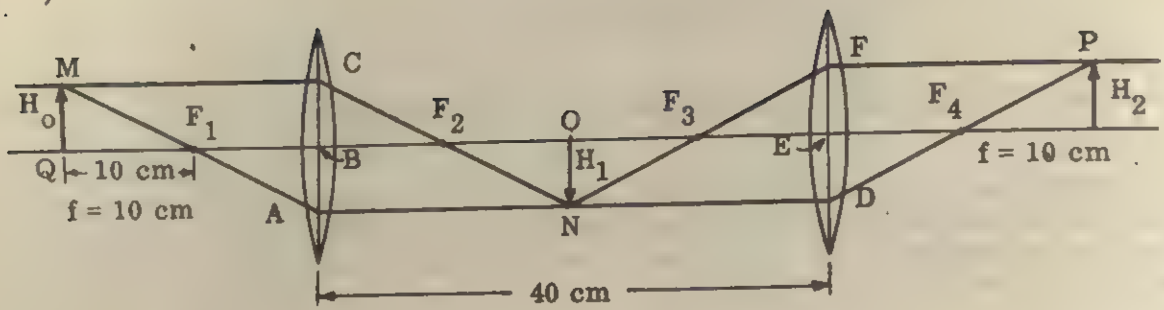
PROBLEM 8

Prove that if two identical converging lenses of focal length 10 cm are placed 40 cm apart, the combination will form an upright image of an object that is 20 cm away from the first lens and the magnification will be 1.

This problem can be done by good students with Section 3. Alternatively, it can be used in class to introduce Section 6, or it can be assigned after Section 6 is studied.

In this problem the main challenge to the students is in drawing a correct ray diagram, since the subsequent geometry is relatively simple. Most of them will draw the principal rays through the foci, although the principal rays through the lens centers would simplify the solution.

a)



Before you proceed to a final quantitative proof, the over-all picture should be pointed out to the students. The object of height H_o has an image of height H_1 formed by the first lens. This image in turn acts as an object for the second lens which forms a second image of height H_2. The image H_1 can act as an object for the second lens because light rays come from it as if there were a real object there. This may be a convenient spot to review the meaning of an optical image.

There may be questions about the meaning of the principal ray diagram. The principal ray MAN helps locate the image H_1 formed by the first lens. The principal ray NDP helps locate the final image H_2. It happens that a real light ray could travel along MANDP. Notice that MCN also locates H_1 and NFP locates H_2, but that no light ray could follow the path MCNFP. In fact, the ray NFP, though useful for constructional purposes could not really exist since it would not have come through the first lens. Notice in particular that the principal ray NFP for the second lens is not a principal ray for the first lens. The diagram (b) shows an actual bundle of rays that forms the arrowhead. Note that some principal rays are among them and others are not. A principal ray can be used to do a construction even though it may not represent a real light ray passing through the optical system.

b)

The proof is as follows:

$$\frac{BA}{BF_1} = \frac{H_o}{QF_1}.\quad \text{But, } BA = H_1,\ BF_1 = f = 10 \text{ cm, and } QF_1 = 20 - 10 = 10 \text{ cm.}$$

Then $H_1 = H_o$. It is then easy to show that OB = QB = 20 cm. Then the second lens is handled exactly the same way to show $H_2 = H_1 = H_o$. The ray diagram shows the final image is erect.

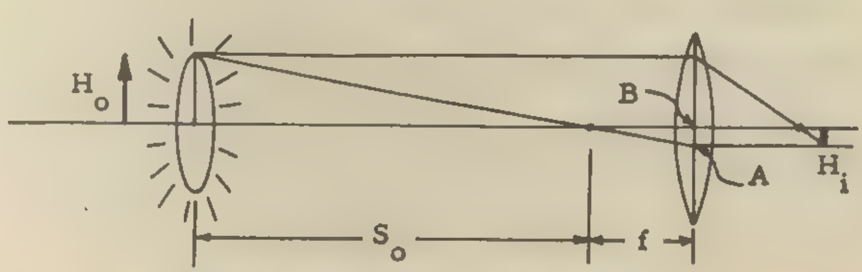
PROBLEM 9

(a) Prove that the size of the image of the sun produced by a convex lens is proportional to the focal length. What is the constant of proportionality?

(b) How large an image of the sun (diameter. 1.4×10^9 m) will be formed by a lens of focal length 1.0 meter?

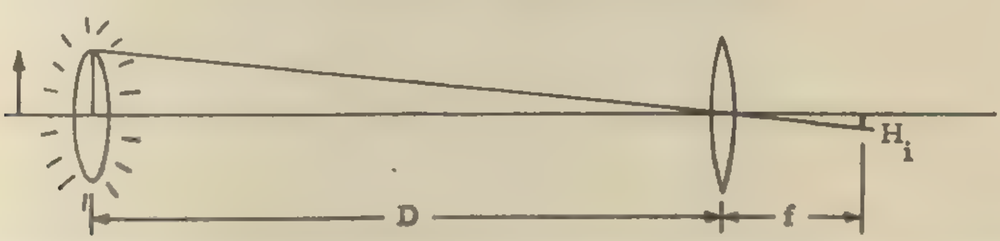
(c) What will be the ratio of the size of the images of the sun formed by a lens of 10 cm focal length and a lens of 10 m focal length?

This problem can be done with Section 3 or used as an introduction to telescopes in Section 6.



By similar triangles $\frac{AB}{f} = \frac{H_o}{S_o}$, and since $H_i = AB$, $H_i = f\frac{H_o}{S_o}$. Since H_o and S_o are constants, H_i (the image size) is proportional to f. The distance from the earth to the sun is 1.5×10^{11} meters, $H_o = 1.4 \times 10^9$ meters (using the diameter for H_o gives H_i as a diameter instead of radius), and f = 1 meter. $k = \frac{H_o}{S_o} = \frac{1.4 \times 10^9 \text{ m}}{1.5 \times 10^{11} \text{ m}} =$ $\underline{0.93 \times 10^{-2}}$.

Note: The use of the central principal ray simplifies this problem.



Once it is pointed out that the image is at the principal focus of the lens--and hence a distance f from it, similar triangles give directly $H_i = f\frac{H_o}{D}$.

b) $H_i = AB = 0.93 \times 10^{-2} \times 1 \text{ m} = \underline{9.3 \text{ mm}}$.

c) The ratio of the sizes of the images is just $\frac{f_1}{f_2} = \frac{10 \text{ cm}}{1000 \text{ cm}} = \underline{10^{-2}}$.

PROBLEM 10

How large an image will be formed of an artificial satellite (53 cm in diameter) passing at an altitude of 500 miles, if it is photographed with a camera whose focal length is 10 cm? Would you expect an actual photograph to show a larger or smaller image than the size you have calculated?

Use the same construction as for Problem 9 and obtain $H_i = f\,\frac{H_o}{S_o}$.

$$H_i = 10\text{ cm}\ \frac{53\text{ cm}}{500\text{ miles} \times 5280\text{ ft/mi} \times 12\text{ in/ft} \times \frac{100\text{ cm}}{39.37\text{ in}}} = \underline{6.6 \times 10^{-6}\text{ cm}}.$$

The actual photograph would give a larger image:

(a) One developed grain on the film is larger than this.
(b) Diffraction effects. See later chapters.
(c) Lens defects such as spherical aberration.
(d) The speed of the satellite is fast enough to blur the image into a streak even for very high shutter speeds (the satellite moves about 5 meters in 1/1000 sec), thus, in 1/1000 sec the image spot would move a distance across the film nearly 10 times the image diameter.

This second part of the problem can be used to stimulate a short class discussion if you have time. It is not essential, but reminds students of lens defects and other factors which influence images. If you mention the streaking of the satellite you might add that the time exposures needed for photographing stars requires that the telescope mount have a correcting mechanism which counteracts the earth's motion, keeping the telescope trained on the same field of view. Or you might ask students how they would photograph a faint star and see if they can "invent" this correcting mechanism.

PROBLEM 11

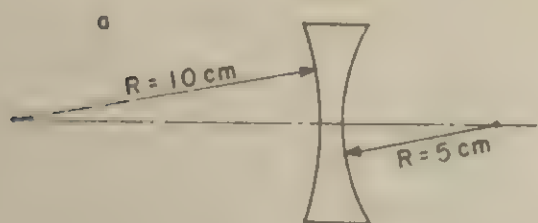
(a) What is the focal length of the lens in Fig. 14-18? (Index of the glass is 1.50.)

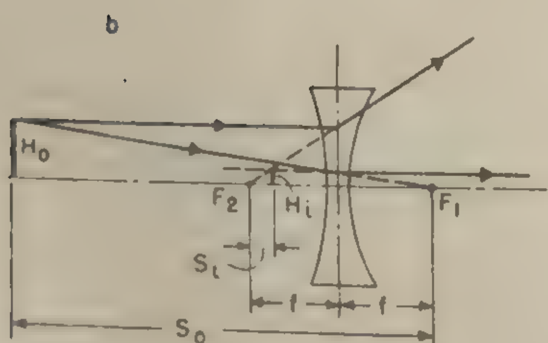
(b) By sketching the paths of some light rays, show what the lens does to incident light parallel to its axis.

(c) From the ray diagram of Fig. 14-18 (b), show that $S_oS_i = f^2$. Notice from which focal points S_o and S_i are measured.

(d) What happens as you move the object toward the lens? Can S_i ever get bigger than S_o? Is the image ever bigger than the object?

(e) How would you find (experimentally) the focal length of a diverging lens?



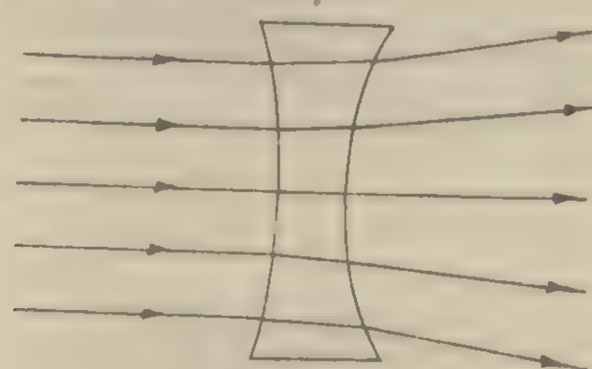


This is a rather difficult problem in the application of the lensmaker's formula and ray diagrams to a diverging lens. In principle it could be assigned after Section 3, but it is probably better to wait until your class has a thorough knowledge of ray diagrams. The students need not memorize the results of this problem.

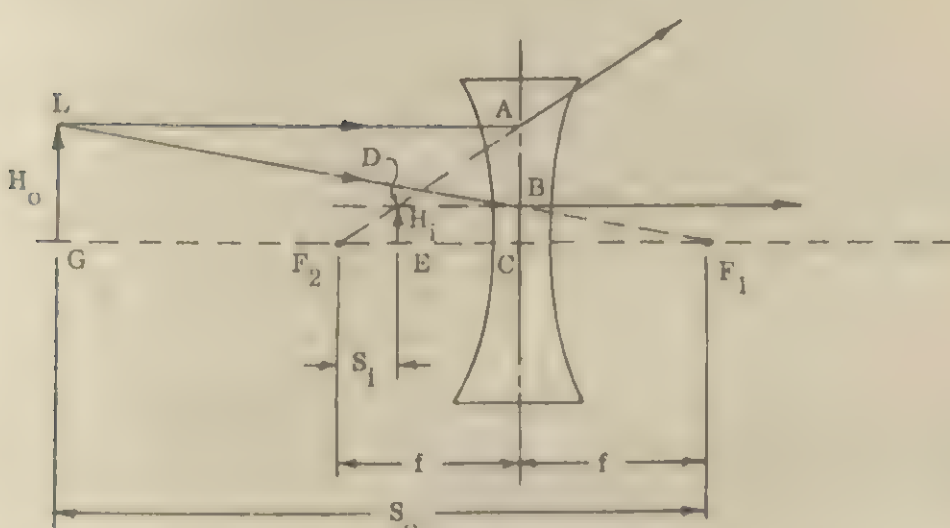
a) By the lensmaker's formula, with $n = 1.5$; $R_1 = 10$ cm, $R_2 = 5$ cm and since both surfaces produce divergence, (i.e., they make the lens thinner in the middle) we add $\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. $\frac{1}{f} = (1.5)\left(\frac{1}{10\text{ cm}} + \frac{1}{5\text{ cm}}\right) = 0.5 \times \frac{3}{10} = \frac{3}{20}$; $f = \underline{6.67\text{ cm}}$.

Since this is a diverging lens we often write $f = -6.67$ cm.

b)



c)



Triangle F_2DE is similar to triangle F_2AC. therefore, $\frac{H_i}{AC} = \frac{S_i}{f}$. Since $AC = H_o$,

$$\frac{H_i}{H_o} = \frac{S_i}{f}. \qquad (1)$$

Also, triangle F_1BC is similar to triangle F_1LG, so that, $\frac{BC}{H_o} = \frac{f}{S_o}$. Since $BC = H_i$,

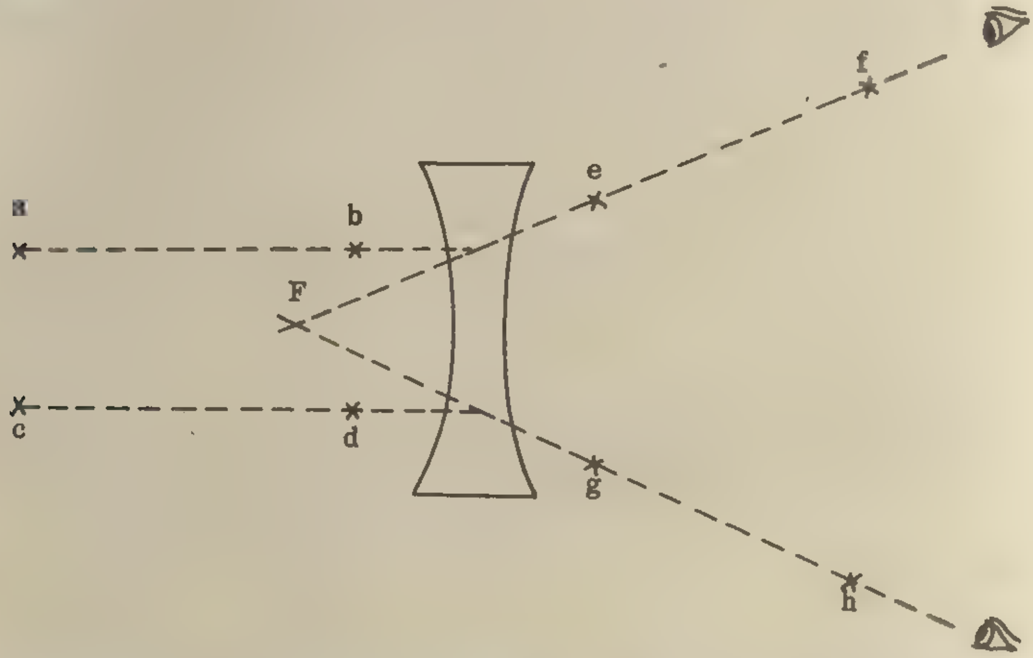
$$\frac{H_i}{H_o} = \frac{f}{S_o}. \qquad (2)$$

Combining (1) and (2), $\frac{S_i}{f} = \frac{f}{S_o}$, $S_oS_i = f^2$.

With a converging lens, a ray from the object drawn parallel to the principal axis is refracted by the lens so that it passes through a principal focus. It is from this focus that the image distance S_i is measured. Similarly, the ray parallel to the axis of a diverging lens is refracted by the lens and appears to have come from a point on the axis, one of the principal foci. It is from this point that the image distance is measured. In a like manner, a ray from the object directed towards the other principal focus is bent parallel to the axis upon passing through the lens. It is from this focus that the object distance is measured.

d) In the diagram for part c), as we move H_o toward the lens, B moves up toward A. The lens always causes rays to diverge. Hence, the image is always on the same side of the lens as the object. But since B can never get higher than A, $BC = H_i$ can never be greater than $AC = H_o$. As H_i moves up the line F_2A we see that S_i approaches F_2C as a maximum; while S_o approaches CF_1 as a minimum. Thus, S_i can never get bigger than S_o.

e)



Place four pins a, b, c, d, so that ab and cd mark parallel rays entering the lens. Sight from the other side and place the two pins e and f so as to line up with the apparent line a and b. Place two more pins, g and h, so as to line up with c and d. The lines ef and gh, when extended back, cross at the principal focus, F.

PROBLEM 12

Assume your distance of most distinct vision is 15 cm. What is the maximum magnification that can be obtained with each of the following convex lenses when used as a magnifying glass or simple microscope?

(a) $f = 30$ cm,
(b) $f = 10$ cm,
(c) $f = 1$ cm,
(d) $f = 1$ mm.
(e) Graph the maximum magnification as a function of "f."

This "formula" problem on the magnification of a simple microscope is intended to give students a feeling for the amount of magnification possible with lenses of different focal lengths. Answers will vary depending on the value taken as the "closest distance of distinct vision". In the solution below, we have used 15 cm.

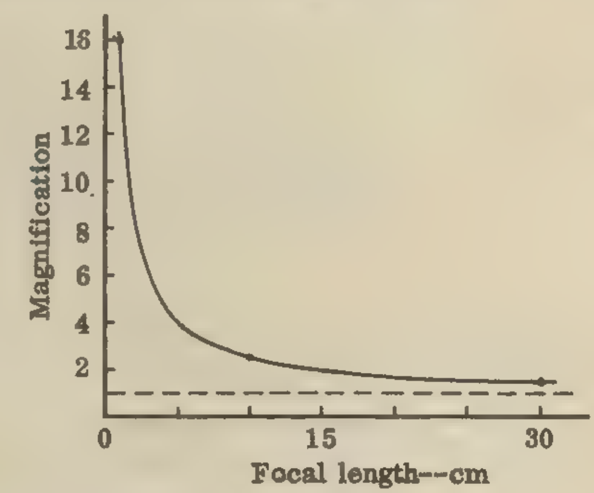
a) mag. $= \frac{15}{f} + 1 = \frac{15}{30} + 1 = \underline{1.5}$

b) $\underline{2.5}$

c) $\underline{16}$

d) $\underline{151}$

e)



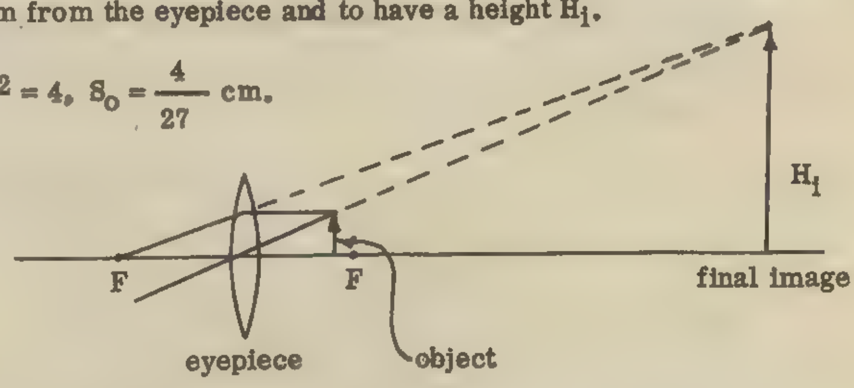
PROBLEM 13

Assume your distance of most distinct vision is 25 cm. A compound microscope has an eyepiece of 2.0 cm focal length and an objective of 4.0 mm focal length. The distance between objective and eyepiece is 22.3 cm. What is its magnification to two significant figures?

This problem can be solved with ray diagrams using similar triangles. Here, instead, we shall use formulas.

Consider the eyepiece and work backwards from the final image. We shall consider the final image to be 25 cm from the eyepiece and to have a height H_i.
Then $S_i = 25 + 2 = 27$ cm.

$$S_oS_i = f^2 = 2^2 = 4, \ S_o = \frac{4}{27} \text{ cm.}$$



The "object" for the eyepiece is thus $2 - \frac{4}{27} = \frac{50}{27}$ cm from the eyepiece. The relation between H_o and H_i is $\frac{H_i}{H_o} = \frac{S_i}{f}$, and $H_o = \frac{f}{S_i} H_i = \frac{2}{27} H_i$.

In order to reduce confusion when we now refer to the objective lens, we have used prime marks to indicate H_i, H_o, S_i and f for the formation of the image by the objective lens. The student should see that the "object" for the eyepiece is the image formed by the objective lens ($H_o = H_{i'}$). This image is 22.3 cm $- \frac{50}{27}$ cm from the objective lens.
$S_i = 20.45$ cm $- f = 20.45$ dm $- 0.4$ cm $= 20.05$ cm.

$$\frac{H_{o'}}{H_{i'}} = \frac{f'}{S_{i'}} = \frac{0.4\ \text{cm}}{20.05\ \text{cm}} = \frac{1}{50.1}$$

But, $H_{i'} = \frac{2}{27}\ H_i$, so $H_{o'} = \frac{2}{27 \times 50.1}\ H_i$, and $H_i = 675\ H_{o'}$. To two significant figures, the magnification is 680.

PROBLEM 14

Using the microscope of Problem 13, with the same adjustment, we see an amoeba. With a ruler, we measure the size of the virtual image by looking at it with one eye and at the ruler with the other. On the ruler the amoeba appears to be about 10 cm long. About how big is it really?

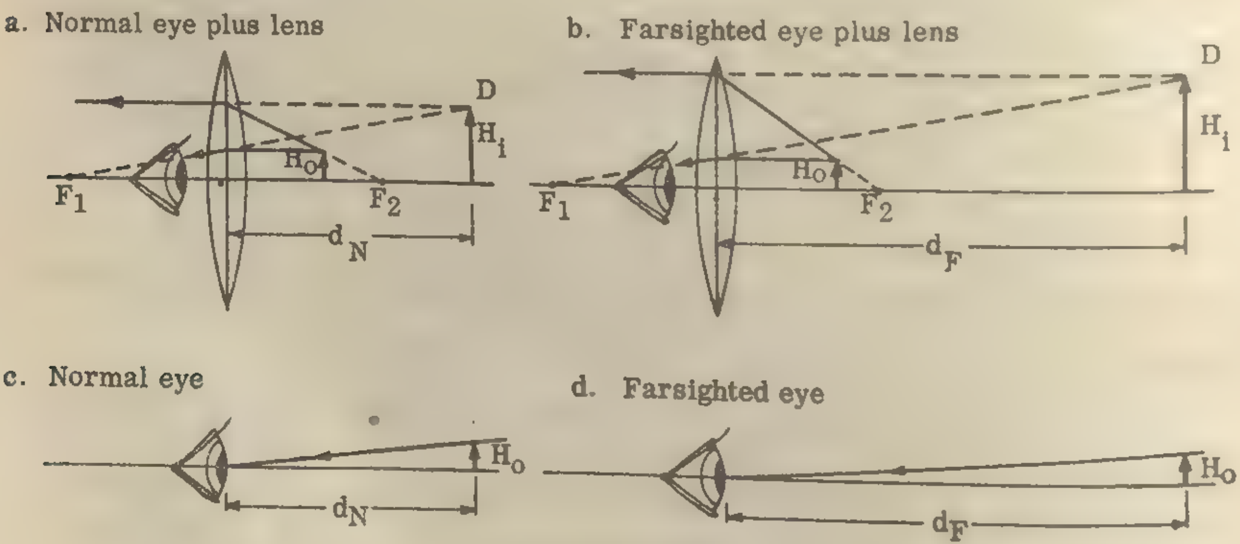
If we have a 680-fold magnification, an image 10 cm long means that the length of the amoeba is actually $\frac{10}{680}$ cm = 0.015 cm.

PROBLEM 15

For the maximum magnification of an eyepiece, we found $\frac{d}{f} + 1$ where d is taken as the distance of most distinct (or closest distinct) vision. If your eyes can accommodate to see distinctly at 15 cm, we should write $\frac{15\ \text{cm}}{f} + 1$ as the magnification of a simple magnifier for you. Also, if you can accommodate to images no closer than 35 cm, $\frac{35\ \text{cm}}{f} + 1$ would apply. Why does the magnification go up for someone who accommodates poorly at small distances? Does he see more detail than someone who can accommodate closer? Be prepared to discuss this question in class.

This is a good problem for class discussion.

In this problem the reference is to a virtual image formed by a converging lens, (Figures 14-10 and 14-11).



For a given object, the top of the image will always lie along the line F_1 D. Thus when either person places his eye close to the lens and moves the object to give a clear image he will see an image which subtends the same angle as the other person. See Figures a and b. On the other hand without the lens the person who accommodates poorly will see a smaller image than the other person when they each view the object at the distance of most distinct vision. See Figures c and d. Thus, for the person with poor accommodation the angle has been magnified more by the lens than for the person with good accommodation. Eyes which could accommodate at infinitesimally small object distances measured from the eye would not need a magnifier to see infinitely large images.

Through the magnifier both will see the same amount of detail as far as angular size is concerned, but since the object is farther from the lens for the person with poor accommodation the image will be less intense and thus some detail may be lost.

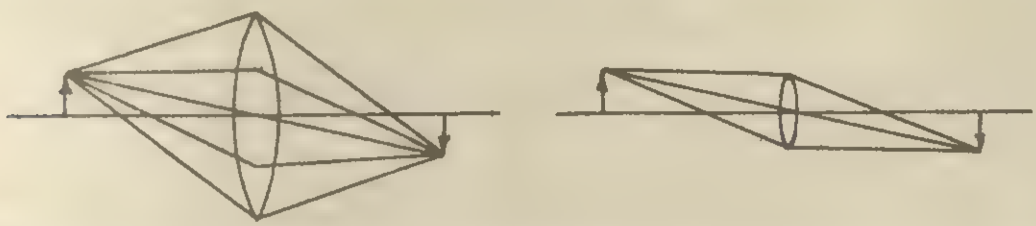
PROBLEM 16

Two lenses both have a focal length of 20 cm, but one has a diameter four times that of the other. Draw sketches of the two lenses and tell how the images they form differ.

a) Since the diameter of the lens does not affect the ray diagrams, the positions and sizes of images will be the same. The image formed by the larger lens would be 16 times brighter than that formed by the smaller one. For many applications this is an important factor. In making the lens it would be much more difficult to keep the larger lens free of aberrations and faults.

If the lenses were used as magnifying glasses the principal difference would be that more could be seen through the larger lens. It would have a larger field of view.

b)



Note the much greater amount of light concentrated by the larger lens on the image.

PROBLEM 17

(a) What is the ratio of the focal lengths of a crown-glass lens for violet light and for red light? (The index of refraction for various colors is given in Table 4, Chapter 13.)

(b) Is the ratio the same for all kinds of glass?

This problem leads to the ideas behind the making of achromatic lenses.

a) Using the lensmaker's formula, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, we have

$$\frac{f_v}{f_r} = \frac{(n_r - 1)}{(n_v - 1)}$$

From Chapter 13, Table 4, page 221, for crown glass, $n_r = 1.513$, $n_v = 1.532$,

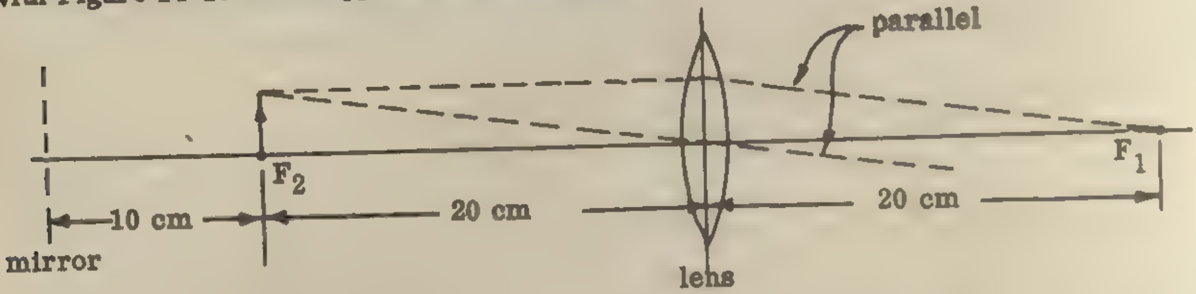
$$\frac{f_v}{f_r} = \frac{0.513}{0.532} = \underline{0.964}.$$

b) The ratio is not the same for different kinds of glass. The difference between different glasses makes possible the kind of correction for chromatic aberration shown in Figure 14-14.

PROBLEM 18

A lens of focal length 20 cm is placed 30 cm from a plane mirror and an object is placed on the axis 10 cm from the mirror. Where will the image of the object be found?

This problem could, in principle, be done after Section 3, but it is rather difficult and requires that students have a good grasp of ray diagrams and image formation. Students will need to recall that light from a point at F is refracted so as to be parallel upon leaving the lens (or rediscover this through drawing the principal ray through the center of the lens). The diagram is shown below (students should be warned not to work with Figure 14-19 which applies to Problem 19).

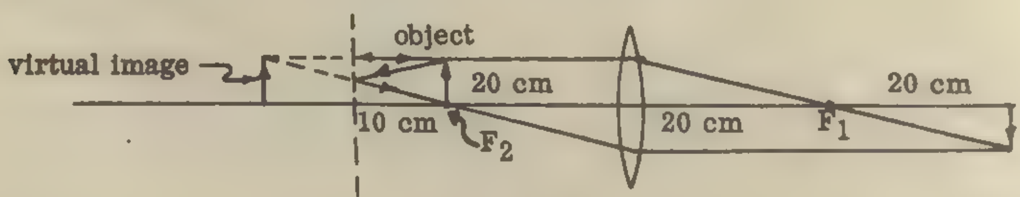


The first challenge is to draw a ray diagram for an object at the focus, F_2.

The principal ray parallel to the axis and going through F_1 is simple. The ray from the tip of the arrow through F_2 is perpendicular to the axis and would not reach the lens. Thus they need the ray through the middle of the lens. This would be parallel to the first ray after it went through the lens. Hence, no image is formed by light which goes from the object directly through the lens. (Some students may say the image is formed at infinity; although when physicists speak of an "object at infinity" or an "image at infinity", they mean by this merely that rays are parallel. For inexperienced students, parallel rays have much more meaning than infinity since the concept of an image is connected with the intersection of two light rays.)

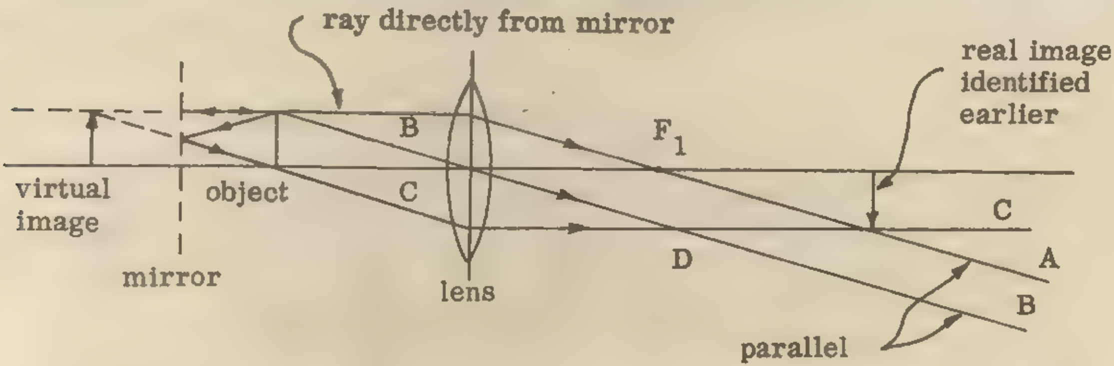
The second part of this problem involves deciding what happens to rays which leave the object, reflect from the mirror, and finally come through the lens. In order to proceed, the student must know or rediscover that rays which leave the object and hit the mirror return from the mirror as though they originated from the virtual image of the object. The lens then forms a real image of this virtual image.

Once they realize this, they can easily construct the image.



Using any two of the three principal rays, one can easily find that the image formed by the lens is 40 cm from the lens or 20 cm from F_1.

There is one additional point that might be made in this problem. Students could be asked if there is another image where the ray going directly from the object through the lens crosses the ray going from the object to the mirror through the lens. For example:

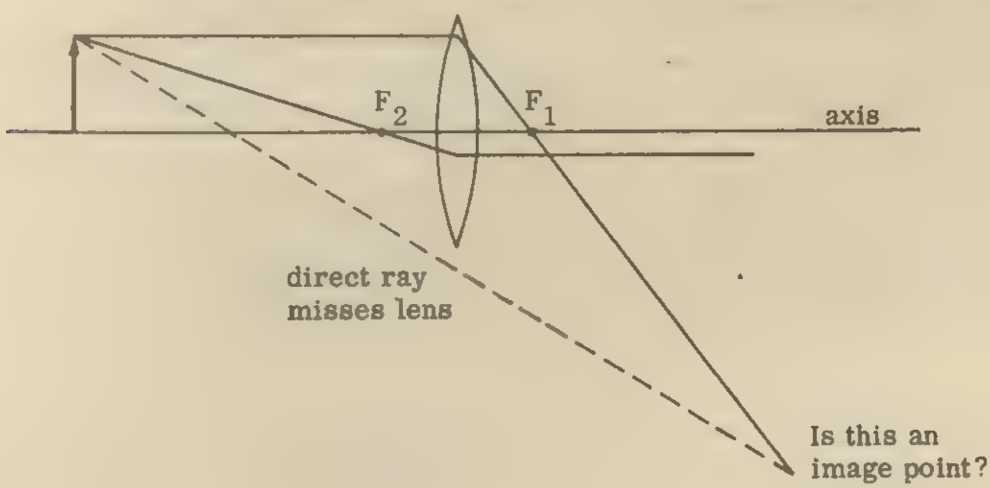


The question is whether there is an image at D where ray B crosses ray C. Some students will think that there is another image here, but they will realize that there is not if you ask them about other rays leaving the tip of the arrow and lying between rays A and B. They will see that these other rays produce a family of lines parallel to rays A and B beyond the lens. Hence if there were an image at D there would be one also at other points along C (indeed, at any point that could be reached by both the reflected and unreflected rays).

After discussion, some students will point out that you have an image only if many rays cross, and two are not enough. Finally you can lead them to a much better statement such as "You get an image when rays all of which go through the same optical elements cross". You might use different colors of chalk to show rays leaving the same point but going through different elements.

Thus, the rays leaving the object and going directly through the lens might be drawn with white chalk. Those going to the mirror and then through the lens can be drawn in red chalk. Those leaving the object but missing the lens completely can be drawn in orange chalk.

(If you ask students whether you get an image where one ray which has gone through a lens crosses a ray which has not, they will see easily that rays should go through the same optical elements.

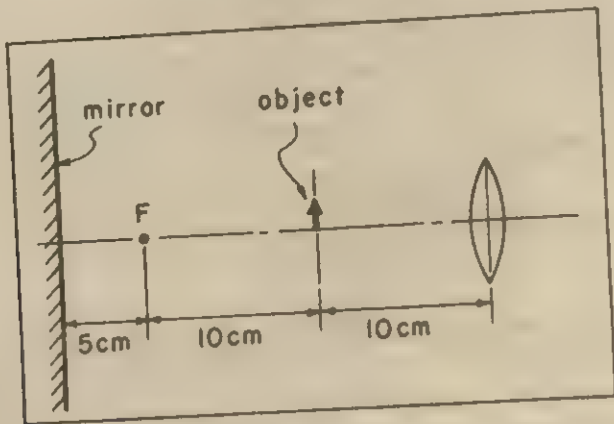


PROBLEM 19

Where are the images of the object in Fig. 14-19? Can you see all the images if you look through the lens

(a) with your eye near the lens?

(b) with your eye far from the lens?



This is another hard problem involving the formation of images by a plane mirror and a lens.

If the direct rays from the object to the lens (and not those going first to the mirror) are considered, they form a virtual image (enlarged and erect) at F. This image can be found easily using a standard ray diagram. The image could be seen by the eye if the eye were any place to the right of the lens (provided, of course, that the eye intercepted light rays originating from the object and passing through the lens).

A second image is formed by rays which go from the object to the mirror, and then back through the lens. The position of this image can be found most quickly by constructing the virtual image produced by the mirror (15 cm behind the mirror) and using it as a new object for the lens. Using principal rays and similar triangles, it is easy to see how the image appears 40 cm to the right of the lens and is the same size as the object. If an eye were less than 40 cm from the lens, convergent light would hit the eye and the eye could not focus this convergent light on the retina. On the other hand, if a normal eye were farther to the right of the real image than the distance of closest vision, it could see this image.

Chapter 15 - The Particle Model of Light

The central purpose of this chapter is to give students an idea of what a physical model is, how it must be tested, and to indicate both the usefulness and limitations of a model. Perhaps more important, but harder to express (and teach) is the spirit of searching for deeper understanding and seeing a little of the adventure and challenge in developing a physical theory.

Thus far Part II has looked at light phenomena, noting regularities, the applicability of laws, and the consequent predictability of certain phenomena. In Chapter 15, we try to encompass these phenomena in a more comprehensive "picture" of light. In addition to exploring (and finding partially inadequate) the particle model of light, this chapter serves as a bridge between geometrical optics (reflection and refraction) and waves (Chapter 16 through 19). The material on waves necessarily returns at points to physical optics, and, in addition, considers those phenomena (diffraction and interference) which are characteristic of waves.

CHAPTER CONTENT

In looking for a theory of light, a simple particle model is seen to account for the laws of reflection and refraction and to satisfy the inverse-square relation which describes the variation of intensity with distance. The model predicts that light exerts pressure on the surfaces it strikes. It also predicts that heating should be associated with absorption. Experiments verify both the existence of light pressure and the association of heat with absorption. Now we begin to find a few flaws. It is possible but awkward to explain the partial refraction and partial reflection that occurs at the surface of refractive material. There seems to be no simple particle model for diffraction. Finally, experiment shows that a particle model for light would require the speed of light in a refractive material to be higher than in air or vacuum. This is contradicted by experiment. While a particle model would account for a number of light phenomena, it is awkward or fails for other phenomena. The model must be modified or a new model found.

CHAPTER EMPHASIS

Because models give coherence and direction to physical thought, you should try to give your students an idea of what a model is. You will want to encourage discussion and speculation, but you will have to be on guard against wild conjecture. With your help, students will be inventive and will enjoy talking about possible mechanisms. Remember that students have not yet studied mechanics formally and therefore may have some wrong ideas about how particles act. While you can correct misimpressions, you will not want to discourage students by criticising serious suggestions too sharply or abruptly.

If necessary, most of the chapter can be treated principally as a reading assignment. It is not important that students learn specific facts or analyses. First priority in class discussion should be given the general import of the chapter. This suggests using some of the problems as a basis for class discussion.

COMMENT

No matter how much you have tried to avoid it, your class has probably engaged in some prior discussion of the nature of light. Some students may "know" that light is waves, or "know" that light is both waves and particles. Other students with little previous knowledge of light may be completely unbiased and ready to accept anything they read in Chapter 15 as fact. Both kinds of students will need a little "stage-setting" before they begin Chapter 15.

As an example, there was the case of a student in this course who, before he had completed the chapter, bet a student from another physics course that the speed of light in glass was higher than the speed in vacuum. Using the particle model for light and mechanical model for refraction the PSSC student showed that his hypothesis had to be correct and collected the bet! Later he learned that reparations were in order. It is, of

course, delightful that the student so clearly appreciated the arguments of the early parts of the chapter that he could convince the "doubter". However, it ought to be possible for students to follow the reasoning and appreciate the rise and decline of a model without a conviction that the particle model is the last word.

As is partially illustrated in the above example, students can read most of this chapter as "fact" supporting the complete validity of a particle theory of light. In this case, the denouement may leave some students with the impression that the chapter is trivial – it sets out the ridiculous idea that light is like a baseball and bothers to analyze that idea in detail only to show it is wrong. Other students already "know" about waves, so why bother with buckshot?

The key is that students should understand that the purpose of this study is not to prove or disprove a specific model or theory of light at one fell swoop. The purpose at this point of the development is simply to begin to develop a model of light. We take a model which seems reasonable (as the particle model did, historically) and examine the process of fitting the model to observed data--the process of testing a theory. At the conclusion of this chapter, we should not have the notion that the particle model is worthless. Although it is not completely adequate, we do not necessarily discard it. (Indeed, in Part IV we will consider how a particle theory does contribute to our understanding of light.) In this chapter we take the first step toward the development of a model for light.

Thus, the most important "stage-setting" you can do, no matter how you choose to develop the chapter in detail, is to make clear that the purpose of studying this chapter is more than learning the detail of the arguments for or against a particular theory; it is also to analyze how a theory is tested.

You will need to push students into going beyond the details and thinking about the concept of a model. Why do we search for a model? (Make them realize that if light does behave like particles in some respects, then we may look for other particle characteristics in light, and thereby perhaps discover something about light we had overlooked.) Finding even a tentative model usually suggests both future investigations and the reinterpretation of known data.

* * *

It is desirable to soften the impression that light must be either a particle or something which is (in some undefined sense) different from a particle. You can help students by substituting for such questions as "Is light made up of particles?" and "What evidence do we have for light being particles?", such questions as "Does light behave like particles?" and "What evidence do we have that light behaves like particles?".

We all know that light is neither a particle nor a wave. Light is light. In some cases it behaves in a way similar to particles (or, as we say, its behavior agrees with the particle model). In other cases it acts the way waves do (i.e., it conforms to a wave model). When we say that light is a "photon", we do not resolve this duality in any real sense. We simply invent a new word to describe something which has both particle and wave characteristics.

* * *

No attempt is made in Chapter 15 to marshal all the arguments in favor of a particle model. The key arguments for the particle-like nature of light (the photoelectric effect and the emission of light by atoms) are deferred until Part IV where they can be considered in a meaningful context. In this chapter we ask only whether a particular, very simple, particle model would work. When this simple model fails, we look for another simple model. This leads to the consideration of waves.

* * *

NOTE: Some teachers feel that, for the introductory work with waves in Chapter 16, it is worth the time and trouble to hang a long brass coil on string supports. This may be especially desirable if you do not have a smooth floor (See Experiment II-7 and the

yellow pages of the Guide for this experiment.) If you decide to suspend a coil you will want to start early because it takes some time and because you will want a bit of practice before you use it with students. You will find comments on its suspension and use on page 16-9 of this Guide for Chapter 16.

SCHEDULING CHAPTER 15

	14-week schedule			9-week schedule		
Subject	Class Periods	Lab Periods	Exp't	Class Periods	Lab Periods	Exp't
Secs. 1, 2	1	2	II-5 II-6	1	1	II-5
Secs. 3, 4, 5	1	-	-	1	-	-
Secs. 6, 7, 8	2	-	-	1	-	-

RELATED MATERIALS FOR CHAPTER 15

Laboratory. Experiment II-5, The Refraction of Particles, should precede class discussion of Section 2. See the yellow pages for suggestions.

Experiment II-6, The Intensity of Illumination as a Function of Distance, is optional. If done, it should precede class discussion of Section 3. See the yellow pages for suggestions.

Home, Desk and Lab. There are several problems in which students are asked to devise laboratory experiments, or to discuss assumptions made in laboratory experiments. Problems 12, 15, 16, and 19 are of this type. Finally there are some problems in which students are to discuss the predictions of various models of light. Problems 2, 3, 8, 11, 18, and 20 are of this nature. Of these problems, the ones you assign probably should be discussed in class.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 15-9, detailed comments and solutions on page 15-9 to 15-18.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
2	1*	2			
3	7	3*, 4, 5, 6, 8, 9	10	3*, 6,* 8*	5, 6
5	12*	11	13	12*	
7	19*	14*, 15, 16	18	17, 19*, 20*	

Films. "The Pressure of Light", by Professor Jerrold R. Zacharias of M. I. T. is intended for use with Section 4. The film gives remarkable visual evidence that light exerts a pressure. It also shows how some experiments, designed to prove one theory, will introduce new problems and avenues of investigation. Running time: 21 minutes.

"The Speed of Light", by Professor William Siebert of M. I. T. is appropriate in connection with Section 7. The film shows an experimental measurement of the speed of light in air and an experimental comparison of the speeds of light in water and air. Running time: 23 minutes.

Science Study Series. At this writing, "Michelson, A Biography" is being printed as a part of the Science Study Series.

Introduction

PURPOSE To introduce the concept of a model.

CONTENT The straight line motion of light and the noninteraction of intersecting beams do not contradict the idea that light behaves like particles. If particles moved fast enough and were small enough, they might behave like light.

EMPHASIS Treat briefly.

COMMENT Do not dwell on the question of why a fast particle does not bend appreciably. The students are not familiar with the magnitude of the earth's gravitational pull, but they know that a bullet goes "pretty" straight.

Some students may think of a beam of small particles on an "everyday" scale -- for example, pellets from a shotgun. In order to stretch students' imagination a bit, propose (if they don't) that they imagine a relative particle size and distribution in a light beam such that, if enlarged so that the particles are the size of BBs, then the average spacing between particles is, say, the distance between the earth and the sun.

We are not concerned here with the actual state of affairs in light. If some student insists on a factual summary of what size light really is, you can tell him that at this point the only way we have of judging the "size" of light is that intersecting beams do not interact. A correct statement about the "size" of light requires a most sophisticated understanding. The photons that are described in Part IV, Chapter 33 are effectively as broad as the beam of light of which they are particles.

Section 1 - Reflection

PURPOSE To draw a simple inference from a model.

CONTENT a. All reflection phenomena, including complex mirror effects are summarized by the two laws of reflection.

b. Ideal ball bearings bouncing off resilient surfaces obey the two laws of reflection. (Real ball bearings come close, except for gravity.)

c. A particle model of light can account for both specular and diffuse reflection.

EMPHASIS Treat briefly.

COMMENT We now have an upper and a lower limit on the size of the "particles." They must be small enough that they do not bump into each other in intersecting beams, but they must be larger than the irregularities of the surface of a good mirror.

DEMONSTRATION A carom or a plastic ring (the core for the winding of Scotch Tape) snapped by thumb and finger along a smooth surface and against a reflecting wall visually demonstrates the equality of the angle of incidence and reflection.

Section 2 - Refraction

PURPOSE To show how models sometimes lead to new ideas for experiments, and that with certain assumptions, a particle model could account for refraction.

CONTENT a. All refraction phenomena, including lenses and prisms, can be summarized in the two laws of refraction.

b. One particular particle model can be constructed in which the particles obey Snell's law. This model, however, requires that the particles speed up when they enter the region in which they bend toward the normal.

c. This model therefore suggests performing experiments which measure the speed of light in different substances to see if light behaves like the particles of this model.

EMPHASIS Treat briefly, but allow enough time so that students can observe the refraction of a rolling ball either in class demonstration or in laboratory. Experiment II-5 should be done before treating this material in class.

COMMENT Since students are not expected to know enough mechanics to calculate what will happen, it is important for them to understand intuitively the results of experiments such as II - 5 and as shown in Figure 15-4, page 240. It probably will not be at all obvious to students that the speed on the lower level, v_ℓ, does not depend on the angle of incidence, which is pointed out in the text. Thus, performing the experiment is important.

You may return to this point in Volume 3 when energy conservation is discussed. At this stage, however, you can only indicate the qualitative dependence that n increases as h increases, and that n decreases as v_u increases. Some students may want more detailed explanations, but you will have to point out that these can come only after you have studied mechanics.

Section 3 - Source Strength and Intensity of Illumination

PURPOSE To review the inverse-square law, and to see if the particle model of light predicts this behavior.

CONTENT a. The total amount of visible light given off by a light source, (i.e., the strength of this source) is specified by comparing it to an arbitrary standard, "the candle". (A 100 watt light bulb has a source strength of about 120 candles.)

b. The intensity of illumination on an object is the amount of visible light falling on a unit area of the object. The intensity of illumination can be specified by giving the number of candles of source strength that must be placed one foot away to give the same intensity of illumination; this is called the number of foot-candles.

c. The intensity of illumination, I, is inversely proportional to the square of the distance from the source.

d. The particle model would predict that the intensity of illumination will follow the inverse-square law.

EMPHASIS The principal idea to get across is that a simple particle model is consistent with the inverse-square law. If you have time to do Experiment II-6 "The Intensity of Illumination as a Function of Distance", it should be done before discussing this material in class.

If you have time, you may want to treat this section more fully. If you want the students to learn about source strength and intensity of illumination, you will have to give them short definitions such as those given above. (The definition of intensity of illumination does not appear in this volume; it can be found in Chapter 4, Section 3, middle of left column, page 44.) You should probably also give them simple examples using realistic values.

DEVELOPMENT Unless you give them practice and help, many students will do problems involving the inverse-square law by using the formula blindly. They may make unnecessarily long and complicated calculations because they do not understand the basic proportionality. For example, if you give the problem: "What is the intensity of

illumination on an object 4 feet from a uniform light source, if the intensity is 20 foot-candles at 2 feet?", many students will first solve for the source strength, k. They will find that the source is 80 candles and that it would therefore give an intensity of $80/(4)^2 = 5$ foot-candles at 4 feet. If you use essentially the same problem except giving the distances in centimeters or meters or miles, some students will start to recalculate without realizing they already have the answer. Some even will convert the distance to feet so that they can find the source strength in candles, instead of simply dividing by the ratio of the squares of the distances.

Give the students several examples and make sure that they realize that if the distance increases by a factor m, the intensity of illumination decreases by a factor m^2. Start with numerical problems like:

a. "If the intensity of illumination is 18 foot-candles at 5 feet, what is it at 15 feet?"

b. If there are 3 foot-candles of illumination at 10 feet, how many are there at 2 feet?

Then proceed to more abstract examples like:

c. If the intensity of illumination is 2 foot-candles at d, what is it at two-thirds of d?

As a final example, you might give the problem:

d. If there are x foot-candles of illumination at d, how many foot-candles are there at r? You may find some students who think the answer depends on which is larger, d or r.

COMMENT Some students may wonder why the points shown on the graphs in Figure 15-6, scatter. The most reasonable answer is that experimental errors were made in reading the intensity or the distance. One occasionally might expect deviations to arise because most light sources are not spherically symmetric (no deviations occur if there is spherical symmetry) but such errors would usually give a uniform trend of increasing deviation (to larger or smaller intensities) as the distance became smaller. Since some points lie above the straight line and others below, the error is more likely one of measurement. (The more astute students might guess that since the errors seem biggest where r is the smallest, the error was most probably made in measuring distance.)

Section 4 - Light Pressure

PURPOSE To show how models can be used to make predictions.

CONTENT The particle model predicts or implies that light exerts pressure. Experiments show that this is correct. The pressure from ordinary light sources is extremely small; however, light pressure in stars can be tremendous. (Students are not familiar with the quantitative concept of pressure, but they recognize that particles hitting a surface will push it.)

EMPHASIS Treat briefly. Showing the film "Pressure of Light" will make the point.

CAUTION As indicated in the film, a standard radiometer with one side silvered and one side black may seem to be pushed by light. However, this is not the case. If it were responding to light pressure, the silvered surface (which reflects) would be pushed harder than the black surface which absorbs. The momentum change is larger when the light bounces back. Instead, the black surface is pushed harder because the radiometer responds to heat. There is residual gas (even though, by ordinary standards, the radiometer bulb is evacuated). This gas is heated more near the black surface (which absorbs heat as well as light) than near the silvered surface. The pressure of light is so small that the greater pressure of the heated gas overcomes it, and the radiometer rotates with the black surface receding.

SUPPLEMENTARY INFORMATION (For the teacher, not the student.) For a typical light source, 100 foot-candles falling on 1 ft^2 corresponds to about 6 watts. This corresponds to 6 joules/sec or 6 newton-meters/sec. The force is the rate of change of momentum. Since the momentum of light is its energy divided by $c = 3 \times 10^8$ m/sec, the force is $\frac{6 \text{ newton-meters}}{\text{sec}} \times \frac{1}{3 \times 10^8 \text{ m/sec.}} = 2 \times 10^{-8}$ newtons. The force corresponding to 100 ft.-candles is therefore 2×10^{-8} newtons; if this is the illumination on 1 $ft.^2$, the pressure is 2×10^{-8} newtons/$ft.^2$ if the light is absorbed. If the light is reflected, the pressure is double this or 4×10^{-8} newtons/$ft.^2$.

Section 5 - Absorption and Heating

PURPOSE To show again that predictions can be made from a model.

CONTENT The particle theory predicts that a substance which absorbs light is heated. This is found experimentally.

EMPHASIS Treat very briefly.

COMMENT You will need to treat this section briefly, merely noting from rough analogy that particles could account for absorption and heating. The students will not know much about heat. Most of them will not know that lead would get hotter than steel if both were pounded, but this is an easy experiment to do in class or at home. If you start to discuss this in any detail, they will want to know more about lead, steel, and heat; about why pounding produces heat; etc.

Further, the heating of a transparent object is not easy to demonstrate.

Section 6 - Some Difficulties With the Particle Theory

PURPOSE To show that the simple particle theory cannot explain all phenomena of light, and that models often must be made more complicated or abandoned.

CONTENT a. The existence of partial refraction and partial reflection at one surface is hard to explain using the simple particle picture.

b. Diffraction cannot be explained simply with a particle model.

c. If a model becomes too complicated, it is of little use; if each experimental fact requires a new, unpredicted change in the model, the model has limited value.

COMMENT Do not try to kill completely the particle model here; leave it somewhat in abeyance. The point you need to make is that for a particle model to work, it would have to be more complex. Possible explanations of partial refraction were indeed given by physicists when these difficulties with the simple model first arose. As far as students know now, diffraction could be explained by assuming interaction between light particles and the edges of a hole or slit.

Although diffraction and interference are the two least particle-like phenomena, do not dwell on them now. In Chapters 18 and 19, the students will learn more about diffraction and will appreciate then that a relatively simple wave model can explain these phenomena. Once a simple wave model exists, it will be much less worthwhile to try to get a complicated particle model which also works.

Section 7 - The Speed of Light and the Theory of Refraction

PURPOSE To show how the speed of light was measured and to show how a measurement can refute or force changes in a theory.

CONTENT a. The speed of light was measured using a stroboscopic principle.

b. The speed of light is slower in a medium whose index of refraction is high.

c. The speed of a light signal in a medium is the speed in a vacuum divided by the refractive index n.

d. The simple particle model of light is not adequate.

EMPHASIS This section can be treated quite briefly insofar as the particle model is concerned. If you have the time, you might want to discuss briefly how one measures the speed of light. The film "Speed of Light" will be interesting both because of the evidence it presents and because of the experimental techniques employed.

DEVELOPMENT After the main ideas are clinched, this is a good spot to review the stroboscope and some simple kinematics. Students usually enjoy thinking about measuring a speed as fast as that of light. To be sure they understand, ask them how fast Cornu's wheel (page 245) has to turn in order to get the first maximum return from the distant mirror. If they used d = 23 km, the light path is 46 km, and taking the speed of light as 3×10^5 km/sec. the light signal traverses the path in 15.33×10^{-5} seconds. One revolution should take 200 times as long if there are 200 slits, and hence one revolution would take 3.066×10^{-2} seconds. The wheel would be turning about 32.6 rev/sec, or about as fast as the fractional horsepower motors found in home workshops. But then Cornu's wheel had to go 28 times as fast in order to see 28 maxima!

Michelson's apparatus had only 8 sides or the equivalent of 8 slits (Figure 15-8), but he could rotate it faster, and he could use narrow slits to define the light beam and viewing direction. Hence he could get very high precision.

You might ask students what the minimum rotation speed of Michelson's mirror was. The path of 2×22 miles is about 70.4 km; hence the light requires about 2.35×10^{-4} sec. Since this would be the maximum time for 1/8 revolution, one revolution must take at most 1.88×10^{-3} seconds. This corresponds to almost 32,000 revolutions/min as the minimum rotation speed.

Section 8 - The Status of the Particle Model

PURPOSE To review ideas about the use of models.

CONTENT a. This simple particle model does not work.

b. It could be modified or abandoned.

c. (A particle model, more complex than ours, is still quite important.)

COMMENT Do not dwell on the incorrectness of our particle model. You probably should point out explicitly that this is a very simple model. You might even add that the students should know more about mechanics and how particles behave before they try to make a more complex model, except as they play with new models in the exercises. Do not introduce the photo-electric effect or other phenomena which demand some kind of particle model. Instead, just lead into Chapter 16 by asking what else, except a particle, moves.

Chapter 15 - The Particle Model of Light

For Home, Desk and Lab - Answers to Problems

There are several problems in which students are asked to devise laboratory experiments, or to discuss assumptions made in laboratory experiments. Problems 12, 15, 16, and 19 are of this type. Finally there are some problems in which students are to discuss the predictions of various models of light. Problems 2, 3, 8, 11, 18, and 20 are of this nature. Of these problems, the ones you assign probably should be discussed in class.

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 15-9 to 15-18.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
2	1*	2			
3	7	3*, 4, 5, 6, 8, 9	10	3*, 6, 8*	5, 6
5	12*	11	13	12*	
7	19*	14*, 15, 16	18	17, 19*. 20*	

SHORT ANSWERS

1. 4.89×10^8 m/sec.
2. See discussion on page 15-10.
3. See discussion on page 15-10.
4. See discussion on page 15-10.
5. Home project.
6. See discussion on page 15-11.
7. a) 6.4 candles.
   b) 4.4 ft-candles.
8. See discussion on page 15-13.
9. See discussion on page 15-13.
10. a) 27.8 ft-candles.
    b) 50 ft-candles.
11. See discussion on page 15-14.
12. See discussion on page 15-14.
13. a) $(0.8)^n$
    b) See graph on page 15-15.
    c) See graph on page 15-15.
    d) 10
    e) See discussion on page 15-15.
14. 833 m.
15. See discussion on page 15-16.
16. See discussion on page 15-16.
17. See discussion on page 15-17.
18. See discussion on page 15-17.
19. See discussion on page 15-17.
20. No.

COMMENTS AND SOLUTIONS

PROBLEM 1 — The index of refraction of carbon disulfide is about 1.63. What should the speed of light be in this liquid, according to the particle model of refraction given in Section 15-2?

According to the particle model, the velocity of light in a medium of refractive index n is

$$v = nc = 1.63 \times 3 \times 10^8 \text{ m/sec.} = \underline{4.89 \times 10^8 \text{ meters/sec.}}$$

During discussion, remind the students that this formula comes from the particle analog of Experiment II-5. Depending on how you are developing the particle model, you may wish to remind them that this is an incorrect answer.

PROBLEM 2

Occasionally there occurs in the heavens an explosion of a star, producing what is known as a super nova. The star suddenly becomes many times brighter than before. As you know, the stars are so far away that the light from them takes many years to reach us. An explosion that we observe must have occurred a long time ago, and the light has been traveling toward us ever since. We see the explosion as a bright white light, not as a series of different colors arriving at different times.

(a) What does this show about the speed of light of different colors in vacuum?

(b) Try to suggest a particle model for dispersion in prisms consistent with the single speed of light of all colors in vacuum.

This is a straightforward problem; the student is to analyze a simple fact about light in terms of the properties of a model.

a) Since the explosion appears as white light and not a sequence of colored flashes, one can conclude that the speed of the various colors of light in a vacuum is the same (within the resolving time of the eye) assuming that all colors were released at the same time.

b) A particle model for dispersion, consistent with the single speed of light of all colors in vacuum, might include some kind of variable force acting at the interface of the dispersive medium. This force would have to accelerate differently the various particles for different colors.

PROBLEM 3

Can you explain the different strengths of light sources in terms of particles of different size? How about the decrease in intensity caused by inserting a partially absorbing sheet of matter in a light beam? Can different intensities at varying distance from a source be accounted for in this way?

The purpose of this problem is to give students experience in testing a model by comparing it with the facts learned from experiment.

The problem asks us to assume that a 200 watt light bulb, say, emits particles which are larger than a 100 watt light bulb. We then have to assume either that

1. a subsequent decrease in intensity by absorbers or inverse-square is due to fewer and/or smaller particles per square meter per second, i.e., intensity of illumination is due to two factors, size and number of particles, or that

2. intensity of illumination is due to the size alone, and has nothing to do with the number of particles per square meter per second. In this case, the inverse-square law must be blamed on a decrease in the size of particle.

One can find little that is logically wrong with (1), except that it seems very odd that two candles that are far apart each emit particles of a particular size, but when placed close together so as to form a single more intense source, they emit larger particles. In fact this is so odd we are likely to discard this theory.

Assumption (2) must be discarded, because if the particles get smaller with time or distance from the source, why doesn't the beam of light converged by a lens get dimmer, instead of brighter?

PROBLEM 4

What assumptions do we make in using the inverse-square law to determine star distances? (See Section 4-3.)

The principal assumptions made in determining the distance of a star by the inverse-square law were:

a) The inverse-square law applied over very large distances for which we could not test it.

b) There was no absorbing material ("cosmic dust") between the star and us. (There actually is some.)

c) The most questionable assumption is that stars of the same color have the same intensity and that the sun is taken to have the same intensity as most other stars.

d) Euclidean geometry holds in outer space.

PROBLEM 5

Make a simple photometer for comparing light sources. Place aluminum foil between two identical paraffin blocks and hold the blocks together with tape. When light falls on this photometer from one side the end of the block of paraffin on that side appears bright. When two light beams of equal intensity of illumination fall on the two sides, the ends of the paraffin blocks appear equally bright.

(a) To check that the photometer functions properly we turn it over so as to interchange the positions of the two paraffin blocks. Why?

(b) With such a photometer you can find out when two light beams are of equal intensity; but if the intensity of the light falling on the two sides differs, we cannot get a quantitative measure of the relative intensities of illumination. Why?

The simple photometer can be used to check the inverse-square law by checking one light against 1, 2, 3, 4, etc. identical lights at various distances on the other side. It can be used to see whether an ordinary electric lamp emits the same amount of light in all directions.

a) If the two paraffin blocks are not identical, the ends of the blocks could appear equally bright despite the fact that the intensities of light falling on the two sides were not equal. Interchanging the blocks would now exaggerate the difference in intensity on the two sides and the ends of the block would differ greatly in brightness. Various combinations of blocks can be tried until two are found that give matched brightnesses even when the positions of the blocks are interchanged.

b) We can tell whether one paraffin block is brighter than the other, but not how much brighter, because our eye cannot judge intensities quantitatively.

In building the photometer there is a possible pitfall. (You may not want to warn students in advance, so you can see how many of them discover it.) If ordinary household aluminum foil is used, the photometer will not be symmetric. That is, if both sides are equally illuminated, when the photometer is turned over it will no longer show the two sides equally bright. The reason for this will be clear if you examine a piece of aluminum foil. The two sides of the foil are not equally reflective. One side is shiny, and the other side is duller. In order to make a good photometer, use a folded sheet of foil as a separator, with the shiny side out on both sides.

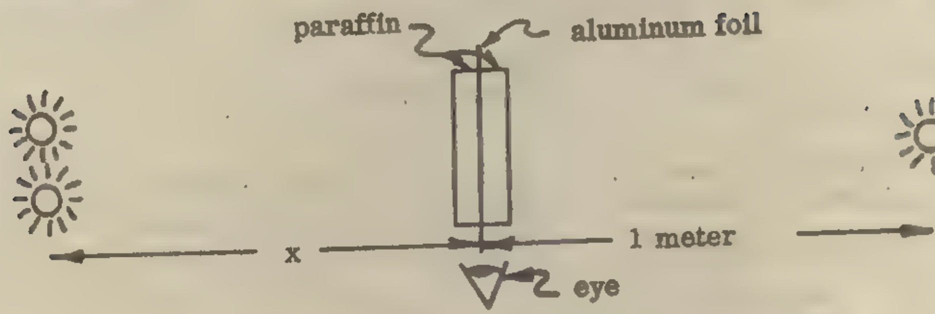
PROBLEM 6

Devise a procedure for checking the inverse-square law with the photometer of Problem 5.

This problem can be assigned as a home project, or as a thought problem.

There are at least two easy methods of checking:

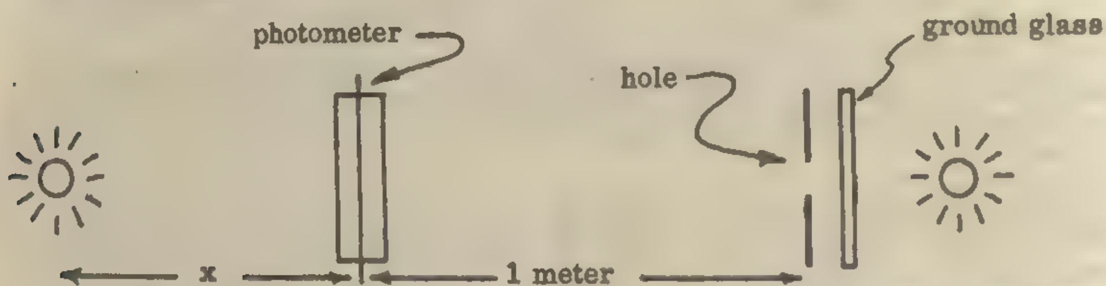
1. Multiple sources.



If several nearly identical sources are available, several of them may be placed on one side of the photometer at a distance x from it, and balanced by another at a standard distance – perhaps one meter – on the other side. If one source balances at $x = K$ meters (K will be 1 meter if the source is identical to the standard one) two sources will balance at $x = K\sqrt{2}$ meters, three at $K\sqrt{3}$ meters, etc.

A good class discussion may be generated by inquiring whether the standards must be identical to the others. How should one check to make sure the bulbs used have the same intensities? If n bulbs are supplied from a single battery, they may load the battery causing each bulb to become dimmer. How could this be checked?

2. Size of opening.



Provide a uniform source by placing a piece of ground glass-- or tissue or waxed paper-- in front of a bright light bulb. Then place a shield in front of it that can have holes of various areas cut in it. As the area is doubled, the intensity from that side is doubled, and the other light will have to be placed at a distance $1/\sqrt{2}$ times its original distance from the blocks. In fact x^2 multiplied by the area should be constant.

Again a good class discussion can be generated by asking how one might check the operation of this device. One might use two openings of the same area but different shapes, etc.

PROBLEM 7 It is found that a 40-candle source placed 3.0 feet from a light meter gives the same scale reading as an unknown source 1.2 feet from the meter.
(a) What is the strength of the unknown source?
(b) What is the light intensity in foot-candles read on the meter?

This is an easy problem involving the inverse-square law. Part (a) requires no knowledge of illumination units; Part (b) does.

a) $I = \frac{40c}{(9\text{ ft})^2} = \frac{K}{(1.2\text{ ft})^2}$, $K = \frac{57.6}{9} = \underline{6.4\text{ candles}}$.

b) $I = \frac{40c}{(9\ ft)^2} = \underline{4.4\ \text{foot-candles}}$

In class discussion be sure to point out that part (a) could be worked if the 40-candle source had had a strength of 40 "gizmos", and no one knew what a "gizmo" was except that it measured the intensity of a light source.

PROBLEM 8

What does the particle theory predict about the intensity produced by an *extremely* weak light source — so weak that only a few particles are emitted per second? How might you test this prediction?

The particle theory predicts that light from very weak sources has so few particles that the light appears to come in bursts. This aspect of the particle theory can be tested by letting light from a weak source fall on a very sensitive light meter and looking for large fluctuations in the reading. Of course, if no fluctuations are found, then light may still be particles--only smaller ones than were supposed when the experiment was designed. (One of the nice things about this particular "thought" experiment is that indeed, when we look at a sufficiently weak source, we do count individual "particles" of light. It is hard to reconcile this observation with a wave model.)

PROBLEM 9

In the use of radar, a beam of radiation is sent out from a source. Some of this radiation falls on a distant object, such as an airplane, is reflected by the object, and is detected when it returns to its starting point. The radiation used behaves like light. Assuming that the source of the radiation is equivalent to a point source of light and that the object reflecting the radiation is a diffuse reflector, can you convince yourself that the intensity of the returning radiations varies inversely with the *fourth* power of the distance from the source to the reflecting object?

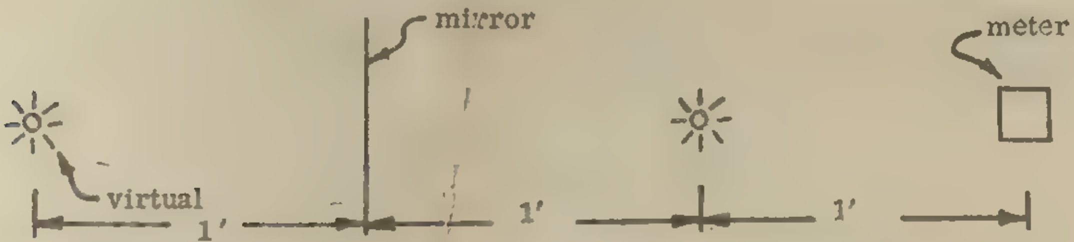
The intensity of "light" arriving at the distant object is proportional to $1/r^2$, i.e., $I = K/r^2$. The amount of "light" it reflects is proportional to the incident intensity. It therefore becomes a source of strength CK/r^2 But the intensity of "light" arriving back at the radar station is $I = \frac{CK/r^2}{r^2} = \frac{CK}{r^4}$ as was to be shown.

Some students will remember a similar problem--Problem 13 for Chapter 4.

PROBLEM 10

A light meter calibrated in foot candles and a mirror are 2.0 feet apart. A very small 25-candle-power source is placed midway between them. What is the reading of the meter if the mirror is
(a) a plane mirror?
(b) a concave mirror with a diameter of 6 inches and a focal length of one foot?

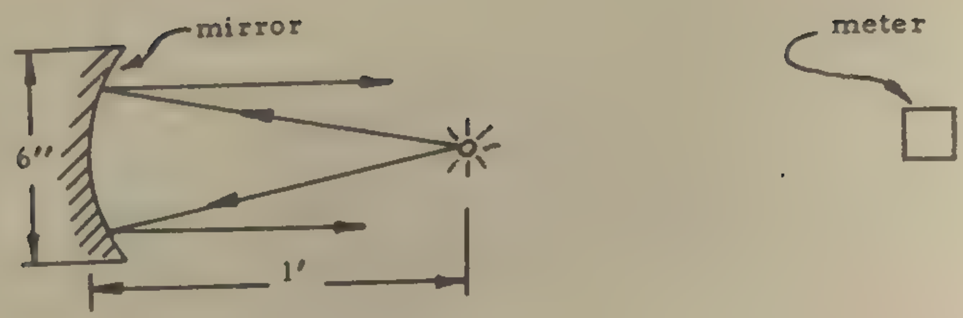
a)



We can replace the mirror by a virtual source 1 foot behind it. Then,

$$I_{\text{at meter}} = \frac{25C}{(1\ ft)^2} + \frac{25C}{(3\ ft)^2} = \frac{250}{9} = \underline{27.8\ \text{foot-candles.}}$$

b)



Since the light source is at the principal focus of the concave mirror, the rays after reflection are parallel to each other. This gives a cylinder of light six inches in diameter that does not change in intensity with distance if scattering and absorption are neglected. For the short distances involved in this problem, scattering and absorption of light in clear air would be very small indeed. Thus the intensity in this cylindrical beam of light is the same as at the surface of the mirror itself.

$$I_{\text{at meter}} = \frac{25C}{(1\text{ ft})^2} + \frac{25C}{(1\text{ ft})^2} = \underline{50\text{ foot-candles}}.$$

PROBLEM 11

A sensitive thermometer placed in the different parts of the spectrum from a prism will show a rise in temperature. This shows that all colors of light produce heat when absorbed. But the thermometer also shows a rise in temperature when its bulb is in either of the two dark regions beyond the two ends of the spectrum. How can the particle theory account for this?

The particle theory would account for this heating by saying that there are particles of light not detected by the eye which are refracted by a prism and which cause a rise of temperature in a thermometer placed at the extremes of the visible spectrum.

PROBLEM 12

A thermometer is placed in the beam of light coming from a lamp. Its reading increases until it becomes steady at 26°C. When a piece of ordinary window glass is placed between the lamp and the thermometer, the reading drops to 23°C.

(a) What can you conclude about the nature of the light coming from the lamp?

(b) If the window glass is replaced by Corex glass (see Section 11-5), would you expect the thermometer reading to be above or below 23°C? Above or below 26°C?

(c) Can you be equally sure of each of your answers?

This experiment indicates that some light is absorbed by the window glass. A careful experiment would show that the glass does not absorb or reflect this much visible light, and hence the experiment might tell us that some "invisible" light was coming from the lamp.

Since it is known that Corex glass passes some light which window glass does not, we would expect the temperature to be above 23° C. Since it must absorb some light, we would expect the temperature to be below 26°. We can be quite sure of the latter answer. We cannot be quite so sure of the former, since we do not know <u>all</u> about Corex glass. It does pass more ultraviolet light than ordinary glass, but it might absorb more infrared. We do not know.

PROBLEM 13

A calibrated light meter shows that one glass plate transmits 80% of the light from a fixed source.

(a) What fraction of the light will be transmitted by two, three, etc. up to ten such plates?

(b) Draw a graph of the fraction of light transmitted as a function of the number of plates.

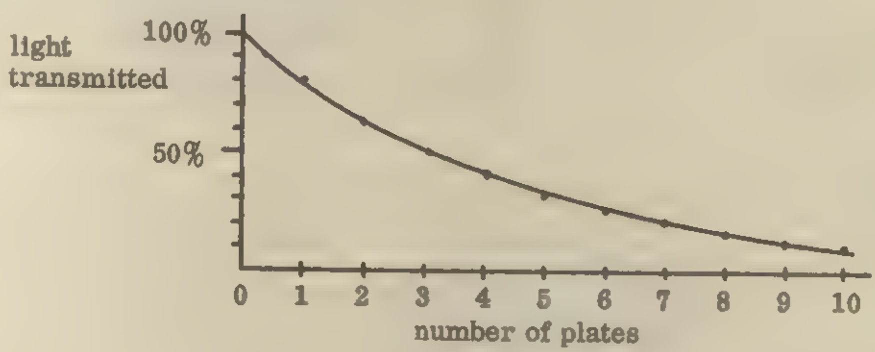
(c) Plot the logarithm of the fraction of light transmitted as a function of the number of plates.

(d) How many plates are needed to absorb 90% of the light?

(e) Be prepared to discuss the effect of reflection on the accuracy of your results.

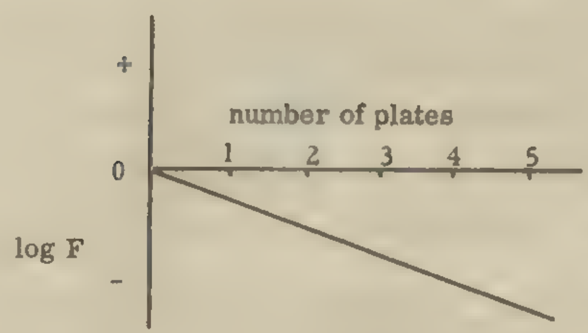
a) If 80% is transmitted by 1 plate, then 80% × 80% = 0.8 × 0.8 = 0.64 or 64% is transmitted by two, and $(0.8)^n$ by n plates.

b)



c) $F = (0.8)^n$

$\log F = n \log (0.8)$



d) $0.1 = (0.8)^n$

$-1 = n\,(-1.0000 + 0.9031)$

$n = \dfrac{1}{0.0969} \approx$ 10 plates.

e) Reflection affects the results in several possible ways.

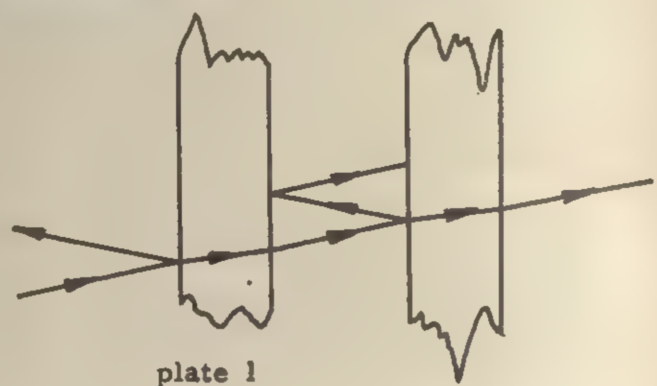
1. If the 80% includes losses by reflection, and reflection were eliminated, clearly the intensities all increase; but the answers remain qualitatively the same.

2. If the 80% includes the losses by reflection and the glasses are stacked so tightly together that the inner surfaces seal optically (this can be done), there will be no

reflection losses at the inner surfaces, and the transmitted intensity will be greater than without this method of stacking.

3. If the 80% includes the losses due to reflection, there is another effect. Suppose 10% were reflected at an air-glass interface. Then when two plates were stacked, 10% of 80% of the light would reflect off the second plate.

Of this light 10% would reflect off the first plate, and thus 10% of 80% would approach the second plate. Of this 80% would be transmitted. Thus in addition to the 0.8×0.8 there will be a $0.1 \times 0.1 \times 0.8 \times 0.8$ or about a 1% correction. The resulting intensity variation will not be quite a straight line when plotted as a logarithm.



PROBLEM 14

Using the data given in Section 15-7 about Cornu's application of the Fizeau method, compute the shortest distance from the rotating disc to the mirror that will permit the returning beam to pass through the opening immediately following the one through which it started.

Section 7 states that in Cornu's stroboscope there were 200 openings and that it could turn at 54,000 rpm.

$$\text{Time for 1 revolution} = \frac{1}{5.4 \times 10^4} \text{ min.} = \frac{6}{5.4} \times 10^{-3} \text{ sec.}$$

$$\text{Time for } \frac{1}{200} \text{ of a revolution} = \frac{6}{1.08} \times 10^{-6} \text{ sec.}$$

$$\text{Distance light travels in that time} = \frac{1.8 \times 10^3}{1.08} \text{ meters} = 1.67 \times 10^3 \text{ meters.}$$

$$\text{Distance to mirror} = \frac{1.67 \times 10^3}{2} = 8.33 \times 10^2 \text{ m} = \underline{833 \text{ meters}}.$$

PROBLEM 15

It might be imagined that on each reflection the speed of light decreases slightly. Assuming that you can measure the speed of light (see Section 15-7) describe an experiment designed to test the assertion that reflection does not change the speed.

Instead of using one mirror, one could make the light reflect 100 times before it returned to the toothed wheel. In this way one could determine whether the speed of light diminishes on reflection. There are many variations, some very clever, for making this experiment more accurate, but it is not worthwhile spending class time discussing them.

PROBLEM 16

Can you modify your proposed experiment (Problem 15) to test the assertion that the speed of light in vacuum is independent of previous refractions?

To test the hypothesis that the speed of light in vacuum is independent of previous refractions one lets the light go through a number of refractions before it first goes through the toothed wheel.

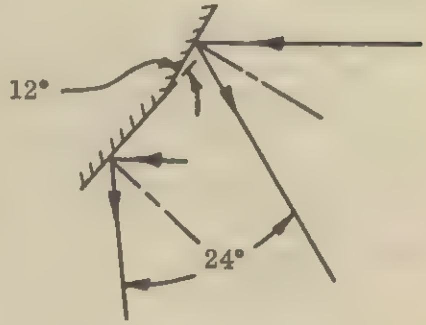
PROBLEM 17

In the Foucault-Michelson modification of the Fizeau method, rotating mirrors are used in place of the rotating wheel. (Fig. 15-8.) Assuming that mirrors are rotating 500 times per second and that the distance from them to the fixed mirror is 10.0 km, show that a mirror face rotates through an angle of 12° before it reflects the beam a second time and that the angle between this reflected beam and the original direction of the light is therefore 24°.

$$t = \frac{d}{v} = \frac{2 \times 10^4 \text{ m}}{3 \times 10^8 \text{ m/sec}} = \frac{2}{3} \times 10^{-4} \text{sec.}$$

$$\text{angular rotation} = \frac{500 \text{ rev}}{\text{sec}} \times \frac{360°}{\text{rev}} \times \frac{2}{3} \times 10^{-4} \text{sec} = 12°$$

If the mirrors turn through an angle of 12°, the normals to the mirror will also change direction by 12°. The angles of incidence and reflection will both change by 12°. Thus there is 24° between the initial and final beams.



24° between initial and final beams

PROBLEM 18

Suppose that someone were to propose to you that sound consists of small, rapidly moving particles that are emitted by a source and that affect your ear when they strike it. What evidence could you use to support or to refute it?

Sound could not be composed of particles emitted by a source and received by our ears. One can show this because sound does not propagate through a vacuum.

Another way to show this would be to make a box and a barrier of material which completely absorbs sound. Such a box would not let any sound out. However, a barrier of this material would let sound get around it. This could not be true if sound were particles, unless they were scattered by the air. If this were so, sound would be absorbed rapidly by the air.

One could check to see if sound obeys an inverse-square law, if it reflects and refracts like the particle model, and whether it exerts pressure. Sound does get around obstacles. For instance, the sound of a truck is heard on the other side of a house. This would refute the particle model. Students may know that the tweeter in a hi-fi set is placed so that it points at the listener, while the woofer for low tones may face in any direction. This shows that low tones, at least, do not always travel in a straight line.

PROBLEM 19

We have said that light particles must be very small. Suggest an experiment to show that they must also have an extremely small mass.

A flashlight on a sensitive balance does not appear to continually lose mass when it is turned on.

Also if light particles had a large mass, they would have a great deal of inertia, and light shining on objects would knock them about.

PROBLEM 20

Is it possible that the slowing down of light as it goes from a vacuum into a transparent material is some kind of friction effect?

If the slowing down of light as it passed from a vacuum into a transparent material was due to a friction effect in the material, the speed of light in air or in glass would not be constant. The speed in glass would depend upon how much glass the light had traveled through.

If we assumed some kind of friction effect that was effective only at the surface of the transparent material, but not within it, we would have to invent some kind of "reverse friction" which would increase the speed of light as it left the transparent material. Otherwise light re-entering a vacuum after traversing a transparent material would be further slowed, and would be refracted away from the normal instead of toward it. We know that the speed of light in a vacuum is constant, no matter what the light has traversed. We know that light entering a vacuum from a transparent material is refracted toward the normal.

Chapter 16 - Introduction to Waves

At this point in Part II, some of the major characteristics of the behavior of light have been described and a first step has been taken to develop a model for the behavior of light. A simple particle model has been seen to be inadequate. In the last half of Part II the course turns to another model — waves. The model must describe the travel of light through air, vacuum, glass and water; it must encompass the laws of reflection and refraction; it must cover the non-interaction of intersecting light beams; it must predict the observed speed of light in various media; and it must explain diffraction effects. The next four chapters explore the characteristics of waves and their applicability in a model for light.

If some students show a little irritation at "developing a model then throwing it away", this is a good sign of their involvement in the subject; the facts of life are that models are hard to give up. The Ptolemaic model of the solar system was clung to for centuries after its inadequacies were shown clearly. In any case, students should not get the idea that, with waves, we are now getting the "right" model. With waves we ? e taking the next step in the development of a model for light. Part IV returns to this problem to show that, currently, particle and wave concepts must be combined.

Students are fairly familiar with the kinematics of particles. They have watched marbles, baseballs, croquet balls and many other objects speed up, slow down, collide, rebound and fracture. Their knowledge of waves is almost sure to be sketchy. True, they have jumped rope, perhaps thrown a loop along a clothesline, watched ripples on a pond, and seen waves rolling in from the ocean. But it is a rare student who has observed these effects with sufficient care to get a good general feeling for the nature of wave motion. Thus, an important part of the development of Chapters 16-19 in both text and laboratory is the patient building of ideas about waves from concrete examples that can be seen and handled. This gives a firm footing for the concept of waves in light.

Chapter 16 introduces the general characteristics of wave motion in a simple and concrete way. The chapter deals with a pulse on a coiled spring (a piece of a wave moving along only one dimension).

Chapter 16 should be covered thoroughly since it is basic to the rest of Part II. Of the many interesting topics in this chapter, principal emphasis should go to such topics as superposition and reflection. These are essential to later work.

Chapter 16 (and the rest of Part II) is critically dependent upon laboratory work. If you can arrange it, you may want to spend extra time in the laboratory. If you have a fixed amount of laboratory time per week, you may need to augment it with classroom demonstrations.

CHAPTER CONTENT

a. A wave is something which travels and yet does not necessarily take any matter along with it.

b. A pulse moving along a coiled spring maintains its shape as it moves. The pulse moves along the spring, but points on the spring move at right angles to the motion of the pulse.

c. When two pulses cross, the displacement is the sum of the displacements of the individual pulses.

d. When a pulse reaches a new medium (such as a heavier or lighter rope or a fixed support), part of it may be transmitted and part of it may be reflected. Exactly what happens depends on the relative properties of the two media.

e. Idealizations are necessary and useful in analyzing many physical phenomena. This is exemplified in the analysis of waves.

f. A wave model may be suitable for light.

SCHEDULING CHAPTER 16

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part II.

Subject	14-week schedule for Part II			9-week schedule for Part II		
	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2, 3	2	-	-	1	-	-
Secs. 4, 5, 6	2	1	II-7	1	1	II-7

RELATED MATERIALS FOR CHAPTER 16

Laboratory. Experiment II-7, Waves on a Coil Spring. This experiment uses the coil spring pictured in the text, giving students a chance to see pulse behavior at firsthand. If scheduling permits, the experiment might be performed in two parts, first, pulse propagation and superposition, and second, reflection. If it is done in one part, it should be scheduled at about the middle of the chapter. See the yellow pages for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages. Detailed comments and solutions on page 16-17 to 16-25 Short answers are not given for the problems of this chapter because all the problems require drawings or discussion.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1	1*, 2			1*, 2	
2	3*	4, 5*		3, 4, 5*	
3	6*	7, 8	9*	6*, 7, 8, 9*	
4	11*	10*, 12*		10*, 11, 12*	13*
5	15	14		15	14

Films. "Simple Waves", by Dr. John Shive of the Bell Telephone Laboratories. Behavior of waves on ropes and coil springs is used to show velocities in differing media and other elementary characteristics; torsion-bar wave machines used to repeat these, and to demonstrate phenomena of reflection and refraction. Running time: 27 minutes. This film can be shown at any point in your work on this chapter. However, since the film demonstrates all of the phenomena discussed in the chapter, many teachers prefer to use it near the end to help summarize and tie together the ideas.

Section 1 - A Wave: Something Else That Travels

PURPOSE To give students the qualitative idea that a disturbance can travel and to introduce the pulse (which is easier to study and analyze than the periodic wave) as a wave of short duration

CONTENT A wave is a disturbance which travels without necessarily taking any material along with it. Some waves are periodic, but single pulses can also be waves.

EMPHASIS Treat thoroughly. Many students have the idea that a wave must be periodic (or repetitive). Be sure that they realize that pulses are waves because they will soon be studying pulses to learn how waves behave.

COMMENT Some students may quibble over a statement such as at the bottom of the first column on page 248: "...every bit of water is left where it used to be...". Students should realize that this is an idealization which can be approached quite closely. Waves washing ashore and waves so high that they break are cases where the water is transported short distances. These are cases which are not close to the idealization which is being discussed. Even in these cases, the water particles are transported over only a small bit of the entire path of the wave.

DEVELOPMENT A good class discussion of this section can center about HDL Problem 1 on page 258. Students are asked to give examples of how they might get a message from one point to another and list each of these as either mass transmission or wave motion. Some of the examples will require discussion before you can decide in which category they belong. (Some students will not know what "mass transmission" means unless you explain that it merely implies that some object has gone from the starting point to the final point.)

Some arguments will occur over whether a given suggestion is an example of mass transmission or of wave motion. Encourage sensible debate because many students will miss the whole point of the categories unless it is clear to them that the crucial question is one of mechanism. For example, if the suggestion involves opening a jar of coffee and letting the odor spread, before one can decide, one must understand what odor is and how it is transmitted. If you know that some gaseous molecules are released from the coffee and eventually go to the nose, odor transmission belongs under mass transmission. But if one were to postulate (incorrectly) that odor is caused by a distinctive oscillatory pattern induced in air molecules and transferred from molecule to molecule (like sound), odor transfer would be wave motion.

Another example which might be either particle transmission or wave motion involves a string running between two windows. If you stretch it tightly and twang at one end, receipt of the twang at the other end is rather clearly due to wave motion. But what if you pull it two feet into your window thereby removing two feet from the coil in the other window? This sounds like mass transmission in a way, but no piece of string went from one window to the other. Do not let students waste time in a case such as this fighting over which one category is the best one. The two categories have not been that well defined! Students have learned what they were expected to learn and can go on to other suggestions or a different class activity.

An example that may amuse your class is that of a long, thin-walled rubber tube which when observed from a distance, clearly has a small region of expanded diameter moving along it. This bulge usually would be an example of wave motion (unless you know that the bulge is caused by an animal running along inside the tube!) It would be rather unimportant and arbitrary to decide whether or not to label it as a wave if the animal were known to be running inside and causing the disturbance.

Some students will almost surely suggest signaling with a light to communicate a message. If this is suggested early, put it in the "questionable" category temporarily. When you have enough examples, return to the classification of light. If the students have gotten the general idea of this section, they should realize that the course has not yet given them enough information to decide. Inasmuch as the course thus far has not found a light particle, and we are just beginning to explore waves, you might place light in the wave category (with a question mark). On the other hand, if the simple particle model had worked, or if a more complex particle theory would work, light might belong in the mass transmission category. Indecision about where light really belongs is one of the reasons for learning more about what waves are like.

COMMENTS Some students may get the idea that a wave cannot involve mass transmission. They will be less confused if you point out that sometimes, incidental to the wave motion,

some mass transfer occurs. A common example is a surfboard carried forward on a water wave. Clearly the surfboard moves, but the waves would exist even though the board were not there. The surfboard is carried by the wave, but it is not the cause of the wave propagation.

Some students may be a bit surprised at the wide variety of phenomena that are called waves. They realize (correctly) that you cannot expect all the characteristics seen in a wave of starting automobiles to carry over to waves on a string, etc. It would be unwise to dwell on which waves have common characteristics.

At this stage we are not defining a wave. Often the term "wave" is reserved for the wide class of phenomena which satisfies the same differential equation, the wave equation. We cannot give students the wave equation, nor can we discuss it. Instead, we choose simple systems in which the wave equation holds, and observe how these systems behave. At this stage, we need only consider that property of a wave which permits the transfer of a signal without involving the sending of a material particle.

CAUTION If you do not choose varied examples now, you may leave the student with the impression that waves must always have a medium through which to travel. The clearest examples of wave motion are those in which a wave travels in a medium which is clearly visible; everyone can see one end of the rope move up, pull its neighboring piece up, and thereby propagate a wave. Hence students are likely to associate a wave with the medium in which the wave moves. They can most easily visualize a mechanical wave whose propagation depends on the elastic properties of the medium. Eventually, however, they will have to face the idea that light is a wave motion without an apparent medium.

One way to avoid overemphasizing the medium is to be careful in your choice of statement. A phrase like, "We can see the disturbance as it moves along the rope" is probably better than "Each piece of the rope which is disturbed in turn disturbs its neighbor giving rise to a pulse moving down the rope". Emphasis on the explanation of the propagation (or the dynamics of wave motion) may cause students to overemphasize the importance of the medium.

Another way to de-emphasize the importance of the medium in which the wave travels (aside from merely warning the class that there can be wave phenomena without a medium) is to give examples in which there is no medium. Students will accept the idea that if you twist a little bar magnet through 180°, it takes time for the effect of this new orientation to be felt. (There is no need to mention that the signal speed in this case is the speed of light. The only necessary fact, that there is a finite speed, will probably be accepted by the students without further detail.) Consider the bar magnet being turned just outside a vacuum chamber. If there were small isolated compass needles at various points in the vacuum, each would start to turn when the effect of the rotated magnet became known. This pulse of "awareness" or the pulse of "potential awareness" which would travel through the vacuum is a suitable example of a wave moving without a medium.

Section 2 – Waves on Coil Springs

PURPOSE To provide a foundation for the study of waves.

CONTENT a. A pulse moves undistorted at constant speed along a tube.

b. The motion of points on the tube is quite distinct from the motion of the wave.

c. The graphs drawn of pulses will be approximations of pulses. Real pulses never have very sharp corners (or "sharp" changes in slope).

EMPHASIS Treat thoroughly. It is important for students to develop, directly from observation, an accurate picture of wave motion. The "content" listed above, including the distinction between the motion and shape of the wave and the motion of the particles of the rope, should be understood clearly. Demonstrations, problems, and laboratory work are particularly helpful.

CAUTION You will not want to discuss in any detail the dynamics of wave motion on a rope. Students may ask about how one piece of rope affects the next, but they will accept the fact that they cannot expect to understand this quantitatively until they know how forces affect motion (Part III).

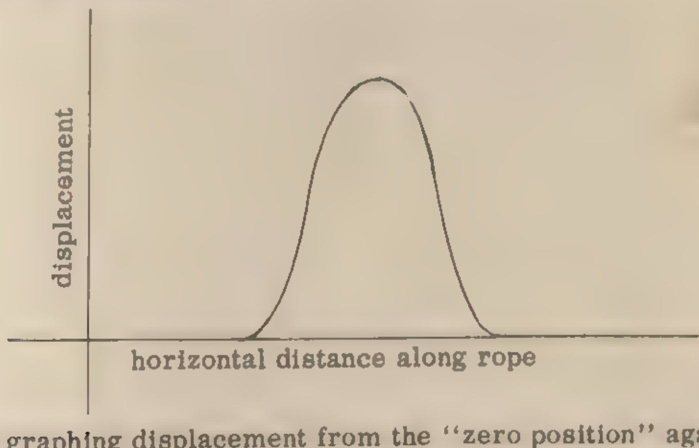
Longitudinal waves are almost sure to come up. Students will think of them, or see them on the coil spring. For the class as a whole it will probably be best to stick closely to transverse waves — explaining that longitudinal waves are just another type of wave motion with which we do not now need to be concerned.

DEVELOPMENT You should demonstrate pulses on a coil spring (or a rope or rubber tube), and have students work with them in laboratory — Experiment II-7, "Waves on a Coil Spring". It is recommended that half of two periods, be spent on this experiment during the time spent on the chapter. If this is not possible, one full period should be allowed in the middle of the time spent on the chapter. (See supplement to this section for details concerning a Slinky hung on strings.)

You can clarify most of the material in this section through a detailed class discussion of Figures 16-3 and 16-4. In order to understand easily superposition in the next section, it is important for students to understand the difference between a picture (or sketch) of a wave and a graph of a property of the wave. Students may have some difficulty with this because in Figures 16-3 and 16-4 the graph closely resembles a picture. You can make this distinction clear to students by examples such as the following:

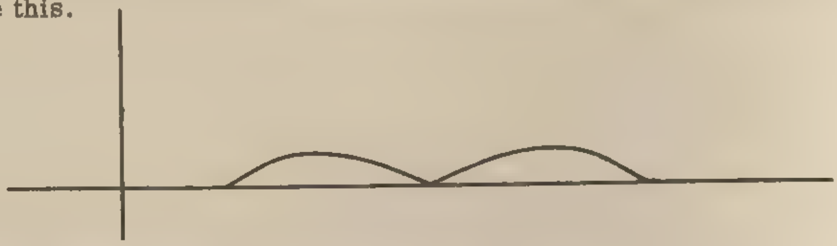
If the following figure

is a sketch of an actual wave (pulse) on a rope, it was correct not to draw in the horizontal axis because the rope cannot be in two places at the same time. In making a graph of this wave we can choose whatever vertical and horizontal scales suit our purposes. We might make a graph that looks like this.



Notice that we are graphing displacement from the "zero position" against horizontal distance along the rope. On another occasion we might watch one point on the rope and graph displacement against time. Both displacement-distance and displacement-time graphs are somewhat deceptive because they have the same general shape as the wave itself. If students still have difficulty with this distinction, you can mention that if the wave looked like this,

a graph of the absolute value of the displacement against a horizontal distance would look something like this.



Now the possible difference in appearance should be clear.

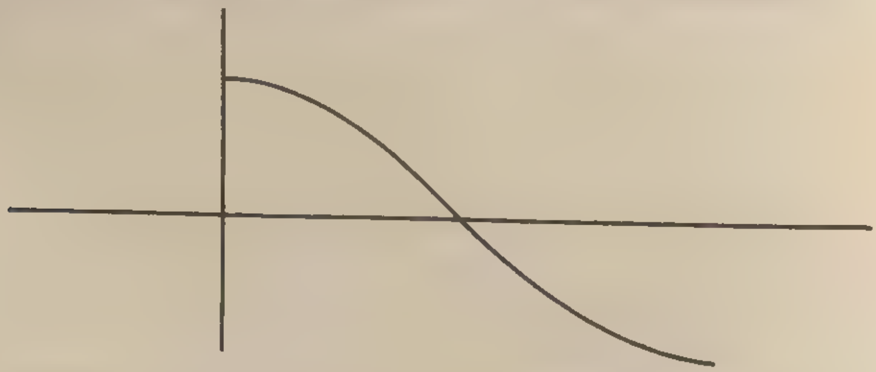
Students should consider Figures 16-1 and 16-2 together with subsequent similar figures as basic experimental data. Studying these figures should be like making careful observations in a laboratory. The following points can be made while you discuss these figures or during discussion of some of the problems.

1) Make it very clear that Figures 16-3 and 16-4 are idealized graphs of displacement of points on the rope from their normal position (plotted vertically) as a function of position of each of these points along the rope (plotted horizontally). Label the two curves with two different times (t_1 and t_2, or 0 and t, or 0 and 1 second, or 0 and 0.1 seconds). If possible, use differently colored chalk for the axis and for the curves. The portion of the axis below the curve is sometimes a source of confusion. Some students mistake the straight-line axis for the rope.

2) Ask students whether the graphs in Figures 16-3 and 16-4 really look like one of the instantaneous frames from Figure 16-1 or 16-2. (You can return to the blurring later.) They should realize that an actual pulse would not have a sharp corner. (It is impossible to produce an actual pulse whose slope changes so abruptly.) The sharp corners are used merely as an approximation convenient for preliminary study.

3) Once you have established that you are working with approximations, you can feel free to give students examples of triangular pulses and other "flat-sided" pulses which they can practice with easily. (Examples later.)

4) In Figures 16-3 and 16-4, what indicates the average velocity of each point? The arrows. Their direction gives the direction of the piece of rope; their length gives the speed. An optional activity (you may want to defer this until able students have had a chance to try Problem 9 on their own) is to graph the velocity of individual points on the rope in Figure 16-3 against horizontal distance. It would look like this.



Practice in drawing such graphs will be useful in solving Problem 9 (a).

5) What is the velocity of the wave? Be sure that the students realize that:

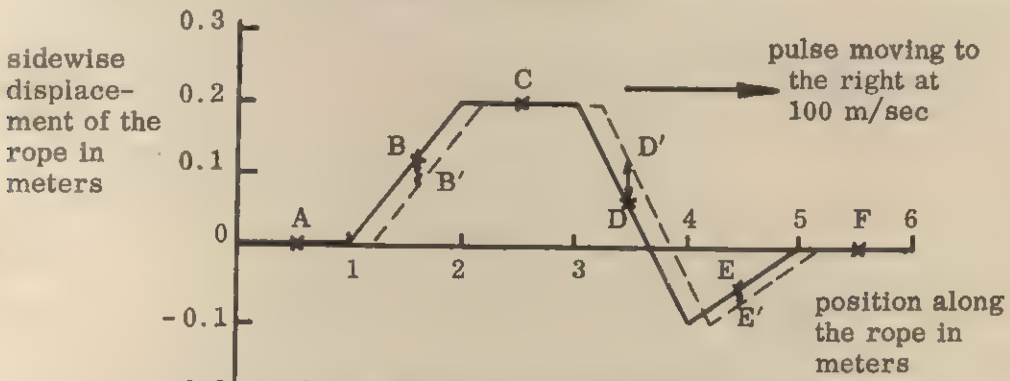
(a) The wave itself must have a single velocity; otherwise its shape would change.

(b) The direction of the wave's velocity is different from the direction of motion of an individual point on the rope.

(c) The magnitude of the wave velocity is not directly dependent upon the magnitude of the particle velocities. If the displacement of each point were doubled, the particle velocities would double but the wave velocity would be unchanged (or almost unchanged).

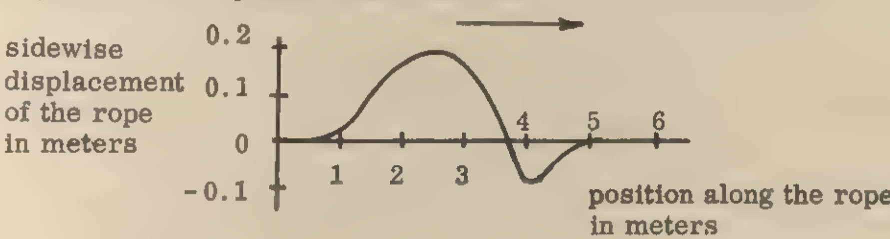
6) Why is there an apparent abrupt change in velocity at the edges of the pulse which is graphed? Is there an abrupt change of velocity (or blurring) in Figures 16-1 and 16-2? Would there be an abrupt change if you drew a displacement graph with rounded corners in place of Figure 16-3?

7) Be sure to include, either during the previous discussion or now, some quantitative work. An example which can be used is shown below.



The graph at one time is solid line.
The graph a little later is dotted line.

Note: If the students object to a complicated pulse like this, remind them it is an idealization. Perhaps the "real" pulse looked like this.

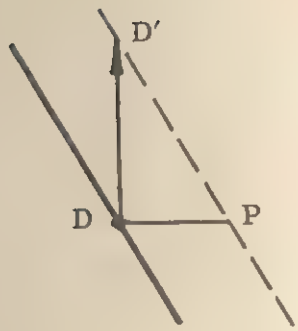


Also note that this is not a drawing but a graph; the vertical and horizontal scales are different.

Consider the qualitative problem first. Ask about points A, B, C, D, and E. Which ones are moving up, down, or standing still? Many students will know immediately. For other students be sure to draw in the dotted line curve showing where the pulse will be 0.002 seconds later. (At this qualitative stage you don't need to specify the 0.002.) It should be clear immediately that points A, C, and F have not moved and, therefore, that they have zero velocity. Point B, during this 0.002 seconds, moved down and therefore has a velocity down. D moved up to D′ and therefore has a velocity up. E to E′ shows that E is moving down.

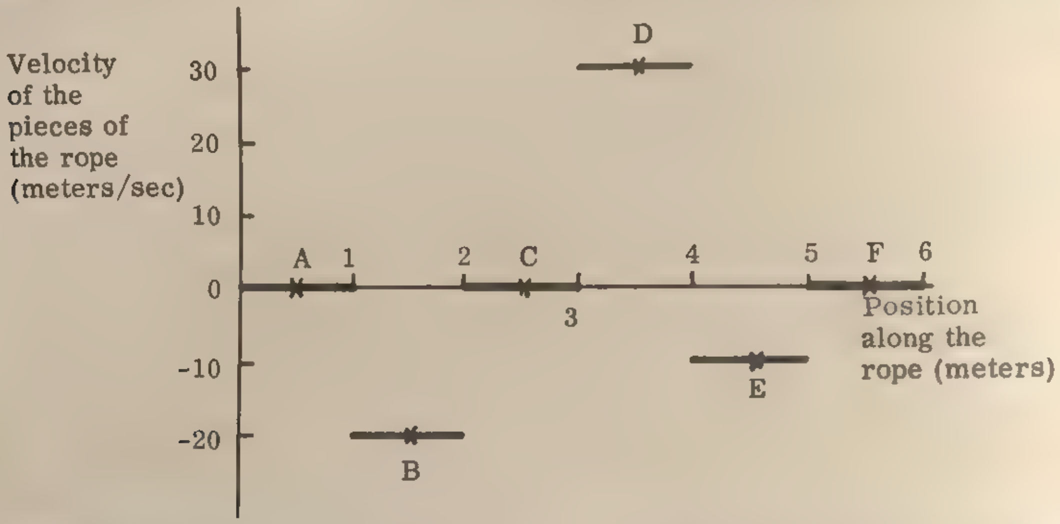
Next you may inquire about which points are moving fastest. It should be clear, just from the drawing, that D moved farthest in the given 0.002 seconds, B next, and E the smallest distance – except, of course, for those points that remained at rest.

The next question is a quantitative one. What is the velocity of point D? To answer this refer to a larger graph:

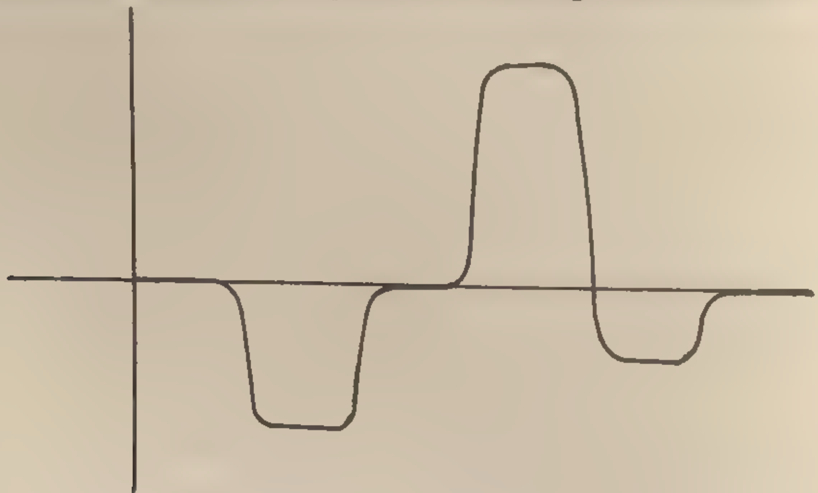


During the given time interval, D moved a distance DD′ while the wave moved a distance DP. In our particular case, the distance DP is 0.002 sec × 100 m/sec = 0.2 meters. Referring to the original graph it is easy to see that $\frac{DD'}{DP} = \frac{0.3 \text{ meter}}{1.0 \text{ meter}}$. Then $\frac{DD'}{DP} = 0.3$. $DD' = 0.3 \times 0.2 = 0.06$ meters. Since this motion occured in 0.002 seconds, point D is moving with a speed $0.06 \div 0.002 = 30$ meters/sec. We can do the same for points B and E, obtaining 20 and 10 meters/sec respectively.

It is now time to point out, if some student hasn't by this time, that the whole piece of rope from 1 to 2 meters is moving with the velocity of point B. The whole piece from 2 to 3 meters is standing still, etc. Now a graph of velocity vs position along the rope can be made.



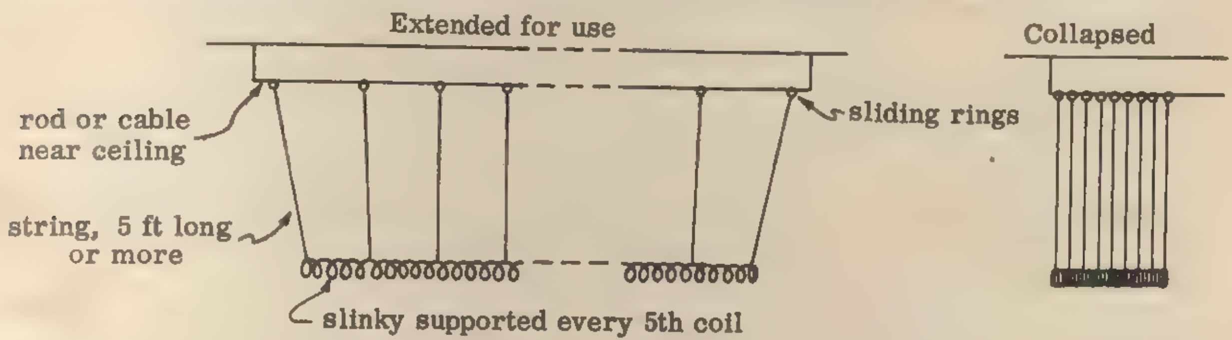
The graph is discontinuous. This is because our wave had sharp corners. Actual waves do not, so that this velocity graph in a practical case might look like this.



(One reason for including this discussion on graphs is that when two waves come together on a rope, their velocities add vectorially as well as their displacements.)

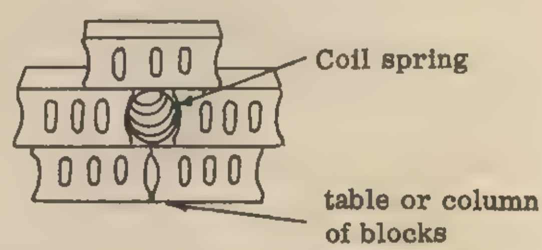
Supplementary Information on Setting up a String-Supported Coil Spring.

The triple-length coil spring "Slinky" (supplied as a PSSC laboratory item) is the most satisfactory coil spring for this purpose. About every fifth or sixth coil should be supported by a string. Pick a spacing, say, every fifth coil, and then stick to this spacing. In order to produce waves of high amplitude which can be seen easily by an entire class, the supporting strings should be five or more feet long. When it is being used, the spring will be extended to a length of about 20 feet. In most cases it will need to be on a collapsible mounting, which can be made by running the individual supporting strings up to curtain rings which can slide along a rod or taut cable near the ceiling.



For good viewing, the coil spring should be about waist high. It can be a little higher if it is necessary to clear tables or benches. Caution: if you use an overhead cable for the rings to slide on, it needs to be a rather heavy one and stretched tightly. Tight clothesline is a borderline case. This is because waves in the coil spring set up waves in the supporting cable which, in turn, influence the wave on the coil spring. If you support the cable at several points, clothesline will do, but then you cannot collapse the coil spring for storage because the supports will prevent sliding the rings together. Another warning: once you have a coil spring extended 20 feet or so, do not let it snap together. It can become so inextricably tangled that destructive surgery is the only recourse.

In order to keep the coil spring extended to about twenty feet, some horizontal force must be applied. The figure above shows the end supporting strings somewhat diagonal. Such slanting supports will not keep the coil spring extended. The ends must be held. A fixed end is easy. Use a table or a pillar of concrete blocks to firmly "pinch" the end. Concrete blocks, either starting from the floor or starting on a table, are particularly convenient because they can be made to grip the coil spring rigidly without damaging it.

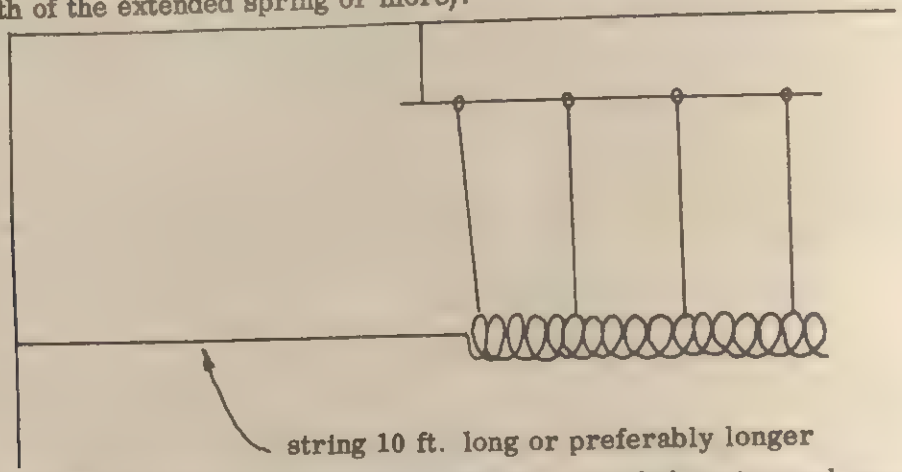


The person who is generating the pulses can stand at the other end. Shake a pulse into the spring and it will travel to the fixed end and reflect (coming back on the opposite side) back toward you. If you want the pulse to reflect again from your end, you must grip the end of the spring as rigidly as possible (rest your elbow on a table, etc.). Even so you will not do as well as as another pillar of concrete blocks. But holding one end with your hand, you will be able to trace a single pulse up and back between six and a dozen times.

Note: To observe these waves, students need to gather at the ends of the spring. A student near the middle of the spring does not see the wave nearly as well as does a student near an end.

Producing a free end is more difficult than producing a fixed end unless you have an

extra long room. Ideally you produce an open end by holding that end with a long string (half the length of the extended spring or more).



If you do not have a room long enough for both the extended spring and an open end, you will need to experiment with compromises involving shortening the length of the spring in order to increase the length of string at the open end.

Since there is very little natural damping in a suspended spring, one of the first problems you will meet is how to make a "disturbed" spring return quickly to rest. Most attempts to force the spring into a rest position with "bare hands" only result in more total agitation. (If you learn to "trap" the wave near one end, you can cut down its amplitude fairly quickly.) What you need is a way of extracting energy from the spring. Hold a large piece of cardboard or similar material loosely against the spring near the middle. Each time the wave crosses the cardboard the spring should scrape back and forth. The motion will die out quickly. If the cardboard is held so tightly against the spring that it cannot move, the motion will not die out rapidly. Experiments with different materials, and different forces of application should result in finding a satisfactory method of rapid absorption.

If you suspend a coil spring, you may want to keep it available for the introduction of Chapter 18.

Supplementary Note on Speed of Transverse Waves on a Rope.

There is no need to give students an analytic expression for the speed of a wave on the rope. They will not yet know enough about mechanics to recognize the units. But you might find the formula useful if students are doing extra work, measuring speeds in the laboratory in order to get a qualitative idea of what the speed, v, depends on. The speed v is given by $v^2 = \frac{\text{Tension in rope}}{\text{Mass per unit length of rope}}$. This shows, for example, that a wave on a coil spring (high mass and low tension) moves slowly. If the tension is expressed in newtons and the mass per unit length is given in kilograms per meter, the speed will have the units meters per second.

In the above relation for waves on ropes we assume that the rope is completely flexible and elastic. Furthermore, we assume that the amplitude of the wave is small enough so that the local tension near the wave shape is the same as the tension on all parts of the rope. (If the stretching were severe and the local tension increased, for example, the velocity would be greatest near the most stretched spot and the wave would not maintain its shape.)

Note on Wave Dynamics: Insofar as you can, you will want to avoid getting into discussions of why a wave moves as it does. Students will need more background in mechanics to make such discussions worthwhile. As background for the teacher, general information on wave dynamics is included as Appendix 7 at the back of this volume.

Section 3 - Superposition: Pulses Crossing

PURPOSE To present the superposition principle.

CONTENT The net displacement of any point which is affected by several independent pulses is the algebraic sum of the displacements which would have been caused by each of the individual pulses. This is the superposition principle: it is a fundamental property of what physicists call waves.

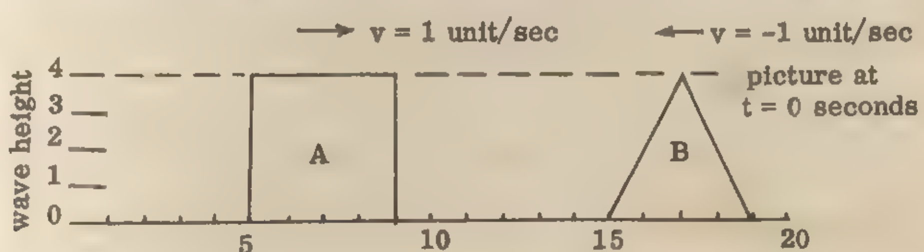
EMPHASIS Students need to understand the superposition principle for displacements in order to appreciate interference in ripple tanks (and thus in light). It is not essential that the work be extended to the superposition principle for velocities. However, if you have time, you may want to extend your treatment of this section (and the chapter) to include superposition of velocities.

COMMENTS In discussing the constructions involving superposition such as those given in Figures 16-6, 16-8, and 16-9 it may be well to check to make sure that students understand that the dashed curves represent the displacement which would exist if each pulse were present alone. If they take pictures in laboratory, they will never see the dashed parts of Figure 16-6 in a picture. This point should be mentioned several times as you work with superposition at the blackboard.

Ask a student to explain Figure 16-8B in order to see how well he appreciates the superposition principle and the distinction between a graph of a pulse and a picture of the pulse. Be sure the class realizes that the dashed curves are graphs of the displacements that would be caused by the separate pulses. A picture of the rope at the instant of part B of Figure 16-8 would be merely a horizontal line.

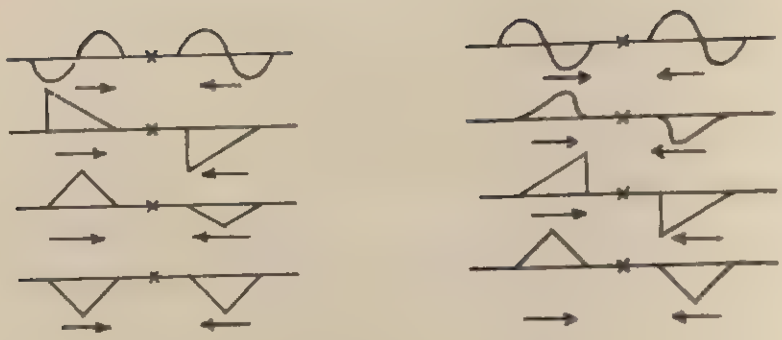
A qualitative perception of the superposition principle as applied to velocities will deepen students' understanding of Figure 16-7 and the related text. Students who have missed the significance of the blurring as an indication of velocity in Figures 16-1, 16-2, and 16-5, may not realize that the coil spring is essentially undisplaced in the middle (fifth) frame of Figure 16-7. Furthermore, an awareness of superposition of velocities will help with HDL Problems 8, 9, and 10.

DEVELOPMENT Give students practice in both wave motion and superposition by drawing two simple wave shapes (rectangular, or triangular, or a mixture) and asking questions such as:



If we had only pulse A, what would be the shape of the rope after 1 second? After 2 seconds? After 10 seconds? If we had only pulse B, what would be the shape of the rope after 1 second? After 2 seconds? After 5 seconds? After 10 seconds? If only pulse A were present, where would the point at horizontal position 12 be after 2 seconds? If only B were present? If both were present? Etc.

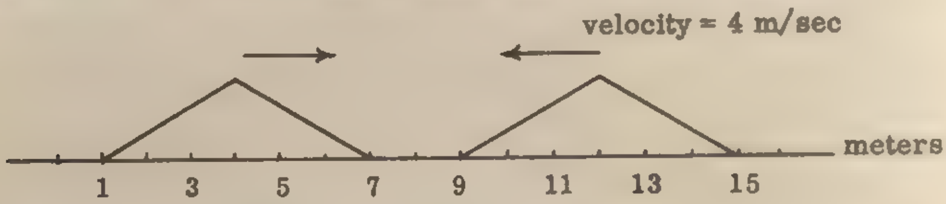
Although most students will follow readily Figure 16-9 and its explanation in the text, they may have only a hazy idea of what similarity must exist between two pulses if the point at which they meet is to remain undisturbed. Since this same question recurs in the next section and at several places in Chapter 18, it is worthwhile to discuss it in class. You may wish to draw several examples on the board and make sure that the students understand which ones do not leave their midpoint on the rope undisturbed. Those on the left do and those on the right do not:



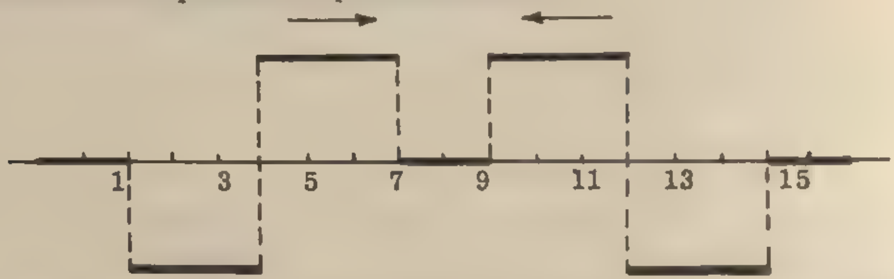
* * *

If you have time, you may want to give the students some of the following treatment of superposition of velocity.

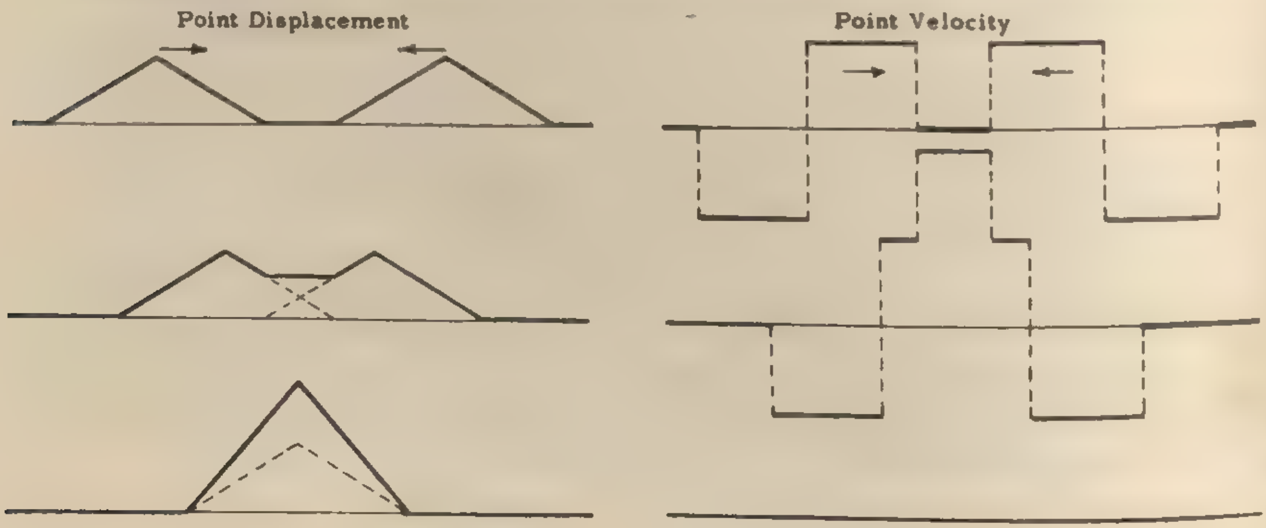
Two triangular pulses moving towards each other give nice examples for studying velocity superposition if you ignore the sharp corners. (Sharp corners imply extremely high accelerations; if the corners were rounded a little, the velocity changes would be physically realizable.) Consider the two pulses shown below moving toward each other:

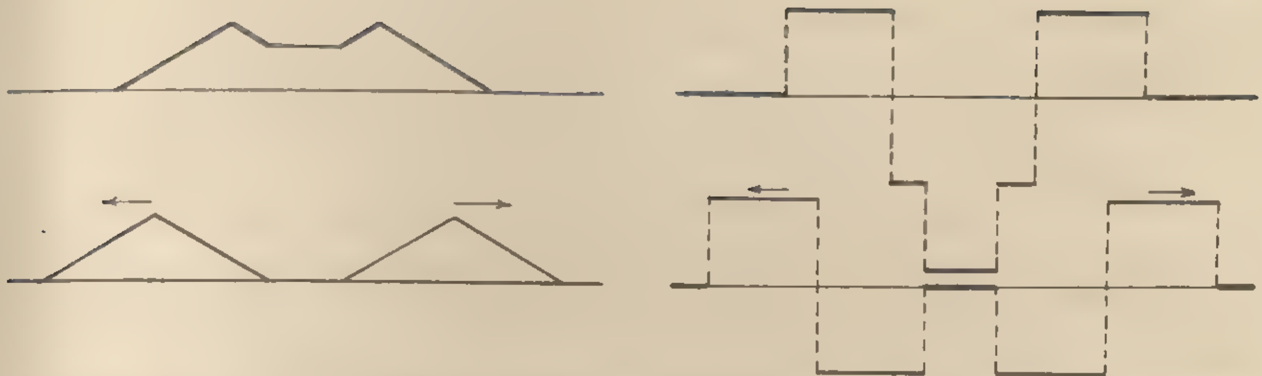


A graph of velocities of points of rope at this time is:



The following sequence of graphs shows points and velocity displacements as the pulses cross each other.



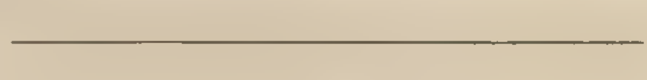


Note that at t = 1 second, the two pulses add to be twice as high, but at this instant the velocity everywhere along the rope is zero! Compare this example with Problem 9 and Figure 16-16.

If the two pulses have opposite polarity,

the velocity graph looks like

As the two pulses cross, there would be a time when displacements would add to zero giving a displacement graph like this

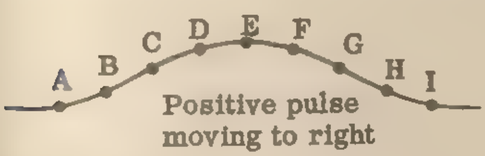


but the velocities would be

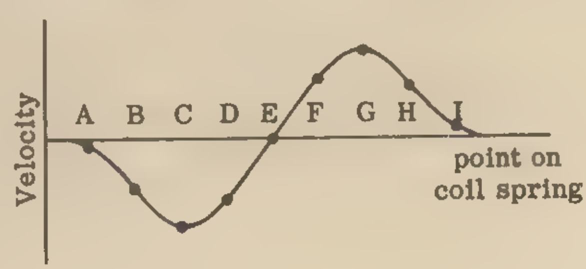
This is the case in Figure 16-7.

* * *

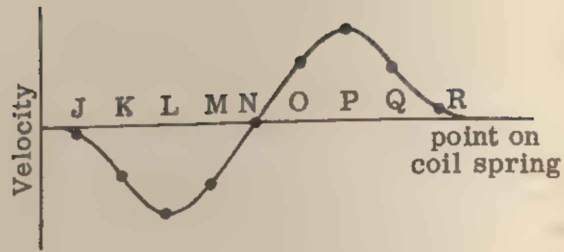
While it is not at all vital, in an extended treatment of this section with able students, you might want to use triangular pulses to show that the particle velocity is proportional to the instantaneous slope. Once this is established, you can give bright students who ask about Figure 16-7 the semi-quantitative explanation below.



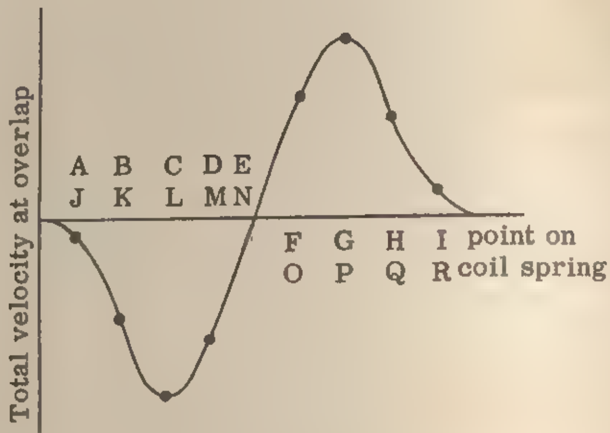
(See frame 1 or 2 top, Figure 16-7)



Negative pulse moving to left



Total displacement at overlap is zero (See frame 5 from top, Figure 16-7)



Section 4 – Reflection and Transmission

PURPOSE To describe (not explain) what happens when a pulse reaches a boundary.

CONTENT In general, a pulse on a coil spring is partially reflected and partially transmitted if it comes to a boundary between different kinds of materials.

a. If the boundary is a rigidly fixed end, the pulse is reflected back upside down.

b. If the boundary is a free end, the pulse is reflected back right side up.

c. If the pulse goes from a "light" medium to a "heavy" medium, the transmitted pulse continues right side up, the reflected pulse comes back upside down.

d. If the pulse goes from a "heavy" medium to a "light" medium, both the transmitted and reflected pulses are right side up.

e. If the two media have nearly the same mass per unit length, most of the pulse is transmitted. If they have equal masses per unit length, there is no boundary (as far as the pulse is concerned) and the whole pulse is "transmitted".

EMPHASIS It is important for students to realize that partial transmission and partial reflection occur at boundaries. They should also know that the polarity of the reflected wave depends on the exact nature of the boundary. They should not be expected to have a good mechanical idea of why the pulses reflect as they do. Observation of actual reflection and transmission of pulses in laboratory work or in class room demonstrations is essential.

CAUTION When you refer to whether a reflected pulse is right side up or upside down, use these phrases, or use "erect" and "inverted", or introduce the term "polarity". Avoid the word "phase". "Phase" is used in Chapter 18 and has meaning only for a periodic wave. If "phase" is used here in place of "right side up", or "erect", or "positive polarity", students may not get a precise notion of phase when they encounter the concept of phase in periodic waves.

COMMENTS While it is important for students to see that the polarities of reflected and transmitted pulses depend upon the nature of the boundaries they encounter, the specific polarity of the reflected pulse on a spring is not important to the later development of waves. However, the idea of change in polarity is interesting in itself, particularly in that it gives the students something else to look for in the laboratory. (When a phase change is invoked to explain the interference pattern from a thin film in Chapter 9, the student needs to carry over the idea that reflection can be quite different at two different types of boundaries. There is no direct analogy between waves on springs, water waves, and light waves which would make the specific polarity of the reflected wave on a rope important.)

When students watch the reflection of waves on, say, a suspended coil spring, they will inevitably press for an "explanation". "Why do waves reflect the way they do at open and closed ends?" Formally, the same old answer applies: "You do not know enough about mechanics yet to understand why." However, students will enjoy inventing (or hearing from you) "common sense" explanations. Such explanations are fine as long as they are treated as little more than mnemonic devices.

Some students may reject an explanation of reflection which suggests that the wall generates a cancelling pulse. "How does the wall know just what kind of pulse is required, and how does a fixed wall know just when to send it out?" Such questions seek to go more deeply into mechanics of reflection than is possible now. As an alternative explanation (the not to be taken very seriously type), consider an "intelligent hand" replacing the fixed wall. If one tries to hold fixed one end of a rope as a pulse comes toward that end, he has to push sideways on the rope an amount equal and opposite to the push on his hand by the pulse. (Take this as straight intuition; don't mention Newton.) Now suppose that he exerted this kind of push on a stationary rope. A pulse of opposite polarity would be generated. Don't go into much detail. Students may have other explanations. See if, in this spirit, students can "explain" reflection at a free end.

Section 5 - Idealizations and Approximations

PURPOSE To illustrate how idealizations and approximations are used in science.

CONTENT Considering a pulse on a perfectly flexible coil spring which has no internal resistance and is kept in a vacuum, is an example of an idealization. Since the effects found with real coil springs do not depend on those of its features which are eliminated in the idealization, the idealization can be used to advantage.

COMMENT Be sure that the students do not confuse an idealization with a simplified experimental setup. Sometimes you use the idealization as a guide to setting up an experiment. At other times you merely use the idealization as a guide for what to observe and what to ignore in the experimental equipment already available. A third use of the idealization, which is not developed in this section, is as a model on which calculations are based.

DEVELOPMENT You can ask students to give examples of idealizations from earlier parts of the course. The point of such a discussion is not to produce a definition of "idealization" or "approximation". Instead, it should be directed to the idea that, in studying or working with complex phenomena, it is highly important to separate the essential from the inessential. Far from being a superficial explanation, an appropriate idealization or approximation is often a powerful intellectual tool for dealing with a complicated matter. Making the right idealization is often the most important part of the problem.

Here is a partial list of approximations that have been used in studying about light:

1. Reflection is either completely specular or completely diffuse.

2. A spherical mirror is like a parabolic mirror.
3. A spherical mirror provides a sharp focus.
4. A single index of refraction applies to all white light.
5. Lenses form perfect images.

Section 6 – A Wave Model of Light?

PURPOSE To provide a transition to the study of waves on the surface of water.

CONTENT Having seen that waves can cross without interfering with each other, and that waves can be partially reflected and partially transmitted at a boundary, we wish to inquire whether waves can be refracted. To do this, we must consider waves which are not confined to a single straight line.

EMPHASIS Treat briefly.

Chapter 16 – Introduction to Waves

For Home, Desk and Lab – Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Short answers are not given for the problems of this chapter because all the problems require drawings or discussions.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1	1*, 2			1*, 2	
2	3*	4, 5*		3, 4, 5*	
3	6*	7, 8	9*	6*, 7, 8, 9*	
4	11*	10*, 12*		10*, 11, 12*	13*
5	15	14		15	14

COMMENTS AND SOLUTIONS

PROBLEM 1

Suppose you look out your window and see your neighbor across the street sitting on his porch. In how many ways could you do something to attract his attention, make him move, or otherwise influence his actions? Which ways involve mass transmission and which ways wave motion?

It is not the aim of this exercise merely to inform students that some phenomena (e.g., sound) are wave motions. The use of this exercise should be to give the students a better idea of how wave motion differs from the motion of an object. It should be made clear that one must know something of the detailed mechanism of a communication process in order to classify it. Shouting to a friend is a good example of wave motion, if, and only if, students realize that individual air molecules oscillate back and forth, but do not travel from mouth to ear.

See Teacher's Guide, for Section 1, particularly the Development. A clear example of mass transmission is throwing a rock. A pulse sent along a rope is an equally clear example of waves. Discussion of the mechanisms of such familiar wave phenomena as sound and radio waves should be avoided at this stage. It will be sufficient to note that, for these examples, it is difficult to find evidence of mass transmission. Many students will be uncertain about telephoning. (The electrical signal which moves along the wire is an electromagnetic wave; the electrons in the wire move only slightly.) Sending the heat from a fire is ambiguous. Radiation is an example of waves; convection involves mass transmission; conduction is most appropriately an example of waves.

Remember that, at this stage, the classification of light is uncertain. Since the simple particle model did not work, "mass transmission" is unjustified. On the other hand, some particle model might work. At least until we get some wave evidence, listing light under waves should be considered as tentative.

PROBLEM 2

What do you think are the important factors that determine the speed of a starting pulse? Will the speed be the same for a line of trucks and a line of passenger cars?

Class Discussion (Can follow Section 1)

This is an extension of an introductory discussion (page 70) in the text. Because the problem can be answered differently depending upon the underlying assumptions, this exercise, if used, is more suited to class discussion than home assignment.

The speed of a "pulse of starting" depends simply upon the time intervals between the successive starts of the cars in the line. This may be influenced more by such things as driving skill, habits of driving safety, and reaction time than by the acceleration of the car ahead. The problem boils down to (1) What does the driver use as a "signal" to start? The instant the vehicle ahead starts? The car ahead covering some "reasonable" distance or getting up to some "reasonable" speed? (2) After getting a starting "signal" how long does it take the driver to get his vehicle in motion? How do alertness, reaction time, safety habits, driving skill, etc. affect this?

Assuming no difference in reaction times of car and truck drivers, one might say that trucks, being more massive, would accelerate less rapidly than passenger cars; consequently the starting wave for trucks might be slower. This conclusion would be true if the drivers of both trucks and cars waited to start until the vehicle ahead got up to a "reasonable" speed, say, 3 mph; or until the vehicle ahead had moved a "reasonable" distance, say, 10 ft. The lower acceleration of the trucks would then cause a longer time interval between the successive starting "signals" for trucks than cars.

PROBLEM 3

If an upward pulse is moving along a length of coil spring from the left to the right, how does a single point on the coil behave?

(a) Does it move up and then down, or does it move down and then up?

(b) Does it make any difference if the pulse moves from right to left?

Although this problem is very easy, it emphasizes that the particle motion and the wave motion are quite independent.

a) A single point on the tube moves up then down.

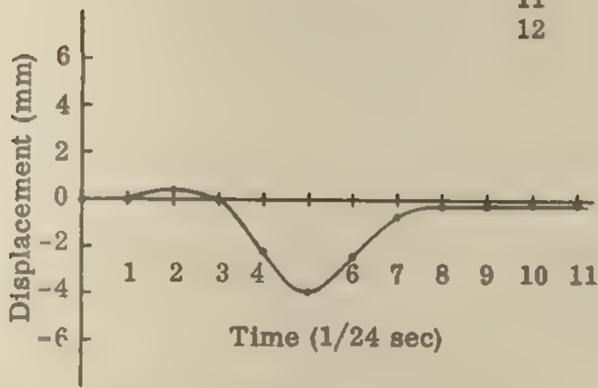
b) This is true whether the pulse moves from right to left or vice versa. The particle motion is determined by the fact that the pulse is "upward".

PROBLEM 4

Fig. 16-2 shows the displacement of a point on a spring as a pulse goes by. Make a graph showing the displacement of this point as a function of time. Plot displacement vertically and time horizontally with $\frac{1}{24}$-sec intervals (the interval between pictures of Fig. 16-2).

Displacements in Figure 16-2 were found by measuring the distances from the bottom of the coil spring to the tip of the reference arrow.

Frame number	Time (1/24 sec)	Distance (mm)	Displacement (mm)
1	0	5.2	0.0
2	1	5.2	0.0
3	2	5.6	0.4
4	3	5.2	0.0
5	4	3.0	-2.2
6	5	1.3	-3.9
7	6	2.8	-2.4
8	7	4.5	-0.7
9	8	5.0	-0.2
10	9	5.0	-0.2
11	10	5.0	-0.2
12	11	5.0	-0.2



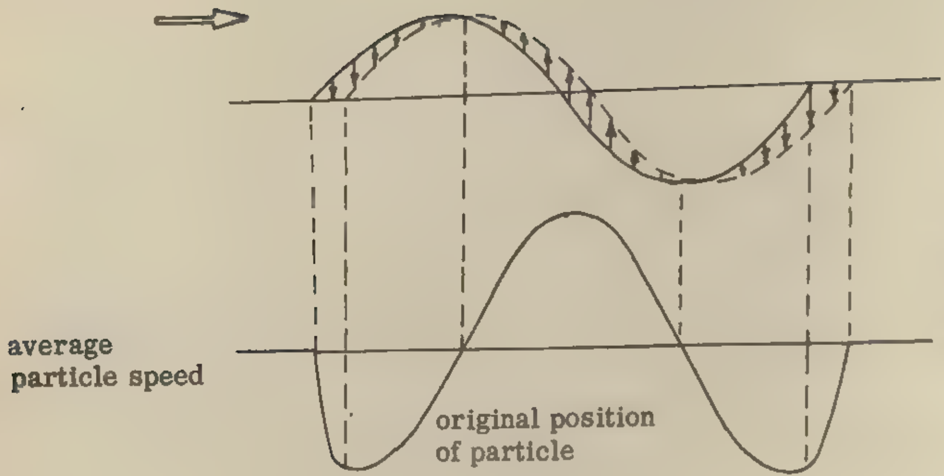
Supplementary note: This problem can be supplemented by considering a pulse, moving from left to right, with a displacement which as a function of distance, looks like

 . Have students draw the displacement of a point as a function of time when this pulse passes by. This will show the similarities and differences between graphs of displacement vs. distance and graphs of displacement vs. time.

PROBLEM 5

Sketch the motion of the spring for the pulse in Fig. 16-14.

This problem provides a good simple test of students' knowledge of the relation between wave shape and particle speed. (See Development in Teacher's Guide, Section 2, Chapter 16.) Make it clear to students whether you want only two successive positions, the "velocity arrows", or a graph of average particle speed as a function of initial particle position.



Note that the average particle speed depends on the assumed time interval.

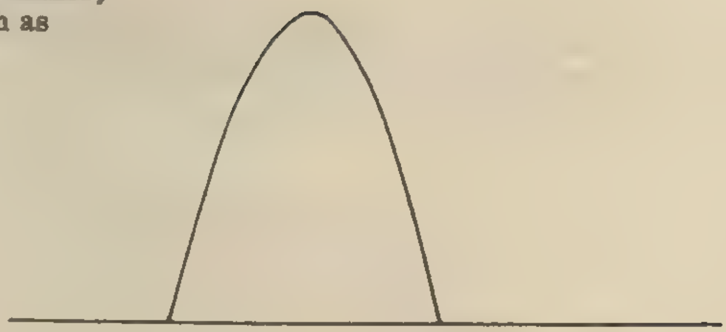
PROBLEM 6

Using the two pulses shown in Fig. 16–15, determine the size and shape of the combined pulse at this moment. Do the same thing for several other positions of the pulses.

Individual pulses as they would look if only one were present at a time.

Add the displacements graphically to get

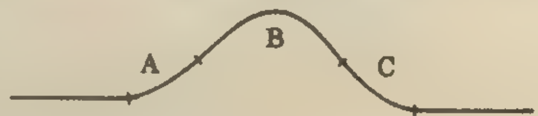
When the pulses are on top of one another, the combined pulse is twice as high as either pulse alone.



PROBLEM 7

The seventh picture from the top in Fig. 16-5 shows two pulses at the moment of crossing. Specify the pieces of spring that are moving and their direction of motion.

In the seventh picture from the top of Figure 16-5, the lack of blur on the left side and only slight blur on the right indicates that, at this instant, there was either no motion or that the motion was very slow.

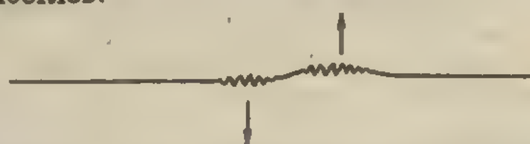


In studying the sequence of pictures (top to bottom of page) it can be seen that sections A and C of the rope have moved downward during the interval between the 6th and 7th frames. In the 7th frame, sections A and C are at rest. Between the 7th and 8th frames, these sections will move upward. Between the 6th and 7th frames, section B moved up and is at rest in the 7th frame. Between the 7th and 8th frames, section B will move downward.

PROBLEM 8

In the fifth picture from the top of Fig. 16-7, which points are moving and in which direction do they move?

Students can do this problem by examining the pictures whether or not they understand superposition of velocity. However, the problem is particularly instructive if the students have been exposed to the superposition of velocities.



As background, look at the fourth and sixth pictures. In picture 5 the points are moving as shown. Note the unblurred part in the middle which is at rest. This is where the velocity changes direction. (Insofar as the two pulses are exact opposites, the midpoint is a node.)

PROBLEM 9

In the sixth picture, Fig. 16-16, we see the superposition of two equal pulses, each of which is symmetrical about its center line.

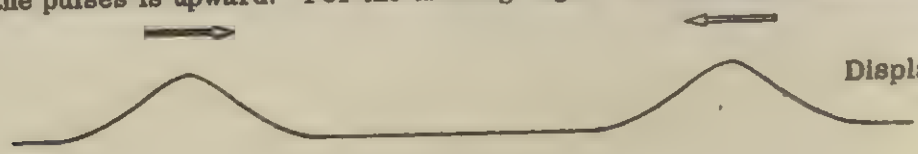
(a) The absence of blur indicates that there is no motion at this instant. Show that this is true by using the principle of superposition.

(b) Assume you deform the coil spring in the same manner as shown in the sixth picture. What will happen when it is released?

<u>Assignment</u> and <u>Class Discussion</u> (Can come after Section 3, but only if you have discussed the superposition of velocities).

Students will not find this problem very hard if they realize that velocities superimpose as do displacements.

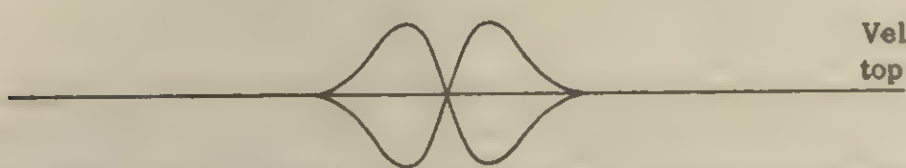
a) It is probably best to draw a graph of velocity of the points of the coil spring vs. position on the coil spring. The student should note that the velocity of the leading edges of the pulses is upward. For the trailing edges, the velocity is downward.



Displacement of the coil spring

Velocity of the particles of the coil spring

"Velocity pulses" a little later



Velocity pulses on top of each other

The principle of superposition holds for velocities as well as for displacements. Add the velocities of the two waves shown in the last figure to get zero. Thus momentarily the velocity of the spring is zero.

b) If the spring is deformed as shown in the sixth picture of Figure 16-16, held at rest, then released, two pulses each half as high as the original deformation, will go out in opposite directions.

This can be made plausible by using the following argument: "Look at Figure 16-16, picture 6. What happens afterwards? Two pulses go out in opposite directions as stated. If we cover up pictures 1, 2, 3, 4, and 5 would we know how the spring came to be deformed as in picture 6? No. Then no matter how it came to be in this condition the same thing must happen afterwards."

Some students may say that we could get the same thing as in picture 6 if a single pulse were traveling along the spring. Although the displacement would be the same there is obviously a difference because if there were only one pulse, parts (especially the leading and trailing edges of the pulse) of the spring would be moving. It might be a good idea to point out that if we wish to predict what will happen at future times to a wave, we must know the displacement and velocity of every point on the spring.

Note that this is a new fact which is by no means obvious. Students might suppose, for example, that the acceleration or some as yet undefined kinematic or dynamic property would help govern the subsequent motion. The essential new fact is that the displacement and the velocity completely determine the future (and incidentally the past) of a wave on a rope if no extra forces act.

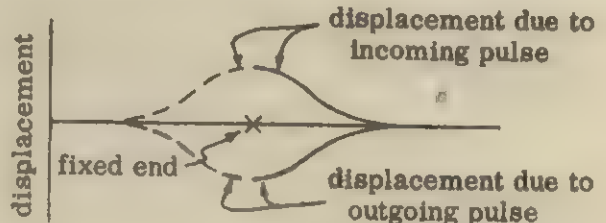
Students will enjoy experiments in the lab to confirm these ideas.

PROBLEM 10 The sixth picture of Fig. 16-10 shows the spring at an instant when the spring is almost straight. Explain why there is an instant when this happens.

In Figure 16-10 there is a symmetrical pulse coming into a fixed end and being reflected. It will look like this.

Halfway through reflection the displacement of the spring is the sum of the displacements of the incoming and outgoing pulse.

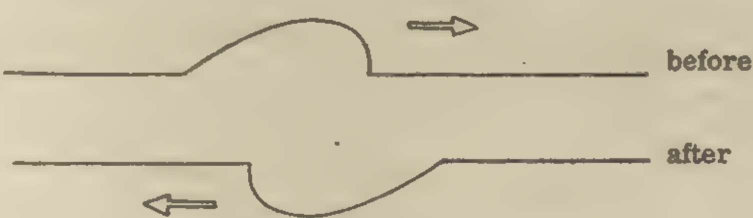
Note that the dotted part of this graph does not represent what happens on the spring. It merely shows "the rest of the pulse".



Note that this graph of displacement should not be confused with a picture of the spring. The spring cannot be in two vertical positions at the same time.

PROBLEM 11 Consider the asymmetric pulse coming from the *left* in Fig. 16-6. Draw the shape it will have after being reflected at a fixed end.

The answer to this is inherent in the first line of Figure 16-9 and the text's description of reflection, particularly the second paragraph of Section 4.

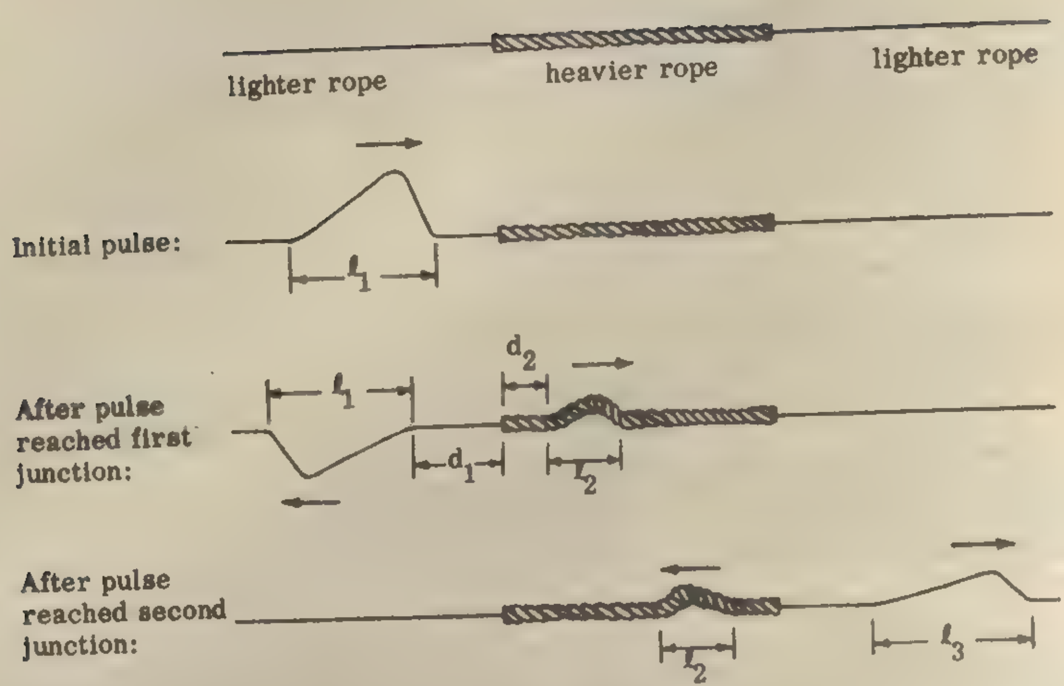


PROBLEM 12 Imagine that you have a medium consisting of three sections of rope: light, heavy, and light. If you shake in a pulse, what will happen?

Although this question simply asks students to restate what is said in Section 4, it is well worth doing. In order to be sure the students understand, ask them to include a sketch.

The pulse will be partially reflected and partially transmitted at each junction. (Do not bother with the exact fraction which is reflected; the greater the difference in the ropes, the greater the fraction that is reflected.)

Note that the speed of the pulse will be less on the heavier rope. Hence d_1 is greater than d_2, and ℓ_1 is greater than ℓ_2. In fact, $\frac{d_1}{d_2} = \frac{\ell_1}{\ell_2} = \frac{v_1}{v_2}$; it is not necessary for the students to realize that this ratio is $(m_2/m_1)^{1/2}$.



PROBLEM 13

Hold one end of a long rope with the other end tied to a rigid support. Stand looking along the rope and generate a wave by moving your hand through three fast clockwise circles.

(a) Describe the wave generated.

(b) Describe the reflected wave.

(c) Describe the motion of a particle of the rope as the wave passes forward and back.

This might be a good test of students' ability to observe effects when they are not sure of the outcome. One difficulty is that it is hard for students to understand why the circular wave reflects as it does. (In particular, if students try to use an argument apparently similar to the one given in Section 4, they will get the wrong answer!) Another difficulty of many students is in finding the right vocabulary to describe a circular wave.

It will be hard to generate a good three-turn wave, but a one-turn or two-turn wave shows the necessary reflection properties.

a) The wave generated is a three-turn "right-hand" helix. It has the same appearance as the threads on an ordinary screw (almost all screws are right-handed) if the screw is held pointing away from you.

b) As viewed by the observer, the rope seems to be rotating the same way in the reflected wave. However, since the wave is moving in the opposite direction, the reflected wave would be called a three-turn "left-hand" helical wave. You would not get this picture by turning a right hand screw around; you would need a left hand screw to serve as a model.

c) As the wave passes, a particle moves over a circular path three successive times. As the wave is moving away from the observer, the particle moves over its circular path in a clockwise direction (as viewed by the observer). As the wave moves toward the observer, the particle also moves in a clockwise direction (as viewed by the observer).

Note that for this circular wave, the particle motion after reflection is in the same sense (circularly) independent of whether the reflecting end of the rope is fixed or free.

PROBLEM 14

Investigate how a pulse on a rubber tube attenuates under various conditions such as on bare ground, grass, sidewalk, and in the air.

It probably will be best to have students make approximate measurements (rather than merely qualitative judgements). Of course, since the attenuation will vary depending on the way the tube touches the surface, there is no point in trying to get a number which expresses the attenuation. On the other hand, the students should have some specific observations to make and specific numbers to record. For example, they might measure how far a pulse travels, or how many times it is reflected back and forth, or the length of time the rope continues to move.

Naturally the attenuation will be the greatest when the friction is largest. Although most students realize that the "roughness" is the main factor, many are uncertain as to which materials present the "roughest" surfaces. Try not to be diverted, at this stage, to a discussion of dynamics or of friction.

PROBLEM 15

If the spring in Fig. 16-1 could be observed with very precise instruments, minute variations in speed would be found. In view of this, are we still justified in making use of the idea of a constant speed for a pulse?

The main point of this problem is to get the student to think about idealizations and approximations.

Whether we are justified in making use of the idea of a constant speed for a pulse depends on how we intend to use this idea. For example, the only place that constant speed is needed in the text (after it is mentioned in the third paragraph of Section 3) is in the discussion of the stationary point formed by two opposite waves. (See the last paragraph on page 252 which is a description of Figure 16-9.) In that case, if the wave speed varied slightly, the point at which cancellation occurred might be slightly displaced from midway between the pulses. Unless the exact location of the point were very important, the constant speed approximation would be justified.

Another effect which might result from a variation of speed is a change in pulse shape. If different portions of the wave travelled with different speeds the wave shape would vary. Once again, unless the exact shape were important for whatever was being considered, this effect need not be considered.

Notice that if the wave shape changes due to attenuation, there is some question of exactly what you mean by wave speed. Merely speaking of the speed of a wave (whether it is constant or variable) implies either that the shape remains exactly the same or (more usually) that the changes of shape are small enough so that corresponding points of the wave can be identified in two positions. Thus, the leading edge, or the peak, or some particular kink must be taken as the reference point on the wave. If the wave shape varied significantly one would speak of the speed of the reference point rather than the speed of the wave. Thus, from the point of view of wave speed, we are entitled to idealize that an actual wave is not attenuated, if we can recognize corresponding points on the wave when it is in two positions.

It is not easy to think of practical situations in which the variation in wave speed along a rope is important. The following example, although artificial, might serve as motivation for a stimulating class discussion, for a laboratory exercise, cr for a demonstration. Assume that you want to find, with very high precision, the midpoint between two points and that you could connect a rope between these points. How could you make the measurement with waves? Guess at a point, put a wave on the rope, and see whether the reflected waves from both ends arrive back at the same time. Would your measurement be affected if one half of the rope was wet? Slightly stiff from paint? Partially rubbing against an object? etc. Which possible causes of error could you catch by interchanging the two halves of the rope?

Chapter 17 - Waves and Light

The purpose of this chapter is to familiarize students with the phenomena associated with wave propagation in a plane. The ripple tank is introduced in the text and in the laboratory as an instrument for observing and experimenting with these phenomena. Observation of the ripple tank gives evidence that waves undergo reflection, refraction, dispersion and diffraction. The analogy between the behavior of water waves and the behavior of light is so strong that it suggests that light may be propagated as a wave disturbance of some kind.

Work on this chapter should be built around the laboratory experiments, using the text to give continuity and to review what has been learned in the laboratory. It is not sufficient that students merely observe "demonstration experiments" with a ripple tank. Students must work in small groups and study each of the phenomena closely in order to get an adequate understanding of wave motion.

The work in this chapter has proved to be among the most exciting in the course. Very few students will begin it with any real familiarity with the ideas presented, despite years of casual observations of water waves. Here is a real opportunity to drive home some of the aspects of "the scientific method". Experience with quantitative and careful observation of wave motion – isolating events, correlating phenomena, generalizing results – can teach students more science than hundreds of pages in books. You will need to be generous with allotments of laboratory time.

CHAPTER SUMMARY

Sections 1 through 4

a) Water waves are surface phenomena. Their crests act as converging cylindrical lenses for light passing normally through them.

b) Segments of wave fronts move along the normal to the wave front.

c) For reflected waves, the angle of incidence is equal to the angle of reflection.

d) A periodic wave is a disturbance pattern which, at any point in space, repeats itself at a regular time interval, called the period.

e) The speed of propagation, the wave length and the period are related by the equation $v = \frac{\lambda}{T}$.

Section 5

a) The speed of propagation of water waves depends on the depth of the water.

b) Snell's law, $\frac{\sin i}{\sin r} = n_{12}$, a constant, applies to the directions of propagation of water waves crossing a region of discontinuity in speed of propagation.

c) The wave model gives $n_{12} = v_1/v_2$, whereas the particle model predicts $n_{12} = v_2/v_1$. Measurements of the speed of propagation of light verify the wave model prediction.

Section 6

a) The speed of propagation of water waves varies with the frequency. This dependence is called dispersion.

b) Refraction of light varies with the color. Thus if light is wave-like in nature, it is possible that color is a function of frequency.

Section 7

a) Water waves do not cast distinct shadows when they pass objects whose size is equal to or less than λ. Therefore, if light is the result of a wave disturbance, since it does cast sharp shadows of ordinary objects, its wave length must be very small compared to a centimeter, for instance. The bending of waves near their edges is called diffraction.

[illegible] less than [illegible] of a wave disturbance [illegible]

[illegible] CHAPTER 17

[illegible] with this chapter. [illegible] laboratory [illegible] if you are trying to [illegible]

[illegible] laboratory [illegible] Experiments [illegible] the laboratory [illegible]

[illegible] for this chapter [illegible] with the [illegible] Experiments which should [illegible]

Section	[illegible] schedule for Part II			[illegible] schedule for Part II		
	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
[illegible]	–	1	II-[illegible] F		1	II-[illegible] F
Sec. [illegible]	1	[illegible]	II-[illegible]	1	1	II-9
Sec. [illegible]	1	1	II-[illegible] F		1	II-10 F
Sec. [illegible]	1		–		–	–
Sec. [illegible]	1	[illegible]	II-11 F	1	[illegible]	II-11 F

[illegible] MATERIALS FOR CHAPTER 17

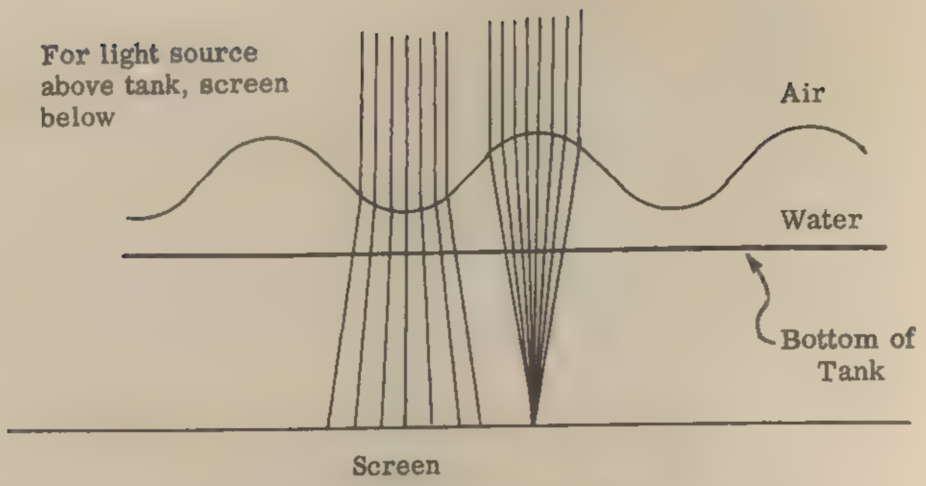
[illegible] Experiments [illegible] — See yellow pages for suggestions.

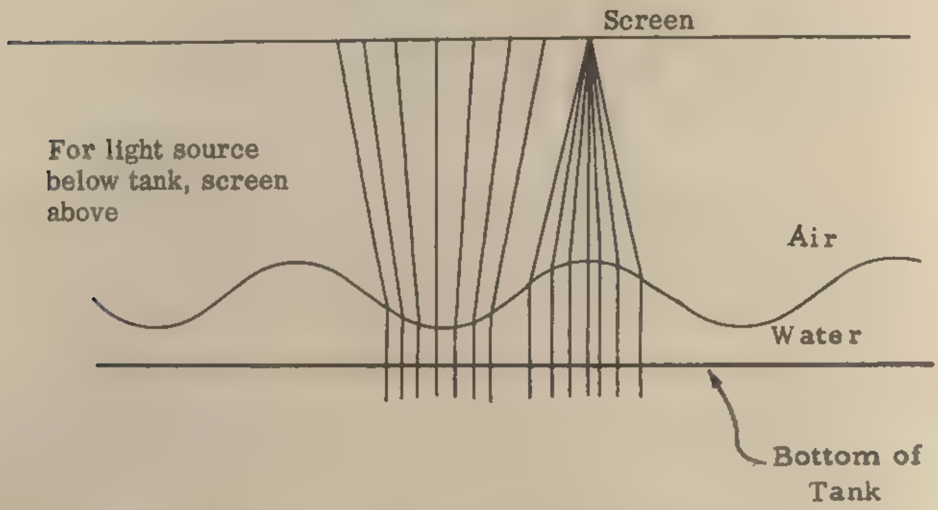
[illegible] according to their [illegible] Those which are [illegible] are marked with [illegible] on page [illegible]

Section	Easy	Medium	Hard	Class Discussion
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]			[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]		16
[illegible]	[illegible]	[illegible]		19

[illegible]

[illegible] of water particles in a water wave [illegible] a water





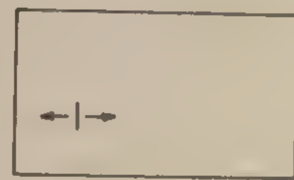
Then ask students, how the "focal length" would change if the distance between crests increased, or if there was a change in the height between crest and trough (which you can call twice the amplitude, if you want to introduce this useful word). Ask them whether they would expect the "sharpness" of the ripple image to change with changes in the horizontal distance between crest and trough. See whether they realize how an amplitude change could compensate for this. (While there is a fairly wide tolerable range, the relationships between amplitude, frequency, lamp distance, and screen distance can be adjusted to give a quite sharp image of the waves.)

The following questions can help drive home the points which should be established by observation in the laboratory.

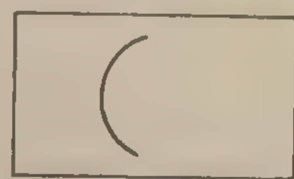
We are looking at a ripple tank — at one point on a crest of one wave (as in the diagram at the right). Where will this point on the wave crest go? Answer: You must show us more of the wave.

Here is a small piece of the wave: Where will this small piece go?

It may go in either of the perpendicular directions.



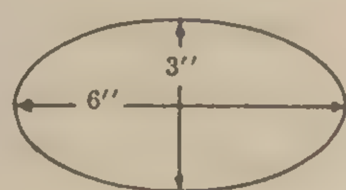
Suppose you have a circular wave. Can you tell which way it is moving? Answer: You can't tell whether it is heading "in" or "out". (A student may justifiably claim that if he saw such a wave in the lab, it would probably be moving out because generators for circular waves which are moving in are rare. But if you tap the edge of a coffee cup, you will see converging circular waves.)



Assume that the wave is increasing in radius and tell where each of the three indicated pieces of it will be after the wave has traveled, say, half an inch (change scale to fit your drawing).

Answer:

Suppose we dipped into a lake an oval-shaped generator with dimensions 3 by 6 inches. How will the wave appear after it has traveled a mile? Answer: It would be nearly a circle. (It would be a mile and three inches by a mile and six inches.) Most students will benefit by the review and the added insight they obtain by considering reflection from the wave point of view. Give them several exercises that they can do graphically or intuitively by thinking of wave fronts. Some students find the wave approach easier than the ray diagram approach, particularly after they have observed reflection of waves in the ripple tank.



In the laboratory you may want to have students try to check things they remember

about the behavior of light with spherical mirrors. Most students (by dipping their finger or a ruler) can easily locate the focal point of a mirror. Have them predict the center of curvature from their knowledge of the focal point and check it experimentally.

It may be wise to point out explicitly the analogy between a straight barrier reflecting a circular wave (Figure 19-9) and a plane mirror, reflecting a point source of light

COMMENT If you have some students who would like a more precise procedure for constructing successive wave fronts, you may want to introduce graphical construction. (This is Huygen's construction, but you need not bother with the name.)

Consider a wave front moving generally "up". How do you find its position a short time later?

Since each point moves along a normal, construct the tangents and perpendiculars at many points.

To find the location of the wave at a short time, t, later, mark the distance vt along each normal. (v is the speed of the wave.) This can be done easily with a compass (or a piece of chalk on a string at the blackboard). After you have these marks, connect them to get the new wave front.

After students understand this construction, you may want to show them the simpler construction which omits the tangents and normal. Simply draw many semicircles from points on the wave front.

If you draw in enough semicircles, the class will see how to draw the new wave front without bothering to construct tangents and normals.

Note: There are two possible advantanges in introducing Huygen's construction at this time.

1. It provides the student with a definite, foolproof way of checking his intuition about how a wave form will behave. Complicated wave forms or complicated reflections and refractions can be managed in a straightforward way.

2. It sets the stage for analyzing diffraction phenomena by introducing the idea that a wave behaves roughly the way a series of point generators would. See Chapter 19, Figure 19-12, page 292.

Section 4 - Speed of Propagation and Periodic Waves

PURPOSE To introduce periodic waves and a method for measuring wave velocity.

CONTENT a. A periodic wave is one which is formed by a disturbance pattern which, at any point in space, repeats itself at regular time intervals. The number of times it is repeated in a unit time interval (usually 1 sec) is the frequency, f.

b. The time interval between repetitions is the period, T. By definition, $f = \frac{1}{T}$.

c. Since waves move with a uniform speed, v, successive identical parts on a periodic wave form will always be the same distance apart. The distance between two such successive points is the wave length, λ. $\lambda = vT$.

d. The speed of waves in a ripple tank can be determined by measuring f (with a stroboscope) and λ (with a ruler), and by using the relation $v = f\lambda$.

EMPHASIS Understanding the relationship $v = f\lambda$ is important, and students should have a good deal of experience with it in laboratory and in problems. Laboratory Experiment II-9 should follow immediately the assignment of this section. It could be done on the second day in this chapter. Then home study and the following class day could be devoted to consolidation of this material.

COMMENT It will probably be worthwhile to spend some class time in a discussion of Figure 17-12 and the associated text materials. If a stroboscope gives successive glimpses corresponding to views A and B in Figure 17-12, it is impossible, on the grounds of that evidence alone, to be sure that the dashed lines drawn in the figure truly connect positions of the same wave at the two successive times. We assume that the wave motion is known to be from left to right. Then in the upper diagram a wave, between glimpses A and B, might indeed have moved a distance equal to $1/3\,\lambda$ as indicated, but it might also have moved a distance $4/3\,\lambda$ or $7/3\,\lambda$ or $10/3\,\lambda$, etc. This means that the time between successive glimpses might have been $1/3\,T$, $4/3\,T$, $7/3\,T$, $10/3\,T$, etc.

Similarly, in the more important case shown in the lower part of the figure, the time between glimpses might, indeed, have been T, as assumed; but it might also have been 2T, 3T, 4T, etc. You may need to remind students that they learned (in Chapter 12) how to find out which is correct. The frequency of operation of the stroboscope shutter can be increased. If the wave pattern cannot be stopped at any higher strobe frequency, then the assumption of a time T, between original glimpses was correct. In other words, the maximum stroboscope frequency, f_s, which gives a "stopped" pattern, gives the frequency of the wave disturbance that is being observed:

$$f_w = (f_s)\ \text{stopping maximum}.$$

Section 5 - Refraction

PURPOSE To show that water waves obey Snell's law.

CONTENT a. The speed of water waves depends on the depth of the water.

b. If the frequency of a ripple tank generator stays constant, there is a single frequency throughout the ripple tank. If there are two regions with different wave lengths λ_1 and λ_2, these are regions with corresponding differences in speed:

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}.$$

c. From our knowledge of waves we can conclude that Snell's law will apply to the

refraction of waves; i.e., $\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$, where v_1 and v_2 are the speeds in the incident and refracting regions respectively. Therefore, $\frac{\sin i}{\sin r}$ = constant for the interface between two particular media, and the "index of refraction" for waves is: $n_{12} = \frac{v_1}{v_2}$.

d. The index of refraction, n_{12}, for light traveling from medium 1 to medium 2, is predicted to be v_1/v_2 by the wave model, whereas, Newton's particle model predicted v_2/v_1. When measurements were made it was found that the wave model prediction was the correct one, thus lending support to the wave-like view of light propagation.

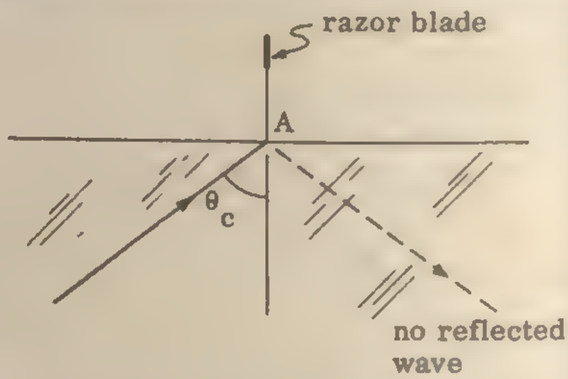
e. At the boundary between two depths of water in a ripple tank, waves are partially reflected and partially refracted. This lends additional support to the wave model, since no particle with which we are familiar behaves in this uncertain way.

EMPHASIS This is very important material. It is probably best presented by beginning with Experiment II-10, Refraction of Waves, in the laboratory. This experiment should give students a good general understanding of the behavior of waves in traveling across two media having different characteristic propagation speeds. After the experiment, this section can be assigned for reading along with a few of the relevant HDL problems. This can be followed by a full classroom discussion of this material.

DEVELOPMENT It probably will be worthwhile to review in class the proof of Snell's law for water waves as it is presented on page 266 of the text. It is important for all students to understand this reasoning.

COMMENT At this point it may be interesting to discuss the phenomenon of total internal reflection of water waves. This is merely a special case of refraction in which a wave, traveling from a medium of slow propagation speed into one with fast characteristic speed, is subject to so great an increase in speed and change of direction, that the wave is turned around and re-enters the original (slow) medium.

Thus in the case of total internal reflection, the disturbance actually penetrates into the medium lying on the far side of the surface at which the reflection occurs. It is interesting to note that in the case of light, a razor blade placed so its edge presses against the surface (air-glass) along the line perpendicular to the paper at A in the diagram on the right will completely cut out the reflected beam.



You might wish to let some interested students try to produce total reflection of water waves in the laboratory. It is not easy. You had better first be sure that your equipment is good enough so that it can be done. You will need to make your "slow" region very shallow, and you will have to run your generator at a very low frequency. (It is essential to be able to run at 8 cycles per second, and 5 cycles per second will be even better.)

Section 6 – Dispersion

PURPOSE To point out that the speed of propagation of waves may depend upon their frequency and to suggest that different frequencies might be responsible for different colors in light.

CONTENT a. The speed of a water wave depends on its frequency; this dependence of speed on frequency is called dispersion. Because dispersion is different at different depths of water, the index of refraction describing waves crossing a boundary between different depths of water depends on frequency.

b. For shallow water and for low frequencies, the dispersion is small.

c. Since the refraction of waves depends slightly on frequency, and since refraction of light depends slightly on color, perhaps different colors of light correspond to waves of different frequencies.

EMPHASIS Treat fully. It will be helpful if students can observe dispersion effects in the laboratory.

COMMENTS One of the most striking demonstrations of dispersion can be given by returning to the ripple tank to demonstrate refraction with an arrangement in which a maximum amount of refraction is visible. (You should be able to produce a clearly visible change in direction of 15 degrees or more. The information in Appendix 8 can help you choose convenient frequencies or depths.) With the ripple tank operating in this fashion, gradually increase the speed of the motor. By the time the speed has doubled or tripled, the waves will move across the barrier undeviated as far as the eye can tell. Students may need to observe these higher frequencies through a stroboscope, although by now many of them will have learned to see high frequency waves by looking through their fingers as they shake both their hands back and forth or by merely blinking their eyes.

This section is not intended to show that frequency and color are definitely related. The principal conclusion is that frequency may be associated with the color of light. This is a plausible guess because the index of refraction of light varies with color and the "index of refraction" of water waves varies with frequency. The section is written for students whose experience is with waves in ripple tanks using shallow water and low frequencies. The key point is that dispersion might depend on frequency; whether the dispersion is small for both water waves and light or whether it is small in one and large in another, does not matter. Only the qualitative aspects of dispersion are important at this point.

Section 7 – Diffraction

PURPOSE To introduce the phenomenon of diffraction of waves. To show the dependence of water-wave diffraction on wave length and to note the diffraction of light.

CONTENT a. For waves passing through openings, the direction of propagation is changed at the edges of the opening. This bending is diffraction.

b. The prominence of diffraction effects depends upon the relationship between wave length and the width of the opening. The effect is pronounced when the size of the opening is comparable to or smaller than the wave length. The effect is small when the wave length is small compared to the size of the opening. (The angle of bending depends on the fraction λ/d.)

c. If the diffraction of light is to be explained with a wave model, the wave length of light must be very small.

EMPHASIS A brief treatment of this material should suffice, but it is important for students to do some laboratory work with diffraction. Experiment II-11 snould precede discussion of this section.

[illegible] the argument used in this [illegible] wave length [illegible]
this section.

1. Water waves show reflection, refraction and dispersion.

2. Light also shows reflection, refraction and dispersion. Therefore, perhaps light is propagated as a wave.

3. Water waves bend at the corners of an obstacle.

4. Light casts sharp shadows. Therefore, the wave model seems to fail.

(5) Do water waves always bend?

6. [illegible] water waves passing through a slit (or around an obstacle) [illegible] the size of slit (or obstacle) is approximately equal to or less than a wave length.

7. [illegible] we can conclude that if the [illegible] than the size of [illegible] experiments with small slits and [illegible]

8. Experiment shows that for sizes less than [illegible] millimeter, bending of light occurs.

9. Thus, [illegible] of the wave model [illegible] a prediction suggested by the wave model. [illegible] support for the wave model of light.

In this argument, we use water waves as a model to explore the [illegible] of light. [illegible] with light. Thus, the [illegible] waves, [illegible] any [illegible]

[illegible] you may want to remind students of the following [illegible]

Chapter 17 — Waves and Light

For Home, Desk and Lab — Answers to Problems

GENERAL. Extensive laboratory work is the key to a thorough understanding of the material on waves. In addition, however, working with HDL problems is essential to get a feeling for the analytic treatment of this material. If you have time, assign a fair number of problems.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 17-[illegible] to 17-[illegible].

Section	Easy	Medium	Hard	Class Discussion
2, 3	1, 2*	3	4, 5*	3, 4*, 5
4	6*, 7, 8			7, 8
5	9, 10*	11*, 12, 13, 14	15*	13, 14, 15*
6		16, 17		16
7	18*, 19	20, 21		19

SHORT ANSWERS

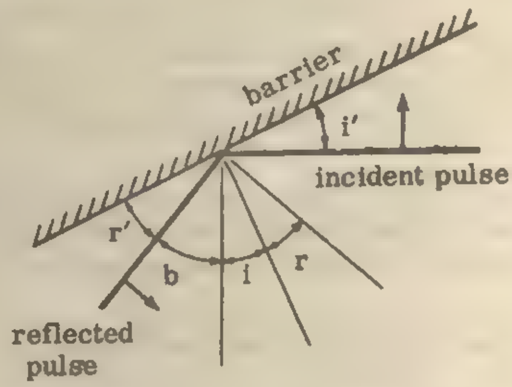
1. 25°
2. 80° to the barrier
3. See discussion on page 17-12
4. See discussion on page 17-13
5. See discussion on page 17-13
6. See discussion on page 17-17
7. (a) [illegible]
   (b) See discussion on page 17-17
   (c) See discussion on page 17-17
8. (a) 30 cm/sec
   (b) [illegible] cm
9. 1.73
10. 1.64 [illegible]
    1.24 [illegible]
11. (a) 1.333
    (b) 20.4 cm/sec.
12. (a) [illegible]
    (b) [illegible] $\times 10^{[illegible]}$ cm/sec
    (c) [illegible] $\times 10^{[illegible]}$ cm
    (d) 1.7
13. (a) Right side
    (b) See discussion on page 17-19
14. (a) 30° [illegible]
    (b) See discussion on page 17-20
    (c) See discussion on page 17-2[illegible]
15. See discussion on page 17-21
16. See discussion on page 17-22
17. See discussion on page 17-22
18. See discussion on page 17-23
19. See discussion on page 17-23
20. See discussion on page 17-23
21. 11 m to 0.032 m

COMMENTS AND SOLUTIONS

PROBLEM 1 In Fig. [illegible] [illegible]

On Figure 17-7 show the angle of reflection r (as in Figure 17-8) by drawing the perpendicular to the reflected pulse. See below.

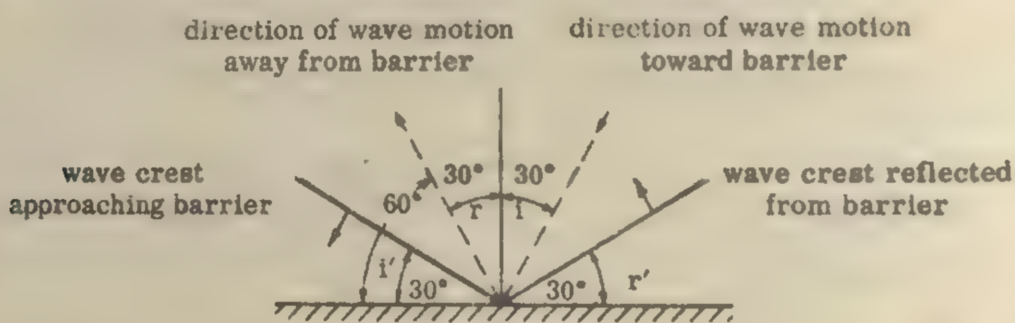
In Figure 17-7 there is a proof that $i = i'$. From experimental evidence, $i' = r'$. Therefore, $i = i' = r' = \underline{25°}$.



PROBLEM 2

A straight pulse approaches a barrier at an angle of 30°. What is the direction of motion of the pulse after reflection? Indicate it on a diagram.

If the wave crest makes an angle i', of 30° to the barrier, then the reflected wave crest makes an angle, r', of 30° to the barrier. Since the wave travels in a direction perpendicular to the wave crest, the direction of travel is at <u>60° to the barrier</u> or at 30° to the normal to the barrier.



PROBLEM 3

Describe the wave motion that results when you dip your finger into the center of a circular tank of water. What would be the motion under ideal conditions?

Make sure students realize that the term "ideal" means "ideal reflector" and "ideal wave propagation". (After working with a ripple tank some students may associate an ideal tank with one which has perfectly absorbing sides.) This is a good problem for beginning a discussion on focusing of waves.

If a finger is dipped into the center of a circular water tank, a circular wave front is produced which travels outward, is reflected, converges back to the center, goes through itself and outward again, etc. Students should note that the direction of propagation of each point on the outgoing circular wave is <u>along</u> a normal to the reflecting surface. Hence the direction of the corresponding segment of the reflected wave is back along the same line, toward the center.

You can see this effect very nicely in a cup or glass of liquid by letting a drop fall at the center.

PROBLEM 4

Suppose we place a barrier in a ripple tank in the shape of an ellipse as in Fig. 17-21. When a circular pulse is generated at point *A*, it reflects from the barrier and converges at point *B*.

(a) From this experiment what can you say about the geometry of an ellipse? (Hint: Consider tiny segments of the circular pulse originating from *A* and see how the ellipse must be shaped so that all segments reach *B* at the same time.)

(b) What will happen if we generate a pulse at point *B*?

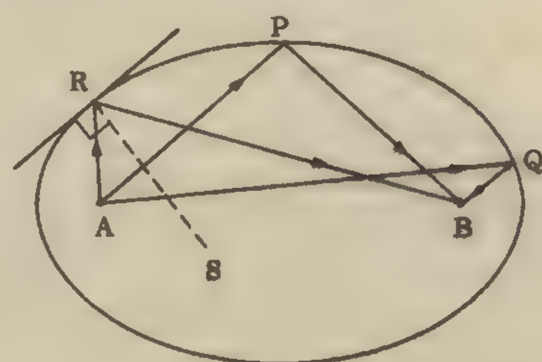
(c) Will such a convergence also happen when you dip your finger in at some point other than *A* or *B*?

This problem asks for the derivation of the geometrical properties of an ellipse from the fact that a circular pulse started at point A is reflected (focused) so as to converge at point B.

a) There are two requirements:

1. A piece of a wave front starting from A toward any point on the ellipse such as P must reflect in the right direction so that it heads toward B.

2. All such pieces of wave front must reach B at the same time.



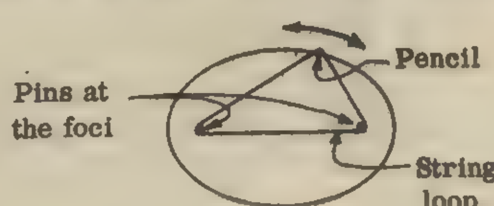
Condition 1 requires that at any point, R, on the ellipse, the angles ARS and BRS are the same size. Condition 2 is the familiar geometric definition of an ellipse: distances such as ARB, APB, AQB, etc. must all be the same. Condition 2 is enough to define an ellipse and condition 1 can be derived from it, but you may not want to go through a proof with the class (see Supplement 2 to this problem). The fact that an elliptical reflector "works" in a ripple tank is adequate verification for students. Points A and B are called the <u>foci</u> of the ellipse.

b) It will converge at A. All reflections are symmetric about the normal to the reflecting surface, and therefore the wave propagation is reversible.

c) No. For points near to the foci, rather good convergence is obtained but not perfect convergence.

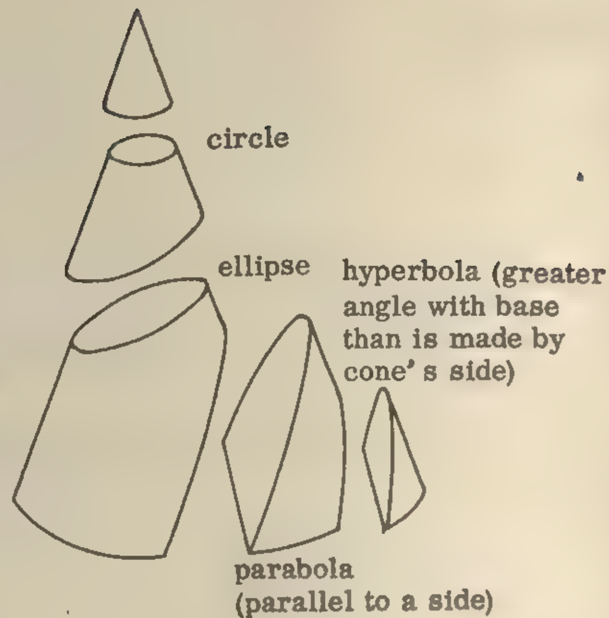
Try this with a circular reflector, for example a coffee cup, letting a drop fall about a centimeter away from the center. The focusing to the point a centimeter on the other side of the center is remarkable.

Students will be interested in seeing reflections from an elliptical barrier in a ripple tank. The barrier can easily be made from a flexible strip of aluminum. It can be formed by fitting it to an ellipse drawn in the familiar manner.



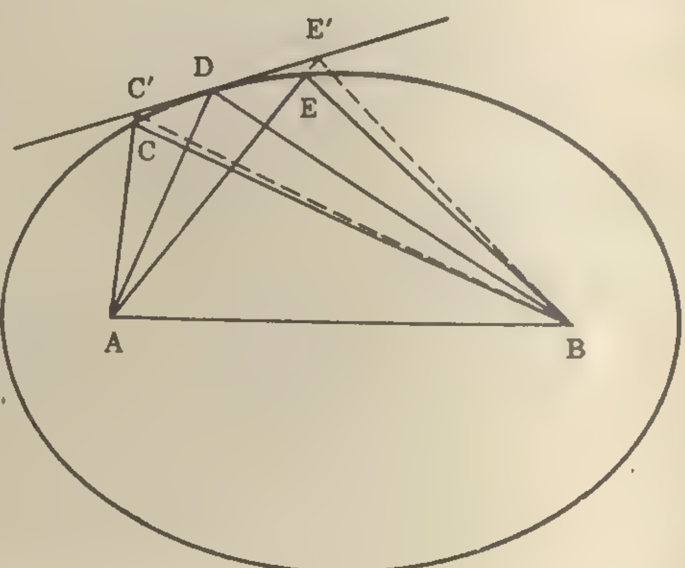
Supplement No. 1 to Problem 4

It is interesting to note that perfect focusing from one point to <u>another</u> point is obtained through using an elliptical mirror, while perfect focusing of parallel light requires a parabolic mirror, and perfect focusing from a point back to the same point requires a spherical mirror. Your geometrically-bent students will recall that these curves are conical sections. As a matter of fact, it can be shown that a hyperbolic mirror is needed for a perfect virtual image of a point which is too close to the mirror for a real image to be formed.

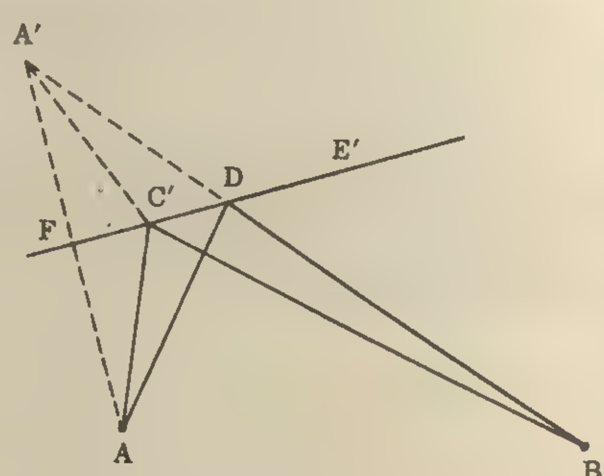


Supplement No. 2 to Problem 4

Here is one proof that lines from the foci of an ellipse to any point on the curve make equal angles with the normal to the ellipse at that point.



Consider the paths ACB, ADB, and AEB. From the geometric definition of an ellipse these paths are equal. Construct the tangent to the ellipse at point D, namely the line C′DE′ and extend the lines AC and AE to meet this line at points C′ and E′. Also draw lines C′B and E′B as shown. Clearly the path AC′B > ACB and path AE′B > AEB. Since paths ACB, AEB, and ADB are equal, it follows that paths AC′B and AE′B are greater than ADB. ADB is therefore the shortest path from A to B which hits the line C′DE′. This information can now be used to prove that $\angle C'DA = \angle E'DB$.

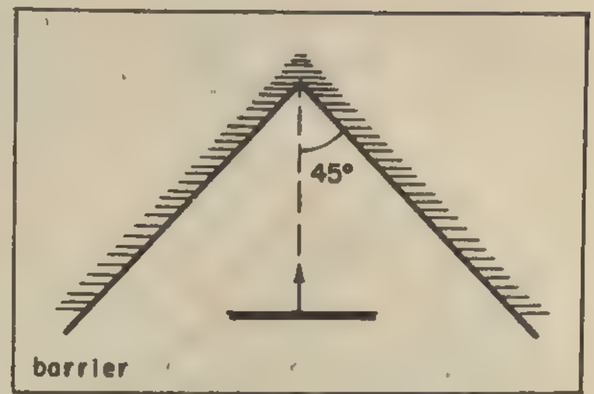


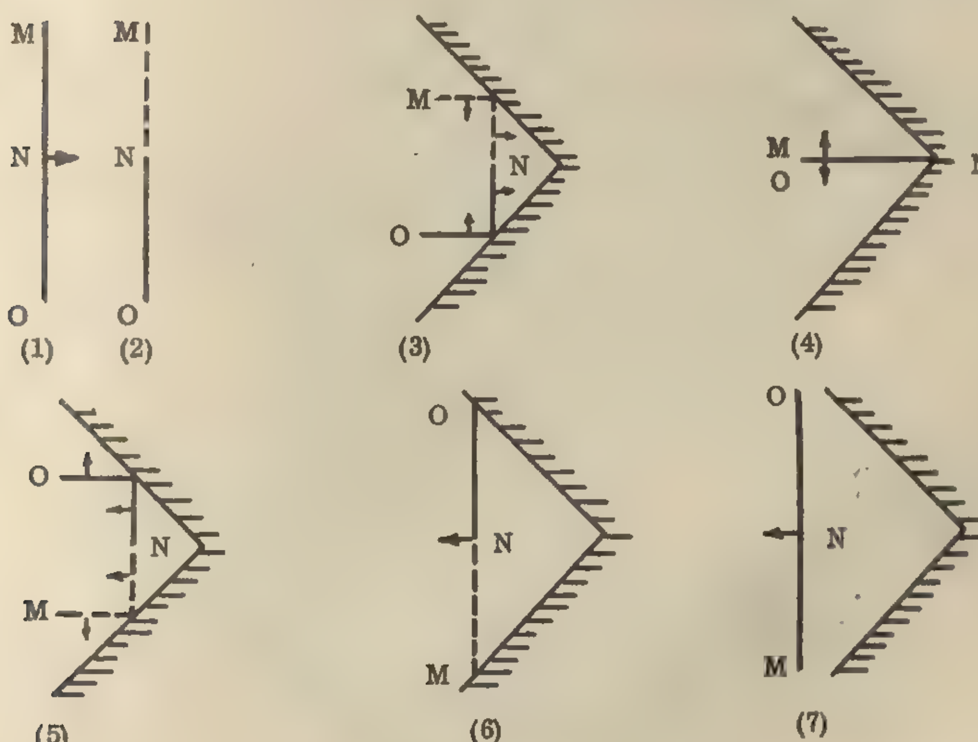
For simplicity we will redraw the line C′DE′ and the ray ADB. Now construct the point A′ an equal distance behind the line C′DE from A. That is, AF = A′F and AFC′ is a right angle. Then path AC′B = A′C′B and path ADB = A′DB. But A′DB is the shortest of all lines from A′ to B when A′DB is a straight line. If A′DB is a straight line, $\angle A'DF = \angle E'DB$. But $\Delta A'FD = \Delta AFD$; hence $\angle A'DF = \angle ADF$ and $\angle ADF = \angle E'DB$. If the normal is now drawn to the line C′DE′ at the point D, two angles are formed which are complementary to $\angle ADF$ and $\angle E'DB$ and thus are equal. This completes the proof.

PROBLEM 5

In Fig. 17-22 a straight pulse approaches a right-angled barrier at an angle of 45°.
(a) How does it reflect?
(b) What happens if the wave is incident at some other angle?

a) The straight pulse is reflected as a straight pulse but interchanged end for end. For a qualitative understanding it is probably best first to consider one-half of the pulse such as ON, in the diagrams below, follow it through the reflection, and then add the dashed half. Leave the proof for part (b).





b) This part of the problem is somewhat difficult. Again consider only the part NP of the pulse in the following diagrams. Point M will follow the indicated dashed track through M′, and point P will follow the indicated path through P′.

Figures 2, 3, 4, and 5 show the position of the pulse segment PMN at intermediate stages. In Figure 2, the pulse is shown after partial reflection. QS is perpendicular to the first mirror and thus parallel to the second mirror. Since $\angle r = \angle i$, the complementary angles PMQ, NMQ, and A are also equal. The angle of incidence at the second mirror is equal to $\angle$PMQ and for simplicity has been labeled $\angle$A in Figure 3 In Figure 4, the angle of reflection is also labeled $\angle$A. In Figure 5, $\angle$SM′N is equal to $\angle$A (the angle of reflection of Figure 4) since their sides are mutually parallel. Thus it has been shown that the reflected pulse is parallel to the incoming pulse, but moving in the

opposite direction and with the ends P and N interchanged. You may wish to discuss the path of point N of the pulse along the line RV to the mirrors and back. This path may be treated as the limit to which the path MXYM′ approaches as point M is moved toward N along the wave front PN.

Note that in a normal reflection from a plane mirror, left and right sides do not cross over. This results in your left hand being your image's right hand. Using a corner mirror, however, you will find that when you move your right hand, your image will also move its right hand. This is one way to see yourself as other see you!

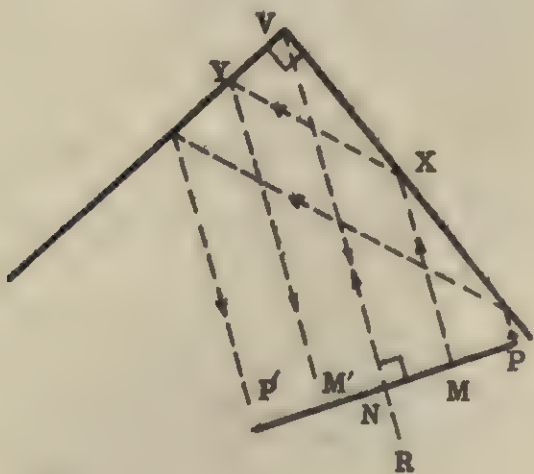


Fig. 1.

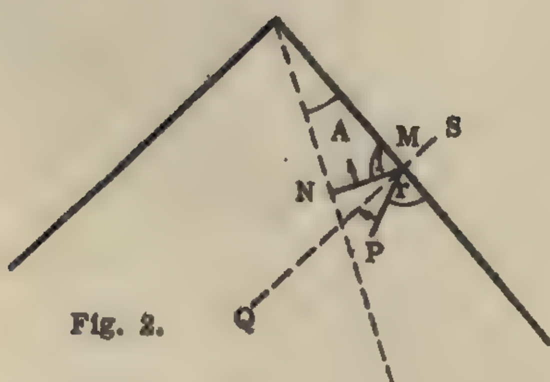


Fig. 2.

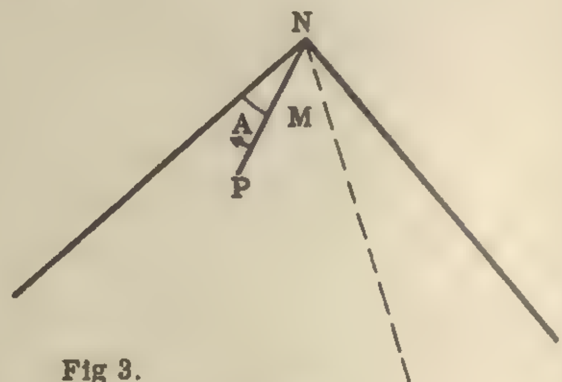


Fig 3.

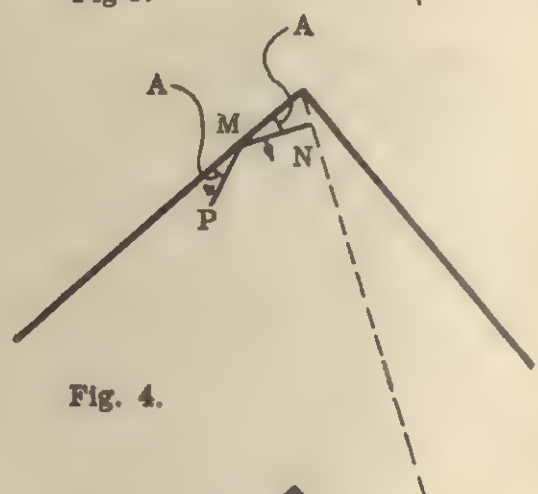


Fig. 4.

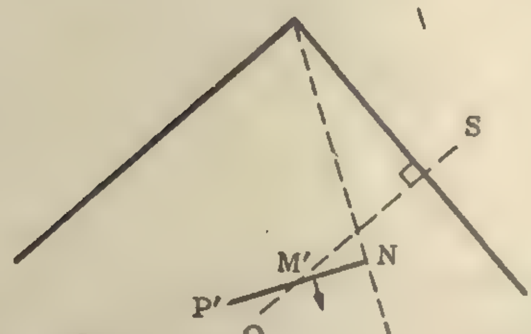


Fig. 5.

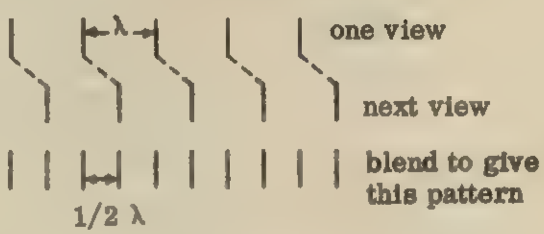
PROBLEM 6

Assume you are looking at a periodic wave of frequency $f = 4$ per sec through a two-slit stroboscope. What do you expect to see if you rotate the stroboscope with a frequency of 1, 2, and 4 revolutions per sec?

One revolution per second of the stroboscope will give two views per second. Between views any particular crest will have moved exactly two wave lengths. Thus the pattern will be stopped with crests one wave length apart.

Two revolutions per second of the stroboscope gives 4 views per second which is the same as the frequency of the periodic wave. You will see a stopped pattern with crests one wave length apart. The pattern appears to be the same as with one rev/sec.

Four revolutions per second giving 8 views per second will give a stopped pattern with crests apparently one half wave length apart.



PROBLEM 7

A point source in the ripple tank produces circular periodic waves. By using a stroboscope to stop the motion, we measure the difference in radius between the first and sixth circular crests and find it to be 10 cm.

(a) What is the wave length?

(b) Why didn't we calculate the wave length by using the radius, say, of the fifth pulse only?

(c) Why do we use this method of measurement rather than take the difference between neighboring crests?

a) $5\lambda = \frac{10 \text{ cm}}{5}$. Therefore, $\lambda = \underline{2 \text{ cm}}$. Some students will make the mistake of thinking there are six wave lengths between 6 crests.

b) At any given instant, it is hard to tell whether there is a wave crest at the center (source). Therefore we may make an error of some fraction of a wave length. In fancier language, we do not know the phase of the wave at the origin.

c) If we make a certain error in locating the position of a wave crest, or in measuring, the error in the wave length is only 1/5 as great when we measure five waves as when we measure one.

PROBLEM 8

(a) In a ripple tank when one pulse is sent every $\frac{1}{10}$ sec, we find that λ is 3 cm. What is the speed of propagation?

(b) In the same medium we send two pulses, the second one $\frac{1}{2}$ sec after the first. How far apart are they?

This problem can be used to trigger a discussion of the formula $v = f\lambda$. It can also be worked logically without direct reference to the formula.

a) $v = f\lambda = 10 \times 3 = \underline{30 \text{ cm/sec}}$.

b) $v = f\lambda$; $30 = 2\lambda$; $\underline{\lambda = 15 \text{ cm}}$.

Or alternatively:

The first pulse travels a distance of $d = vt = 30 \times 1/2$ before the other one starts. It is therefore a distance of 15 cm ahead of the second one. Then they travel along 15 cm apart.

(Note that this problem is intended for use in connection with Section 4 where we still speak of a single speed for propagation in a medium independent of the frequency of the disturbance.)

PROBLEM 9 What is the index of refraction in passing from the deep to the shallow water in Fig. 17-13?

$$n_{12} = \frac{\lambda_1}{\lambda_2} = \frac{0.94 \text{ cm}}{0.53 \text{ cm}} = \underline{1.77}.$$

PROBLEM 10 Measure the index of refraction in Fig. 17-14 by the method you used in the previous problem, and by finding the ratio of the sines of the appropriate angles. Compare the results.

$$n_{12} = \frac{\lambda_1}{\lambda_2} = \frac{0.69 \pm 0.01 \text{ cm}}{0.42 \pm 0.02 \text{ cm}} = \underline{1.64 \pm 0.10}.$$

$$n_{12} = \frac{\sin i}{\sin r} = \frac{\sin (57.5^\circ \pm 1.0^\circ)}{\sin (29.0^\circ \pm 2.0^\circ)} = \underline{1.74 \pm 0.14}.$$

These results check well within the accuracy of the measurements.

PROBLEM 11 A ripple-tank wave passes from a shallow to a deep section with an incident angle of 45° and a refracted angle of 60°.
(a) What is the ratio of speeds in the two sections?
(b) If the wave speed is 25 cm per second in the deep section, what is it in the shallow one?

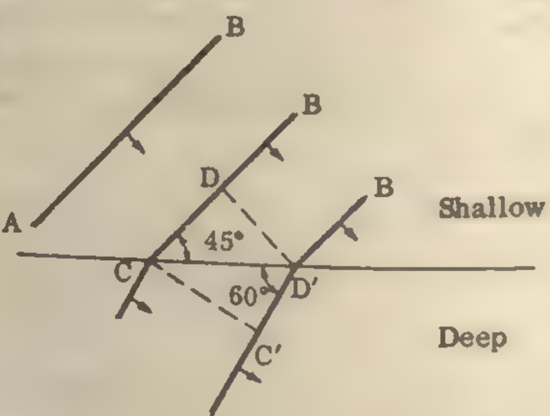
This problem on refraction can be handled as a formula problem or it can be used to discuss the ideas behind the refraction of waves by doing it from basic principles. In order to construct successive wave fronts students will be using a form of Huygen's construction.

a) The wave pulse AB is shown at three successive times. Between the last two, the point D goes to D′ and C to C′. But the time to go from D to D′ is DD′/(velocity-shallow) and from C to C′ is CC′/(velocity-deep). These two times are equal. Hence,

$$\frac{DD'}{\text{velocity (shallow)}} = \frac{CC'}{\text{velocity (deep)}}, \text{ and}$$

$$\frac{v \text{ (deep)}}{v \text{ (shallow)}} = \frac{CC'}{DD'} = \frac{CD' \sin 60^\circ}{CD' \sin 45^\circ} = \frac{\sin 60^\circ}{\sin 45^\circ}$$

$$= \frac{0.866}{0.707} = \underline{1.225}.$$



b) $v \text{ shallow} = \dfrac{v \text{ deep}}{1.225} = \dfrac{25}{1.225} = \underline{20.4 \text{ cm/sec}}.$

PROBLEM 12

(a) A tire on an automobile wheel has a circumference of 7.0 feet. When the wheel is turning 200 times per minute, what is the speed of the automobile in feet per min?

(b) A light wave whose frequency is 6.0×10^{14} per sec is passed through a liquid. Within the liquid the wave length is measured and found to be 3.0×10^{-5} centimeters. What is the speed of light in this liquid?

(c) What is the wave length in vacuum (from which the frequency was calculated)?

(d) What is the index of refraction of the liquid for light of this frequency?

Many students will solve this problem without being aware that they made two assumptions: 1) light is a wave, and 2) the frequency of light is not changed as it goes from one medium to another. You may want to call this to their attention in disucssing the problem.

a) The car travels 7 feet each cycle (one revolution), thus,
7 ft/cycle × 200 cycles/min = 1400 ft/min.

b) The light wave travels 3×10^{-5} cm each cycle of the source, thus,
$v = f\lambda = 3 \times 10^{-5}$ cm/cycle $\times\, 6 \times 10^{14}$ cycles/sec $= 1.8 \times 10^{10}$ cm/sec.

c) $$\lambda = \frac{3 \times 10^{10} \text{ cm/sec}}{6 \times 10^{14} \text{ cycles/sec}} = 5 \times 10^{-5} \text{ cm in vacuum.}$$

d) $$n = \frac{v_{vac}}{v_{liq}} = \frac{3 \times 10^{10}}{1.8 \times 10^{10}} = 1.7.$$

PROBLEM 13

The ripple tank is arranged so that the water gradually becomes shallow from one side to the other. Because of this, on one side of the tank the speed of a wave crest is different from that on the other side. As a result, straight waves become curved (Fig. 17-23). In the picture the pulses are moving toward the top of the page.

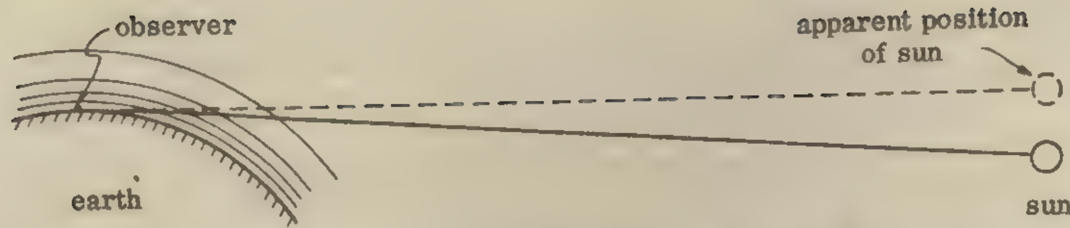
(a) Which is the shallow side?

(b) Does a similar phenomenon occur with light? Be prepared to discuss this in class.

This problem is more suitable for class discussion than for home assignment. The first part of the problem involves the variation of the speed of water waves as the water depth changes. Students can do this part after they have seen, in the ripple tank, that water waves move more slowly in shallow water. The second part of the problem is somewhat more difficult. The student must realize (1) that a similar phenomenon might occur if the index of refraction changed gradually in some medium, and (2) that such situations do exist. They may recall Problem 9, Chapter 13 which dealt with atmospheric refraction.

a) The shallow side is the right side.

b) At first glance one might think of a lens or prism made of glass having a gradually changing refractive index. Such items are not generally made. But there is an example in nature. As light comes through the atmosphere it curves as the index of refraction of the air gradually changes from 1 to its maximum value at sea level. This allows the sun to be seen for a few moments "after" sunset and "before" sunrise.



Atmospheric refraction is also responsible for the "water" mirages that are commonly seen on roads on hot days. The sun heats the road, and the air near the surface becomes hotter (and less dense) than that above it. In this situation, the speed of light is greater near the road surface than it is a few feet above. Then light from a distant source which is headed toward the road in front of the viewer will gradually curve upward. If the angle is proper, the sky can be seen by looking down slightly toward the road. This effect is interpreted by the observer as resulting from reflection from water on the road.

PROBLEM 14

Water waves traveling in the deep section of a ripple tank at 34 cm/sec meet a shallow part at an angle of 60°. In the shallow part all waves travel at 24 cm/sec. When the frequency is increased slightly, the waves are found to travel at 32 cm/sec in the deep section.

(a) Compute the angle of refraction for each case.

(b) Considering the ripple-tank conditions, is it easier to measure the two speeds and find their difference directly or to measure it indirectly by the angular difference found in (a)?

(c) How can we detect small differences in the speed of light?

It should be pointed out to students that the velocity of the wave in the shallow part continues to be 24 cm/sec though the frequency is increased.

a) Refractive index = $\frac{34}{24}$ in one case and $\frac{32}{24}$ in the second. In the first case $\sin 60° = \frac{34}{24} \sin \theta$. $\theta = \underline{38°}$. In the second case $\theta = \underline{41°}$.

b) In order to measure directly the speed of propagation of a periodic wave, it is necessary to make a time and distance measurement, or a wave length and frequency measurement. In a ripple tank, in principle, either of these methods could be used.

One can compare the speed in one medium with that in another by measuring the change in angle of propagation as the wave moves from one to another. Thus if the 24 cm/sec speed in shallow water were known, measurement of the angles of refraction of high and low frequency disturbances could, in principle, yield the 32 and 34 cm/sec speeds. However, this is a very small difference to detect by normal ripple tank procedures.

In the first place, it is very difficult to control the frequency of most ripplers with sufficient precision to render measurable this small difference in speeds. Secondly, the motion of a wave together with its width makes it difficult to specify its position at any instant. If a disk stroboscope is used to "stop" the waves, this difficulty can be overcome and a wave length can be measured if the frequency holds constant. However, even with the use of a disk stroboscope, the angle measurement can be tricky. A change in the apparent direction of propagation results from the interaction of the finite speeds of wave and slit. As the slit travels across the wave, the wave moves and its apparent direction thus is affected.

On the other hand, if one only wishes to show the change of index of refraction with

change in frequency, it is fairly easy to observe qualitatively (in the frequency range which is generally used) that the bending of waves decreases as the frequency increases and that thus, the index must decrease in value.

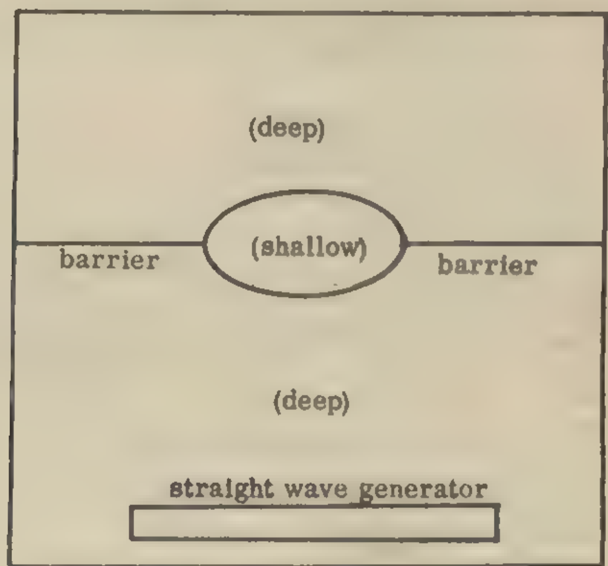
c) In the analogous experiment with optical instruments (a prism spectroscope), angles are very easy to measure, whereas the speed of light is quite difficult to measure, particularly in different media. Therefore, with light the angle measurement is preferred. It enables the observer to calculate the speed of light in any medium relative to the speed in air.

PROBLEM 15

We set up the ripple tank as shown in Fig. 17-24 and generate a periodic straight wave. The resulting wave pattern is shown in the photograph in Fig. 17-25.

(a) Explain what is taking place.

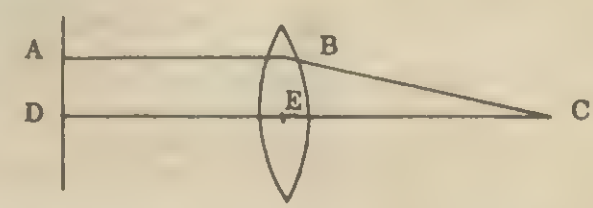
(b) Of what optical arrangement is this a model?



This is an important problem which is well worth class discussion. It affords the opportunity of reviewing lenses and refraction from the wave point of view.

a) The straight wave fronts are bent in such a way that they converge to a "focus" in the deep region behind the barrier.

b) This is a model of a converging lens.



Notice that the wave travels more slowly in the shallow region and that the wave "comes together", i.e., focuses, at a point C whose position is such that the time for each part of any wave front to get to C is the same as that for any other. The time to go from A to B to C is the same as to go From D to E to C. Clearly ABC is the longer path, but this is just compensated by the fact that DEC goes through a greater length of shallow water where the wave travels slowly.

You may want to compare this with Problem 4 of this chapter where it was seen that focusing came about, in the reflection from an elliptical barrier, because it took the same time for all segments of the wave front to reach the focus. In this problem, though

Chapter 18 - Interference

Thus far, the wave model of light agrees with the characteristics of light that were studied earlier. Does the wave model make any predictions, the counterpart of which we may not have seen in our previous experiments with light? The superposition of waves in one dimension showed that when waves passed through each other, regions of complete cancellation, nodes, were produced. Do the two-dimensional waves of the ripple tank similarly show interference?

In this chapter the experimental study of water-wave interference is approached both qualitatively and quantitatively. A student who has seen the interference pattern of a two point source and studied visually the effect of varying the separation and frequency of the sources has a rich background for the understanding of similar phenomena in light, sound, etc. From measurements made of interference patterns in the ripple tank, it is possible to calculate wave lengths. Laboratory work with the ripple tank is therefore an important part of this study. It is also essential that each student does some of the graphical HDL problems such as Problems 3 and 5.

CHAPTER SUMMARY

Section 1 Appropriate periodic waves sent from opposite ends of a spring can produce nodes at certain fixed points as the waves pass through each other.

Section 2 The principle of superposition is used to predict, by graphical methods, the interference pattern that will develop when two point sources generate circular waves of the same frequency and in phase in a ripple tank. Regions of double crests and troughs move outward from the sources and are separated by lines of undisturbed water which also extend outward from the sources.

Sections 3 and 4 The difference in distance from the two sources to any point on a nodal line is an odd multiple of a half wave length. Far from the sources, obtaining the path difference by direct measurement would lead to great inaccuracy. Geometry enables a simple determination of the relation between source separation, position of the nodal line, and wave length. This relation can be verified with measurements which require only moderate accuracy.

Section 5 If the two sources are not in phase the nodal lines shift, bending more sharply around the source which lags behind. The methods developed in the previous sections are extended to arrive at a mathematical relation that includes the factor of phase delay.

Section 6 Summary and Conclusion.

SCHEDULING CHAPTER 18

Section 1 The time required on the ideas of this section will depend on the students' facility with the ideas of Chapter 16. Demonstrations with the suspended spring will be helpful in making concrete the idea of periodic waves. The material must be understood, but try not to spend too much time here. Experiment II-12, a priority experiment, should be done after this section has been discussed.

Section 2 Half the battle is won when the students, from observations of ripple tank experiments, see that two waves can produce zero disturbance at some points. They will follow the text argument more easily if they also perform some of the graphical problems. The first part of Experiment II-12 can introduce this section.

Sections 3 and 4 The analytical arguments of the text can be supplemented by the convincing graphical verifications of Problems 9 (Section 3) and 5 and 10 (Section 4). The second part of Experiment II-12 can be done after these sections have been taken up in class.

Section 5 An examination of the $p = 1/2$ nodal pattern is a quick way to convince students that the relative phase is an important determining factor. This is the section where even the brighter students may have trouble. Do not labor the material, or try for complete understanding unless the going seems reasonably easy. Experiment II-13 can be done in connection with the work in this section.

	14-week schedule for Part II			9-week schedule for Part II		
Subject	Class Periods	Lab Periods	Exp't	Class Periods	Lab Periods	Exp't
Secs. 1, 2	2	2	II-12	1	1	II-12
Secs. 3, 4, 5	2	1	II-13	1	1	II-13

RELATED MATERIALS FOR CHAPTER 18

Laboratory. Experiment II-12, Waves From Two Point Sources, a priority experiment. To be done after first class period on chapter.

Experiment II-13, Interference and Phase, This experiment should be done with Section 5. See the yellow pages for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and laboratory observation, and those which are home projects, are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 18-9; detailed comments and solutions on page 18-10 to 18-23.

Section	Easy	Medium	Hard	Class Discussion	Home Projects, Lab or Demonstration
1	2*			1*	
2	4	3*, 5* 6, 7		3*, 4	4, 7
3	8	5*, 9			
4	13	10*, 11 14	12	11, 12	
5	15	16, 17 18, 19		15, 17	18
6		20		19	19

Films. PSSC films related to Chapter 18 are not yet available.

Science Study Series. "Waves and the Ear", by Willem A. VanBergeijk, John R. Pierce and Edward E. David, Jr.

Sectio 1 - Introduction

PURPOSE To introduce interference through considering periodic waves in one dimension, i.e., periodic waves on a spring.

CONTENT a. Brief review of reflection, refraction, and diffraction indicating the status of the wave model.

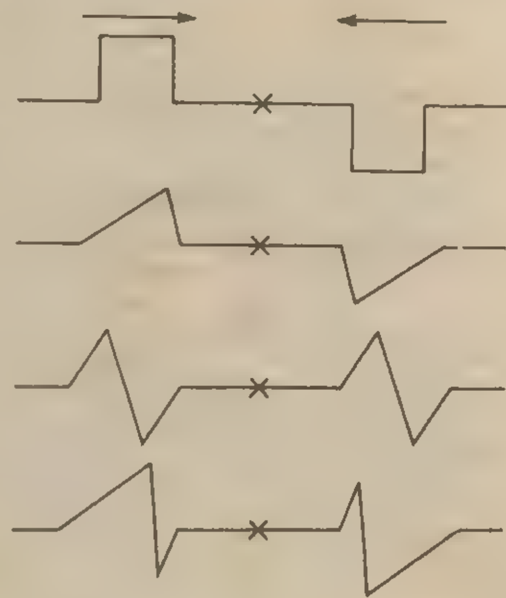
b. The midpoint between two similar, but inverted, pulses on a spring does not move as the pulses cross each other. If a periodic wave is reflected, the points where the incident and reflected waves produce zero total disturbance occur at fixed positions on the spring. These positions are called nodes. The superposition of two waves to produce nodes is called interference.

EMPHASIS This is background material which should be understood thoroughly before proceeding to two-dimensional nodal patterns. The time required to develop this section will depend largely on how well students understand the first four sections of Chapter 16.

DEVELOPMENT The concept of nodes on a coil spring should be introduced with demonstrations with a coil spring, by problems like #2 of HDL, and questions such as the following:

Consider two positive (upward) pulses which are introduced simultaneously at opposite ends of a rope. At which point will there be a maximum upward pulse?

If one pulse generator pushes the rope up at one end while the second pulse is introduced as a downward push at the other end, which way will the center point move when the pulses meet? If the two pulse generators put in pulses of opposite polarity, how must the pulse shapes be related for the rope center to be a node? (Draw some odd-shaped pulses to be sure that students understand that both the shape and size must be right to produce a node.) To be sure students understand, draw one of the pulses (as shown in the left column of the diagrams) and ask someone to draw the pulse which should be introduced at the other end.



Students will get a clearer idea of cancellation if you use some pulses that are not symmetric; otherwise some students may think that you get a node only if each pulse is symmetrical with respect to its own center.

Problem 2 will probably be easy for students after such a demonstration.

* * *

While you have the coil spring set up, it is a good idea to prepare the class for the ideas of path difference and phase difference in two dimensions by using the following exercise. Send a student to each end of the spring and ask each to start a positive pulse when a third student counts three. Then give the class the problem of deciding whether the students really start their pulses at the same time. Even if students do this without supervision in a laboratory, they will rapidly agree that watching the pulses as they add at the midpoint, where the maximum displacement should occur, is the best way to decide. If one student sends a positive (upward) pulse, and the other a negative (downward) pulse, a node should occur at the midpoint.

Ask what could be done to make the pulses cross at a different point. Ask them to decide which student should move (and which way) in order to keep the pulse crossing at the center even though one student starts his pulse later than the other. Be sure that students realize the relation between the point of crossing, the difference in starting time of the two pulses, and the two path lengths.

Section 2 - Interference From Two Point Sources

PURPOSE To develop the concept of nodal lines in surface (two-dimensional) waves.

CONTENT Two point sources generating waves (in phase) of the same frequency (and amplitude) produce a series of lines along which there is no disturbance; these are

called nodal lines. Between the nodal lines, crests reinforce crests and troughs reinforce troughs to give double crests and troughs which move away from the region of the sources.

EMPHASIS The extension of the interference concept to two dimensions, with appearance of nodal lines, is very important to the understanding of the remainder of this chapter and Chapter 19.

LABORATORY A period or even half a period spent in the qualitative study of interference patterns (the first part of Experiment II-12) before the assignment of Section 2 will greatly increase the students comprehension as they read. Encourage students to draw good sketches of what they see in the ripple tank and also to study the patterns with the disk stroboscope.

COMMENT The new element in this section is that the waves are in two dimensions. Conceptually this often presents a big problem to students, although the actual treatment turns out to be rather simpler than might be expected. Experiments with the ripple tank, and graphical exercises, will form an indispensable addition to the text discussion in familiarizing the students with interference in two dimensions.

DEVELOPMENT If the study of this section is preceded by the qualitative part of Experiment II-12, a few minutes spent at the end of the period in a blackboard demonstration of how to begin the drawings for Problems 3 and 5 will give a great deal of additional meaning to the students' study of the textbook. A drawing which shows the simultaneous positions of crests and troughs will enable them to pick out points for cancellation as well as reinforcement. Stress the principle of superposition in this construction. You might use solid lines for crests and dashed lines for troughs. Encourage students to begin such a drawing, exercising care in the proper spacing of the circles, before they read Section 2. A large sheet of paper should be used for the drawing with the sources near the center of the sheet and a wave length of about one centimeter.

The beginning of the next class period might be profitably spent in having students draw the nodal lines on the drawing for Problem 3. This should then lead naturally into a discussion from which you can determine whether they understand what is happening between the nodal lines.

Note on nodal lines where the cancellation is not complete:

We are now discussing waves in two dimensions. A characteristic, not stressed in the text and not qualitatively important in the development, is that the wave amplitude decreases as the wave moves from the point source. (This happens because as the wave spreads out, the same amount of disturbance covers an ever-increasing amount of surface, so its intensity diminishes.) Consequently, unless we look at a point equidistant from the two sources, the two waves will not have exactly the same amplitude. At all points far from the two sources, this difference in amplitude becomes so small as to be negligible. At certain places (the nodal lines) there will be complete destructive interference of the two waves, i.e., the water surface will be undisturbed. On the other hand, consider the water displacement at a point close to where the nodal lines cross the line joining the two sources. The wave from the source closer to the nodal line will, by definition, be out of phase with the wave from the farther source, but it will be greater in amplitude, so that the two waves will not exhibit complete destructive interference. The water surface will not be undisturbed at this part of the nodal line. The disturbance will, however, be a local minimum at this point, i.e., it will be less than at points immediately surrounding it. This is evident in Figures 18-6, 18-9, and the other photographs. The nodal lines are well marked at points far from both sources, but a nodal line which would pass close to one of the sources becomes indistinct, and may even disappear, in that region.

Thus, to be strictly accurate, the nodal lines as calculated in the text are lines along which the water disturbance is a local minimum, but not always exactly zero. However, at points far from both sources, and at points roughly equidistant from the two sources, the disturbance at the nodal lines becomes negligible, and the surface is effectively quiescent along the lines.

In all of our applications of these ideas to light propagation we shall be concerned with points far from the sources, where the above qualification does not apply. It may be advisable, therefore, not to mention this point unless some student, by close inspection of the observed ripple tank pattern or by insight, asks questions.

Section 3 - The Shape of Nodal Lines

PURPOSE To show the relation between path difference and wave length which holds at all points along nodal lines.

CONTENT Maximum destructive interference occurs at points where the difference between the path lengths from two sources is $1/2\,\lambda$, $3/2\,\lambda$, $5/2\,\lambda$, etc. A nodal line is the locus of points for which the path difference is $(n - \frac{1}{2})\,\lambda$, where $n = 1, 2, 3, \ldots$.

EMPHASIS You should stress the crucial role of the path difference. Students can get the qualitative ideas necessary to understand interference in light if they realize that a path difference of $1/2\,\lambda$, (or $3/2\,\lambda$, or $5/2\,\lambda$, etc.) implies destructive interference. It is the concept of path difference, more than the shape of the nodal lines, which should be stressed in this section. The section should go fairly quickly, as a commentary on the detailed construction of Section 2.

DEVELOPMENT The construction displayed in Figure 18-11, and elaborated in Problem 9, will be useful as a class discussion or a home exercise, in emphasizing the role of path difference. Physically, it corresponds to fixing one's attention on a particular part of the wave from one source (a crest, a trough, or an intermediate point) and on the appropriate place on the other wave profile that has just the opposite displacement. The intersection of these two parts of the wave fronts is a point of zero displacement. We plot the successive positions of this intersection as both waves move radially outward.

Section 4 - Wave Lengths, Source Separation, and Angles

PURPOSE To develop the algebra for calculating an interference pattern at large distances from the sources.

CONTENT a. When the distances from the sources are much larger than the distance between sources, the nodal lines become essentially straight lines which, when extended back toward the sources, go through the midpoint between the sources.

b. When the distances are this large, it is impracticable to obtain the path difference by measuring the two paths and subtracting.

c. To a good approximation, the path difference depends on the source separation and the angle made by the straight line portion of the nodal line.

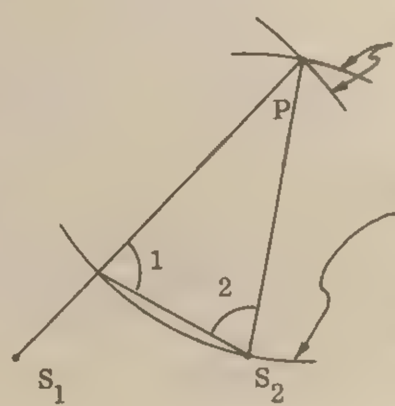
EMPHASIS These ideas will be needed in the next chapter. Be sure students have a good qualitative understanding of the nature of the interference pattern far from the sources, even though you may not take time for the full algebraic development. The last part of Experiment II-12 can be performed at this point.

COMMENT Going from water waves in a ripple tank to a study of light, involves a tremendous change of scale. Consequently, all the experiments with light will involve regions far away from the light sources. It is for this reason that we now examine the nodal lines in distant regions, although they are not so important in ripple tanks. A comment about this point will help students see the reasons for considering the ideas of this section.

DEVELOPMENT Students will appreciate the difficulty of measuring path difference by direct subtraction if they try to do it graphically. Problem 9 and Figure 18-11 present an easy method for constructing the nodal lines in regions fairly close to the sources, but at such distances as of order 5d, or 10d (using the notation of Problem 5), it would become increasingly hard to make circles of sufficiently accurate radii so that their intersections form a smooth, almost straight line. Students will agree that another method is needed for the distant regions. They will find, however, that for ripple tank photographs in the text, direct subtraction is still a useful procedure.

The geometric arguments illustrated by Figure 18-12 (involving the result that for distant points such as P, the two angles (1) and (2) must be close to right angles) will be accepted more easily by students if they verify the arguments with graphical measurements. They should consider series of cases starting with a point P that is close to S_1 and S_2, and progressing to distant points. In fact, if students have followed the graphical procedure of the previous paragraph, the same diagrams can be used. The diagram here is self-explanatory.

It is worthwhile to spend some time on this point, since approximations of this kind are sometimes felt to be strange and "not quite right" by students who have had experience with rigorous plane geometry. Once the point has been accepted, the arguments of the text will be relatively easy. Practice with graphical exercises will be helpful throughout. In addition to Problem 9, Problems 5, 10, and 12 are pertinent and useful.



(a) arcs drawn previously, with $v_1 - v_2 = 1/2\,\lambda$, say.

(b) arc now drawn, radius v_2, centered on intersection P.

(c) straight line joins points of intersection of arc v_2, making equal angles (1), (2).

COMMENT The symbol θ_n may trouble some students because they may think of θ as a variable and may not be familiar with the notation in which a subscript implies a particular value. The idea that θ_n represents a series of particular values, each one determined by the value of n, may need explicit mention.

Section 5 - Phase

PURPOSE To illustrate the effect of the relative phase of sources on interference patterns.

CONTENT a. If two sources of the same frequency do not dip into the water at the same time, they are "out of phase". If one source lags behind the other by a time, t_1, the phase delay, p, is defined as $p = t_1/T$. For example, source 1 lags behind source 2 by $2/3\,T$, $p = 2/3$.

b. If p is not zero, the nodal lines are shifted from their "in phase" position. The formula for the positions of the nodal lines is a simple generalization of the earlier result.

EMPHASIS Your development of this section will depend on how well students have assimilated the previous results. An understanding of this section will add greatly to the appreciation of interference, and the arguments are scarcely more involved than for the in-phase situation. But keeping track of the phase is one more thing to complicate the mental processes, and your less able students may want to give up. The least you should aim for is an awareness that the relative phase of the sources does affect the nodal pattern.

LABORATORY Experiment II-13, Interference and Phase, can be performed at this point.

DEVELOPMENT The definition of phase lag is somewhat arbitrary, and in any particular problem it is a matter of convenience to say which source lags in phase. In the example given in the text, at the beginning of this section, S_2 lags behind S_1 by a third of a period, so it has a phase lag of $p = 1/3$. Since differences in phase of a whole period are immaterial, we may say alternatively that S_1 lags behind S_2 by two thirds of a period, i.e., $p' = 2/3$. To avoid confusion we stick to one convention about phase. Problem 15 may be used to get across the definition of phase lag.

The redrawing of the crests and troughs for the out-of-phase case (Figure 18-14) need not be carried out in detail. If students get the point that one series of circles now has smaller radii, they will readily appreciate that the nodal lines are in new positions, and those students able to follow the arguments in detail will probably be able to go straight to the path-difference discussion. The best way to convince all of your students that phase is a determining factor is to discuss the case $p = 1/2$ as a special case. That the center line is now a nodal line is very clear, since it is the line of equal path differences, so that along it, the two waves are always out of phase. (In the sense of the note at the end of the Guide for Section 2, this is the one case of a true nodal line, i.e., a zero of water disturbance, rather than just a local minimum.)

COMMENT In more advanced physics courses, the concept of phase angle is commonly used instead of phase delay. Students do not need a definition of the phase angle. It may only confuse them if you mention it.

Section 6 - Summary and Conclusion

COMMENT It is important for the study of Chapter 19 that students understand the conclusion stated in the last paragraph of the text. To maintain a fixed interference pattern the phase delay must remain constant. To observe interference in light it is important to consider constancy of phase in the design of the experiment.

Chapter 18 - Interference

For Home, Desk and Lab – Answers to Problems

The graphical exercises can be drawn successfully without elaborate instruments, but they need care. For accuracy in drawing many circles with the same center the paper should be taped onto a drawing board or piece of heavy cardboard. Use of differently colored pencils for different features will make for better understanding.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and laboratory observation, and those which are home projects, are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 18-10 to 18-22.

Section	Easy	Medium	Hard	Class Discussion	Home Projects, Lab or Demonstration
1	2*			1*	
2	4	3*, 5* 6, 7		3*, 4	4, 7
3	8	5*, 9			
4	13	10*, 11 14	12	11, 12	
5	15	16, 17 18, 19		15, 17	18
6		20		19	19

SHORT ANSWERS

1. See discussion on page 18-10.
2. See discussion on page 18-10.
3. See discussion on page 18-10.
4. See discussion on page 18-12.
5. See discussion on page 18-13.
6. They are no longer nodal lines. See discussion on page 18-13.
7. (a) Parallel to the barrier.
   (b) Same, between the rows of double crests. See discussion on page 18-13.
8. Also need to know n.
9. See discussion on page 18-14.
10. Sine of angle and $(n - \frac{1}{2})\frac{\lambda}{d}$ should agree to within 2%.
11. See discussion on page 18-16.
12. (a) 0.24, 0.35
    (b) 2.2 cm, 1.5 cm.
13. See discussion on page 18-13.
14. $\beta = 23.6°$; $\gamma = 36.9°$; $\beta' = 29.3°$; $\gamma = 30.7°$; yes.
15. (a) 1/5
    (b) 2 km/min.
    (c) No change in phase, speed changes to 1.6 km/min.
16. See discussion on page 18-20.
17. (a) 0.3
    (b) 0
18. (a) 5.7°
    (b) 11.5° or 0°
    (c) 10, or 11
19. Interference is just one half of that for two sources 6 λ apart. Image is in phase.
20. See discussion on page 18-22.

COMMENTS AND SOLUTIONS

PROBLEM 1 Summarize the evidence for the wave nature of light.

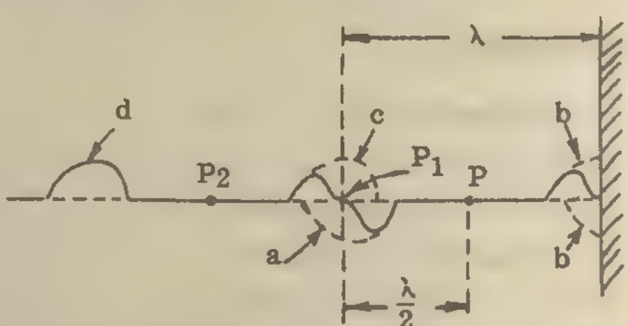
A short discussion of this problem can set the stage for the study of the rest of Part II by reminding students of the evidence we have gathered for wave properties of light.

The point you may wish to steer discussion toward is that diffraction is a characteristic of light that a particle model cannot explain in even a qualitative way. All the other properties — rectilinear propagation, reflection, refraction — can be understood with a wave model, but they also follow from the particle model (with the exception of the wrong velocity relationships the particle model gives for refraction). Interference will also turn out to be clear-cut evidence for waves.

PROBLEM 2 We showed in the text (Figs. 18-1, 18-2, and 18-3) that when pulses are incident periodically on the fixed end of a spring, the point P, a distance $\lambda/2$ from the end, never moves and is therefore a node. Extend the argument to show that
(a) the point P_1, a distance λ in front of the end, is a node,
(b) the point P_2, a distance $3\lambda/2$ in front of the end, is a node.

This easy problem involving path lengths gives good practice for later, more complex, nodal patterns.

a) in order to decide what happens at P_1, λ from the wall, consider the pattern of pulses when the reflected pulse, a, reaches P_1. This would occur at a time T/2 after being in the position shown in Figure 18-3.



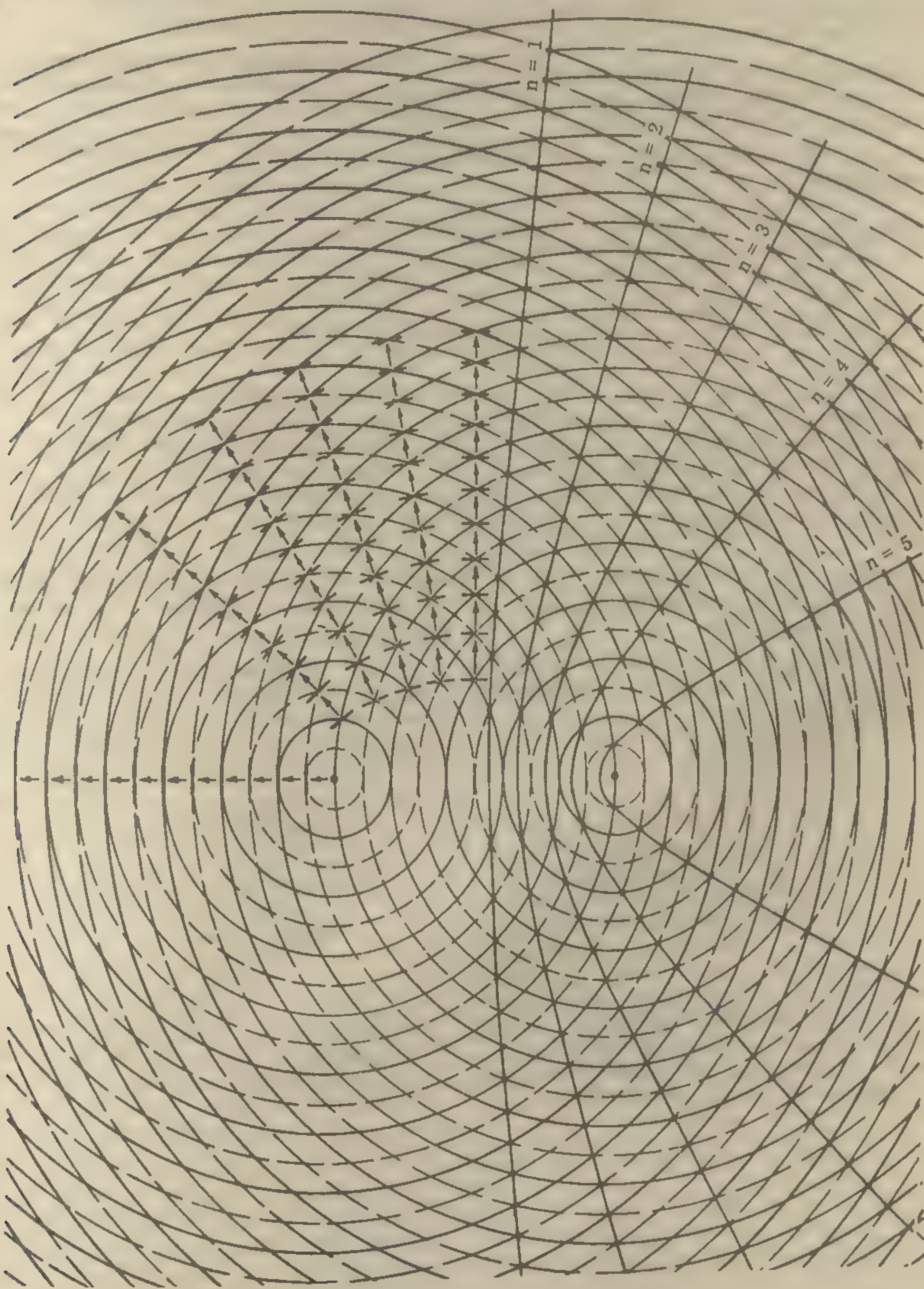
Note that both c and a have moved a distance of $\lambda/2$ from their positions in Figure 18-3. Just as the effects of pulses a and b cancelled each other at P, the effects of pulses a and c will cancel each other at P_1. At a later time, T, pulses b and d will meet at P_1 and their effects will cancel.

b) The point P_2 can be shown to be a node in a similar way. When pulses a and d meet at P_2, pulses b and c will be meeting at P. This will happen at a time T/2 after the diagram shown above or at a time T after Figure 18-3.

PROBLEM 3 Draw the sets of concentric circles and the interference pattern from two sources with $d = 5\lambda$ at the time:
(a) when the generators have just produced crests,
(b) when they have just produced the following troughs.
How have the reinforced crests moved during the time interval between these drawings?

The solution of this problem should help students understand Sections 2 and 3. Encourage students to draw carefully; without careful drawing the regularity of the pattern at the more distant points will be lost.

a) The following diagram has the scale of $\lambda = 1$ cm. The crests at time (a) are represented by solid lines. The troughs for the same time are drawn with broken lines. Cancellation occurs where a crest and a trough cross each other. Points of cancellation have been shown with black dots and these points have been connected on the right side of the diagram to show the nodal pattern on that side.



b) For part (b), a new drawing is not necessary. The same drawing can be used if we now let solid lines represent troughs and broken lines crests. Keeping in mind this change of representation we can see what changes have occured between time (a) and time (b), one half period later.

On the left side of the drawing the short full lines locate the double crests at the time (a), and the short broken lines locate the double crests at time (b). The connecting arrows indicate how the double crests have moved in the time between (a) and (b).

It is interesting to note that as your eye moves from one radial line of double crests to an adjacent nodal line and then to the next line of double crests, you see that the two neighboring lines of double crests are out of phase.

Some students may prefer to use colored pencils to indicate the different parts of this diagram.

PROBLEM 4

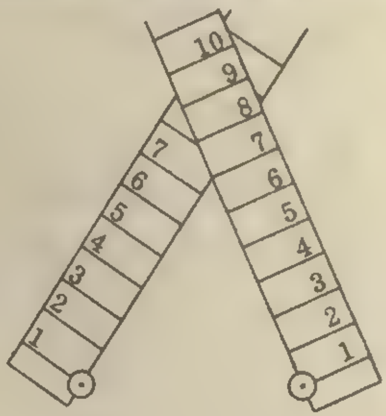
Fold two pieces of ruled paper into long strips about 2 cm wide and hold them as in Fig. 18-19. Imagine that the lines are wave crests. Your fingers then represent the sources of the waves. Notice how the crests from both sources add together. Now by sliding the free ends sidewise, locate nodal lines and moving wave regions.

This home project provides another way of visualizing the construction of an interference pattern from a two-point source. The location of nodal lines with this device will be inexact unless each strip is rotated around a point on its inner edge. Thumb tacks or pins, used as points about which to rotate the strips, will hold the "sources" in fixed positions, making the device easier to handle. To simulate sources in phase, the two strips should be tacked at comparable points (e.g., both tacked at lines).

As well as being a home project, the use of this device might make a good classroom demonstration if the strips (including spacing of the lines) were scaled up to a size adequate for class viewing.

Cancellation, reinforcement, or something in between can be determined for any point on the surface surrounding the sources by bringing the strips together at that point. If comparable positions on the strips (lines, midpoints, etc.) come together, reinforcement occurs and the point so located is in a moving wave region. Where a line on one strip meets a midpoint on the other strip, cancellation occurs and the point so located is on a nodal line.

The strips can be used to search out the nodal lines. If the lines on the strips are numbered as in the diagram at the right, we can trace out the path of nodal lines by locating several points that lie along the nodal line. For example, if we wish to trace the first nodal line, move the strips so that 3 1/2 on one strip touches 4 on the other strip. Mark that point. Then bring 4 1/2 opposite 5; mark, etc. Similarly, the second nodal line can be located by bringing 3 1/2 opposite 5, 4 1/2 opposite 6, etc.



PROBLEM 5

Draw the interference pattern for the case $d = 5\lambda$ on a piece of paper large enough so that you can see the nodal lines become straight at a great distance from the sources. Continue these straight lines back toward the sources and show that they all pass close to the midpoint of the line joining the sources.

If the drawing for Problem 3 was carefully done it may be used in the solution of this problem. However, those students who did not make an adequate drawing before may prefer to start anew and improve their technique. In order to obtain straight nodal lines, the larger circles should have radii of about 9 λ to 13 λ. Therefore, some students may have to draw additional arcs beyond those made for Problem 3. The extension of the straight sections of the nodal lines back toward the sources was not done on our drawing on page 11, because of the confusion it might have caused in the explanation of Problem 3. You may wish to extend these lines in colored pencil on our drawing for Problem 3 in order to see for yourself how nearly they all pass through the midpoint of the line joining the sources.

The student's step by step construction of this final figure will help him clarify the discussion of Sections 2 and 3.

PROBLEM 6

Consider an interference pattern produced by two point generators. What happens at the positions of the nodal lines if we place a third source exactly like the others at the point midway between them?

An exercise in the use of the superposition principle. The particular question asked (referring only to the original nodal lines) is quite easy, but any further analysis of the three-source situation can be very complicated, and students should be warned off. A hint as to the method used in solving the problem asked, would not be out of order.

By the superposition principle, the total displacement at any point is the algebraic sum of the individual displacements, and the addition can be performed in any order. The sum of the displacements due to the two original generators is zero along a nodal line. So the total displacement along this original nodal line is just that due to the third generator; thus it is no longer zero. The original nodal lines are no longer nodal lines.

PROBLEM 7

Draw the crest lines of straight waves incident on a reflecting barrier at 45° and the crest lines of the reflected waves (Fig. 18-20). Indicate the incident direction of motion and the reflected direction. Shade the places where crests cross.

(a) Which way do these shaded double crests move? Indicate with an arrow on your drawing.

(b) Can you find nodal lines in the interference pattern?

In connection with your laboratory work with ripple tanks you should make sure that students see this phenomenon. This graphical problem applies the methods used in the text to a new, but geometrically simpler, situation. As with all of these graphical problems accurate work is required; use of colored pencils to indicate different features will simplify the interpretation of the drawing.

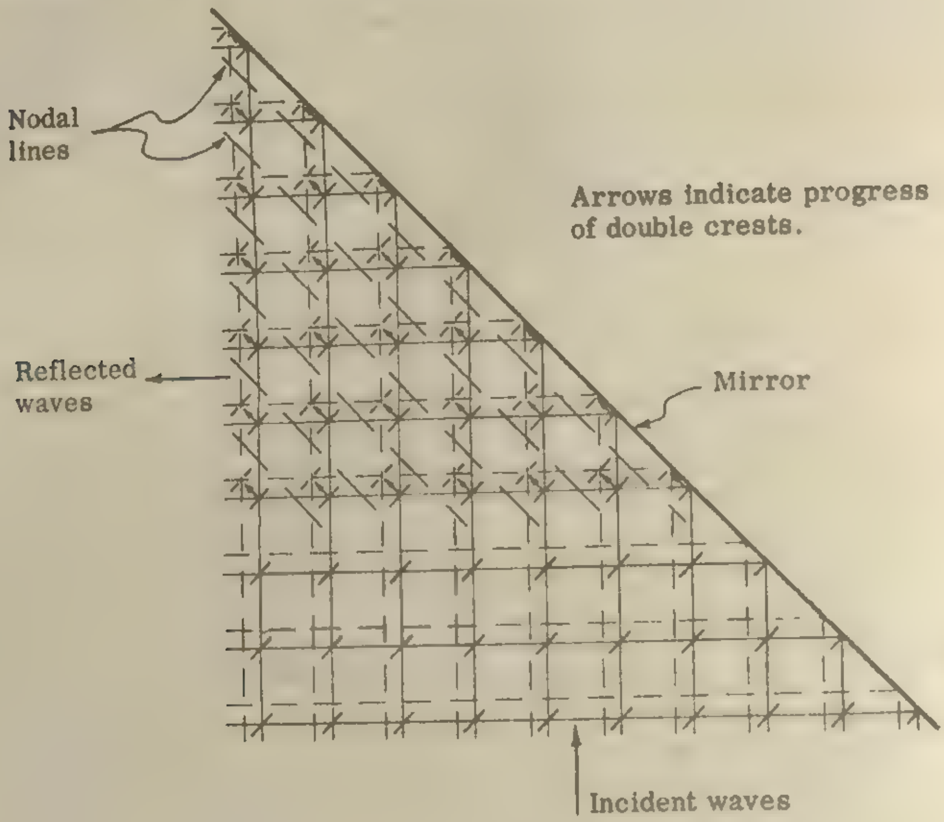
The scale of our drawing is $\lambda = 1$ cm. We use the experimental results of Chapter 17 that crests are reflected as crests, with angle of reflection equal to the incident angle. Therefore in this case the two wave trains are perpendicular. To discover the direction of motion of the double crests, the waves must be redrawn at a later instant. Here, the full lines are the crests at a certain time; the dashed lines are the same crests at a time T/4 later. The arrows indicate the motion of the double crests during

this time. It is clear that there are rows of double crests moving parallel to the mirror. Since each row is exactly out of phase with its neighbor, the lines midway between the rows must be nodal lines. They are indicated by heavy broken lines.

Brief answers are thus as follows:

a) The double crests move parallel to the mirror.

b) The nodal lines lie between the rows of double crests.



PROBLEM 8 You know the distances from a point on a nodal line to the two point sources in a ripple tank. What else do you have to know to calculate the wave length of the waves?

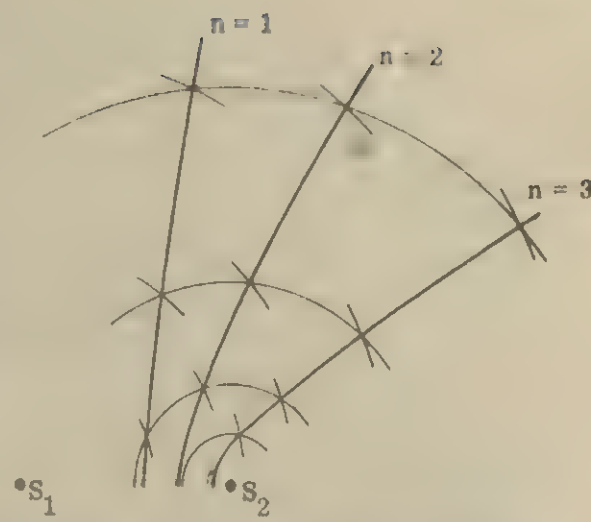
A "quickie" to test the students' understanding of path difference and its significance.

You need to know which nodal line the point is on; i.e., you need to know n. (This is assuming that the sources are in phase, which is intended.)

PROBLEM 9 Construct the nodal lines for two point sources with $\lambda/d = \frac{1}{3}$ by the method of Fig. 18-11. Is this really a different method from that used in Problem 5?

This graphical exercise illustrates the path-difference characteristic of the nodal lines.

The accompanying figure, drawn on the scale $\lambda = 1$ cm, was made with a view to minimizing the number of compass settings required. A large arc of a circle is drawn about S_2, with any convenient radius r. The compass is then set successively to $r + 1/2\lambda$, $r + 3/2\lambda$, $r + 5/2\lambda$; and the intersections of these circles, centered on S_1, with the original arc, are drawn. After a range of values which covers all of the desired space has been tried, the intersections on the line joining S_1 and S_2 are calculated, and marked.



The appropriate intersections are then linked up to form the nodal lines.

The advantage of this method is that convenient values of r can be chosen, with as many or as few as are needed. There is no need to draw a whole series of circles spaced λ apart, as was done in Problem 5 (and 3). One physical interpretation of the path-difference method is that we fix our attention on a particular part of the wave from S_2 (a crest, a trough, or some intermediate point) and plot its intersection with that part of the wave from S_1 which has just the opposite displacement. The intersection is a point on the nodal line, and we plot the motion of the intersection as both waves move outward from their sources. In a sense, this viewpoint is complementary to the Problem 5 (and 3) approach of watching the whole surface at one instant, and observing the lines of zero displacement. The two methods are simply different ways of finding path differences.

PROBLEM 10

Two sources 6.0 cm apart operating in phase produce water waves with a wave length of 1.5 cm. Draw the nodal lines far from the sources. Determine the position of each line by means of intersecting arcs of circles drawn from the two sources. Measure the angle between the second nodal line and the center line of the pattern. Compare the sine of this angle with $(n - \frac{1}{2})\lambda/d$.

This graphical exercise confirms the analytical definition in Section 4 of the angular position of the nodal lines. A careful drawing will be convincing.

The accompanying drawing has been reduced in scale from the full-scale diagram used in obtaining the numerical solution for this problem. The path-difference method (Problem 9) was used to spot points on the nodal lines. In this problem it is the region "far" from the sources that is of interest. In those regions the nodal lines are straight, and when continued back toward the sources, pass through the midpoint between the sources. (Problem 5 verified these results graphically.) Consequently all that is needed is to obtain a point on each nodal line, as far from the sources as is practicable and connect it to the midpoint with a straight line. With the compasses used here (nothing fancy) the largest practicable radius was about 15 cm, i.e., 2.5 d. Some closer points on the nodal lines were obtained, to detect any deviation from straight-line behavior. At a radius of 10 cm the deviation is very slight, which indicates that 15 cm is probably far enough away from the sources. (See the note at the end of the solution.)

Numerical results on $\sin\theta_2$ were as follows: AB = 6.05 cm, AC = 15.84 cm,

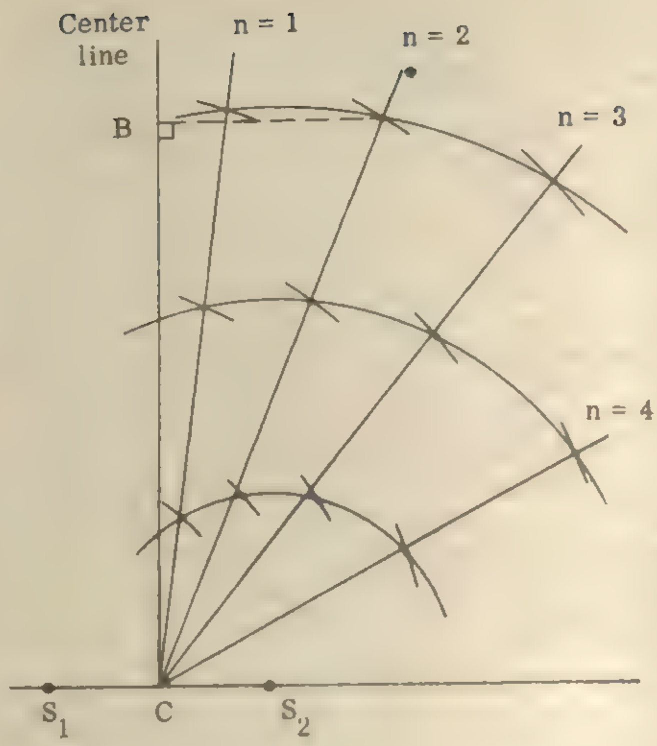
$\sin\theta_2 = \dfrac{6.05}{15.84} = \underline{0.382}$ (graphically). By the path-difference formula,

$\sin\theta_2 = (2 - 1/2)\dfrac{\lambda}{d} = \dfrac{3}{2} \times \dfrac{\lambda}{4\lambda} = 0.375$ (theoretically).

We have a relative "error" of $\dfrac{0.382 - 0.375}{0.375} = 1.9\%$.

A result having this accuracy (not difficult to achieve with rudimentary equipment if care is taken) should be quite convincing. This problem may also help dispel any confusion about n, the number of the nodal line. Whatever the fundamental significance of n, it is unimportant here; $(n - 1/2)\lambda$ is the path difference used in drawing the nodal lines, and the same quantity occurs in calculating $\sin\theta_n$.

[Note on the error: the reason the error turned out to be rather large was not poor drawing technique, but the fact that at a distance of 2 1/2 d the nodal lines are still a small distance from their asymptotes. By analytical geometry one can show in this case that the quantity we measure in our drawing should differ from the angle of the asymptotes by 1.7%. Hence all but 0.2% of our "error" was unavoidable. This points up the inherent weakness of the direct subtraction method. But it is inadvisable to mention this to students in connection with this particular problem; the problem's purpose can be achieved without worrying about the error.]



PROBLEM 11

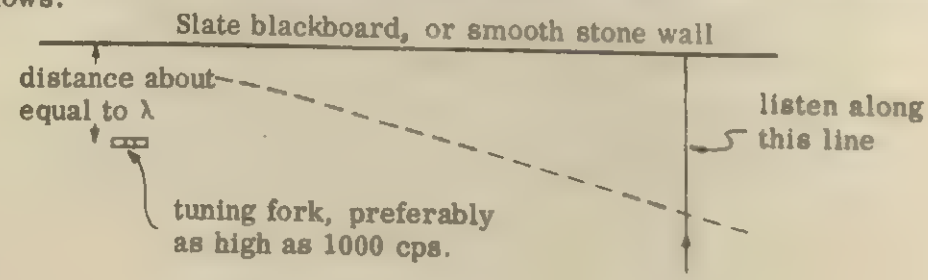
Suggest an interference experiment to prove that sound is a wave phenomenon. How could you use such an experiment to determine the wave length of sound?

This is a good problem illustrating the application of the techniques of this chapter to a different but familiar phenomenon.

While students probably will be able to suggest experiments to determine whether sound is a wave phenomenon, they may not consider the need for their experiment to be laid out with dimensions appropriate to the wave lengths of sound. Unless you are satisfied with a dimensionless experiment, you may need to give students information on the range of sound wave lengths.

The velocity of sound is about 330 m/sec; audible frequencies range from about 50 to about 15,000 cycles per second, and middle C is 256 cps. Wave lengths therefore range from 60 meters to 2 centimeters, with the wave lengths of sounds associated with musical notes around a meter. Students should realize that a single frequency is imperative. They should be able to imagine experiments which will do for sound what has been done for water waves in this chapter.

Warning: Keep the exercise theoretical and speculative. A diversion into actual experiments with sound at this stage would be time consuming, and probably frustrating, since unless the apparatus is well devised, and you have had experience with acoustics, all kinds of extraneous effects can mask the characteristics you seek (as most "high-fidelity" enthusiasts will testify). One simple demonstration which you could have up your sleeve to confound those students who invent complicated apparatus is as follows:



A tuning fork of reasonably high frequency (to have a manageably small λ) held about a distance λ from a slate blackboard, will simulate the situation shown in Figure 18-19 quite well. If a student moves his head along the line indicated, he should hear quite distinct regions of sound and regions of quiet. The position of the nodes (quiet zone) will be enough for an order of magnitude estimate of λ. (Try to strike the tuning fork so as to excite only its fundamental frequency of oscillation. The presence of harmonics can be somewhat confusing.)

PROBLEM 12

(a) From Figs. 18-6 and 18-9 find the ratio of λ/d by using the equation $\sin \theta_n = (n - \frac{1}{2})\lambda/d$.
(b) The dimensions of the photographs are one-fourth the actual size. By measuring d on the photographs estimate the actual value of λ.

Measuring from real patterns is good experience for students; they will see that, physically, the nodal line is not one-dimensional (as practice with earlier problems might lead them to think), but a broad region whose center must be estimated. Although by now they should be aware of experimental error, the fact that variation in measurements may lead to slightly different values should be pointed out.

a) Any of the three nodal lines shown in Figure 18-6 or any of the four nodal lines in Figure 18-9 can be used. Since a center line is hard to locate accurately, the angles were were measured by taking one-half of the angle between the n th nodal line on the right and the n th nodal line on the left.

A typical calculation for the 2nd nodal line in Figure 18-9 is:

$$\theta = 32^{\circ} \text{ (approximately)}. \qquad \frac{\lambda}{d} = \frac{\sin 32^{\circ}}{2 - 1/2} = \underline{0.35}$$

Typical measured values for Figure 18-6 and Figure 18-9 are:

Figure 18-9:

Nodal Line Number	Angle, θ	λ/d
1	10°	0.35
2	32°	0.35
3 (very crude)	55°	0.33

A fair conclusion is that $\frac{\lambda}{d} = \underline{0.35}$ in this case.

Figure 18-6:

Nodal Line Number	Angle, θ	λ/d
1	7°	0.25
2	21°	0.23
3	37°	0.24
4	53°	0.23

A fair average is $\frac{\lambda}{d} = \underline{0.24}$.

b) For Figure 18-6 the measured source separation is about 1.6 cm which implies a 6.4 cm separation of the actual sources. Thus the actual wave lengths were:

Figure 18-6 $\lambda = 0.35 \times 6.4 = \underline{2.2 \text{ cm}}$
Figure 18-9 $\lambda = 0.24 \times 6.4 = \underline{1.5 \text{ cm}}$.

PROBLEM 13

Look up the definition of "hyperbola" and show that nodal lines are hyperbolas.

It may be that the definition of a hyperbola which students will find is just the one we have found for the nodal lines: the locus of points whose distances to two fixed points have a constant difference. From this they will know that nodal lines are hyperbolas, which from a physics point of view adds nothing to their understanding of interference, but enables them to talk to a math student.

There are other definitions of a hyperbola. Unless the students already know, or can easily prove, that they are equivalent to the path-difference definition, they should not spend much time on this problem, but should be content to know where the proofs can be looked up. (Any good analytical geometry text will have the proofs.) Two definitions which they might encounter are given below, with their relationships to ours.

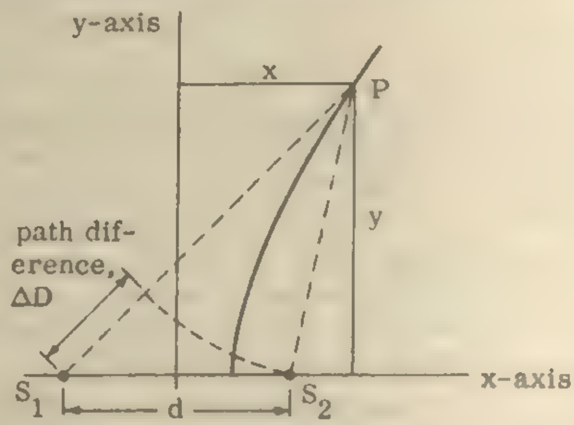
d is the source separation, and ΔD is the path difference, i.e., $\Delta D = (n-1/2)\lambda$. One of the definitions, in terms of rectangular coordinates, is as follows:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Students well versed in analytic geometry may be able to show that the constants a and b are related to d and ΔD as follows:

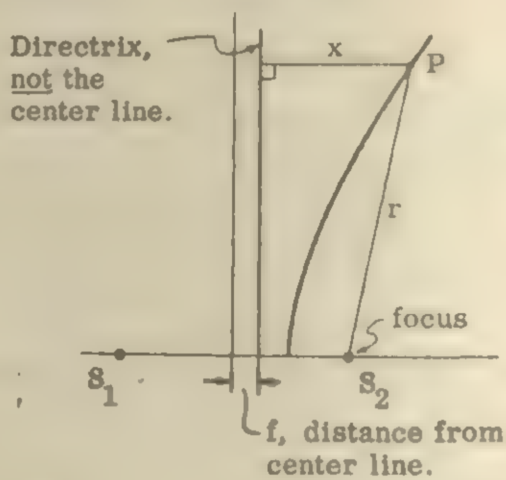
$a = 1/2\ \Delta D,$

$b = 1/2\sqrt{d^2 - \Delta D^2}$. Hence d must be > ΔD for these to be nodal lines.



A less common definition is in terms of r, the distance from a point on the hyperbola to the focus (one of the sources) and its perpendicular distance x to a fixed line called the directrix: $r = ex$, where e, a constant > 1, is called the eccentricity. It is rather more difficult to show that e and f are related to d and ΔD as follows: $e = d/\Delta D$

$$f = \frac{1}{2}\ \frac{\Delta D^2}{d}$$



PROBLEM 14

In Fig. 18-21, $L = 50$ cm, $d = 10$ cm, $\alpha = 30°$. What are γ and β? Find γ and β when $L = 500$ cm. Does this convince you that it is a good approximation to set $\gamma \approx \beta \approx \alpha$ when L is much greater than d?

Let b be the base of the right triangle with the vertex angle = α = 30°. Then b = L sin α.

When L = 50 cm,

b = L sin 30° = 50 cm × 0.5 cm = 25 cm.

$$\sin\beta = \frac{25\text{ cm} - d/2}{50\text{ cm}} = \frac{25\text{ cm} - 5\text{ cm}}{50\text{ cm}} =$$

$$\frac{20}{50} = 0.40;\ \underline{\beta = 23.6°}.$$

$$\sin\gamma = \frac{25\text{ cm} + d/2}{50\text{ cm}} = \frac{25\text{ cm} + 5\text{ cm}}{50\text{ cm}} =$$

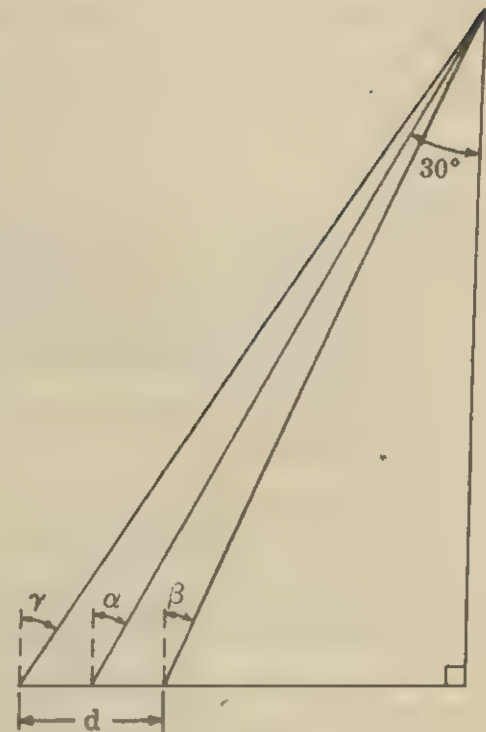
$$\frac{30}{50} = 0.60;\ \underline{\gamma = 36.9°}.$$

When L = 500 cm,
b = 500 cm × sin 30° = 250 cm.

$$\sin\beta = \frac{250\text{ cm} - 5\text{ cm}}{500\text{ cm}} = \frac{245}{500} = 0.490;\ \underline{\beta = 29.3°}.$$

$$\sin\gamma = \frac{250\text{ cm} + 5\text{ cm}}{500\text{ cm}} = \frac{255}{500} = 0.510;\ \underline{\gamma = 30.7°}.$$

Since 29.3° ≈ 30° ≈ 30.7°, students should be convinced that as L becomes very much larger than d, as is true when performing Young's experiment, $\gamma \approx \alpha \approx \beta$.



PROBLEM 15

One red and one blue car are going around a circular race track 5.0 km in circumference. They move at constant speed. Each car takes 2.5 minutes for each lap. The blue car always comes around 0.50 minutes behind the red.

(a) What is the phase delay *p* of the blue car with respect to the red car?

(b) What is the speed of each car?

(c) If the track were only 4.0 km long, would this change the answers to (a) and (b)?

a) Since each car takes 2.5 minutes to complete the lap, the period, T, is 2.5 minutes. A delay of 0.5 minutes is 1/5 of a period. Therefore the phase delay, p, is <u>1/5</u>.

b) The speed of each car is the circumference, 5 km, divided by the time required to go around the track, 2.5 minutes. The speed is $\frac{5\text{ km}}{2.5\text{ min}} = \underline{2\text{ km/min}}$.

c) If the track were 4 km in circumference: the phase delay, p, would be unchanged (p = 1/5) since the phase delay depends only on the timing of the cars, and that is unchanged. The speed, however, is changed: speed = $\frac{4\text{ km}}{2.5\text{ min}} = \underline{1.6\text{ km/min}}$.

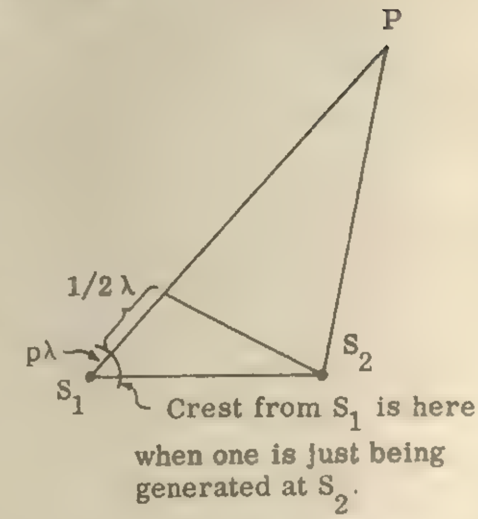
The <u>phase</u> is independent of the amplitude and velocity of the motion. This result also applies to the ripple tank. The pattern we see depends only on p, and not on the velocity of the waves or their amplitude.

PROBLEM 16

Prove that for two sources with phase difference *p*, the first nodal line can be plotted from the equation: path length difference = $(p + \frac{1}{2})\lambda$.

Substituting the value n = 1 into the formula given in Section 5 gives:

$$PS_1 - PS_2 = (p + n - 1/2)\ \lambda$$
$$= (p + 1 - 1/2)\ \lambda$$
$$= (p + 1/2)\ \lambda.$$



The real intent of this problem, however, is to encourage the student to work out a development for one special nodal line. This proof can be made by returning to the argument of Section 3 where it was shown that when the sources were in phase the path difference for the first nodal line was $PS_1 - PS_2 = 1/2\ \lambda$. If S_2 now lags behind S_1 by a phase delay of p, crests would start from S_1 at a time pT (T = period) before starting from S_2. During this time the crests would travel a distance from S_1, equal to $vt = \frac{\lambda}{T} \times pT = p\lambda$. Thus, as the diagram shows, the total path difference from any point on a nodal line to the sources would be increased by this same quantity and would be given by $PS_1 - PS_2 = p\lambda + 1/2\ \lambda = (p + 1/2)\ \lambda$.

PROBLEM 17

Suppose we look at an interference pattern from a great distance L in front of the sources and find that the first nodal line is a distance x from the center line. If $x = .008L$ and $\lambda = .01d$, what is the phase of the sources? (See Fig. 18–13.) If $\lambda = .016\,d$, what is the phase?

Some students may solve this problem without understanding what they are doing, by combining one formula from Section 5, page 231, $\sin\theta_n = (p + n - 1/2)\ \lambda/d$, with the formula related to Figure 18-13, $\sin\theta_n = \frac{x}{L}$. This gives the applicable formula

$$\frac{x}{L} = (p + 1/2)\ \frac{\lambda}{d}$$

and the rest of the exercise is purely algebraic. Students should be encouraged to work the problem from a more elementary viewpoint.

The ideal way for a student to do this problem is to realize that the path difference is $d \sin\theta = d\ \frac{x}{L}$, and to see how this compares with the condition for cancellation of the two waves. In part (a), the path difference is $\frac{\lambda}{0.01} \times \frac{0.008\,L}{L} = 0.8\ \lambda$. Since two sources which are in phase would produce a node when the path difference is $0.5\ \lambda$, the extra $0.3\ \lambda$ in this case implies that the source which is further away is ahead in phase by p = <u>0.3</u>. That the closer source is in phase by p = 0.7 is an equivalent statement.

In part (b), the path difference is $\frac{\lambda}{0.016} \times (0.008) = 0.5\ \lambda$. Hence the sources must be in phase (i.e., p = <u>0</u>).

Of course, unthinking substitution of the given values into the applicable formula will also give p = 0.3 and p = 0, respectively.

A student who tries to derive the formula might get the equivalent formula $\frac{x}{L} = (p - 1/2)\ \frac{\lambda}{d}$. This would give p = 1.3 and p = 1.0, respectively which corresponds to p = 0.3 and p = 0 because p was defined to be a number between 0 and 1.

PROBLEM 18

Suppose that two point sources are generating waves with the same wave length λ. They are placed in the ripple tank a distance $d = 5\lambda$ apart.
(a) If the sources are in phase what angle θ does the straight part of the first nodal line make with the central line?
(b) If the sources have a phase $p = \frac{1}{2}$, what is θ?
(c) How many nodal lines will be produced?

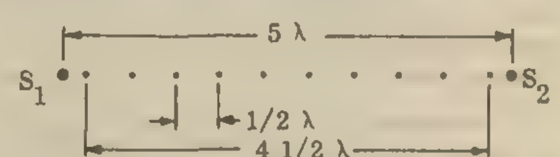
This problem is similar to Problems 3 and 5 except that it includes phase. If you have not already asked for careful drawings of this physical situation, you should. If students have their earlier drawings they now can check graphically their numerical answers to this problem.

a) If the sources are in phase, $p = 0$, and $\sin\theta_1 = \frac{1}{2} \times \frac{\lambda}{d}$ for the first nodal line. Therefore, $\sin\theta_1 = \frac{1}{2} \times \frac{1}{5} = 0.1$; $\theta_1 = \underline{5.7^\circ}$.

(If students interpolate from the sine table on page 636, they will get 5.74°. Students may measure the angle directly from their diagram for Problem 5 and thus get answers that vary by more than one degree from the computed angle.)

b) If the phase difference is $p = 1/2$, students who use the formula (developed in Problem 16) will get $\sin\theta_1 = (\frac{1}{2} + \frac{1}{2})\,\frac{1}{5} = 0.2$. This gives $\theta_1 = \underline{11.5^\circ}$ (or 11.54°). Note that when $p = \frac{1}{2}$, the perpendicular bisector of the line between the sources is a nodal line. This nodal line, which corresponds to $\theta = \underline{0}$, could be called the "first" nodal line. (Remember that what is called the "first" nodal line is quite arbitrary and unimportant.)

c) The easiest way to find the number of nodal lines is to think about the points where the nodal lines cross the line joining the two sources. Since the spacing between these nodal points is $\lambda/2$, there generally will be 10 nodal lines.



However, when $p = 1/2$, both S_1 and S_2 are on nodal lines, and there are 11 nodal lines. In this very special case the middle 9 nodal lines are familiar nodal lines. The last two "lines" lie along the extensions in both directions of the line connecting the two sources. Whether the students speak of these two extensions as one or two lines is quite unimportant.

PROBLEM 19

A point source of periodic waves is placed a distance 3λ in front of a reflecting barrier. The superposition of the incident and reflected waves produces an interference pattern. (Fig. 18-22.) Describe this pattern. Examine such a pattern experimentally in a ripple tank. What is the phase of the image of the source?

This problem on interference combines the ideas of interference with those of an image in a plane mirror providing an interesting link between the ripple tank and optics. If possible this problem should be done experimentally in the laboratory.

A circular wave reflecting from a barrier gives rise to a circular reflected wave which appears to arise from a point as far behind the barrier as the source is in front, i.e., from the "image" of the source. This can be seen clearly in Figure 17-9. Thus the ripple tank of Figure 18-22 has circular waves spreading out from the source <u>and</u> reflected circular waves coming from the image a distance 3λ behind the barrier. An interference pattern results which is identical to that from sources 6λ apart, except

only half of it is seer Such a total pattern would contain 12 nodal lines of which, in this case, 6 could be seen ----- as in Figure 18-22.

The phase of the image will be challenging. Some students may guess incorrectly that a rigid barrier gives a phase inversion because a rigidly fixed end of rope turns a pulse upside down. Students may not know enough about how water moves to be sure whether the barrier impedes the water. Students who think of the water as moving simply up and down may guess, correctly, that there is no phase change.

That there is no phase change in the reflection of Figure 18-22 can be determined by noting that the disturbance along the wall is a maximum, or by noting the succesive spacing of nodal lines and seeing that the barrier is not where a nodal line would be, but is midway between expected nodal lines. If the image had been out of phase ($p=1/2$), the central line, i.e., right along the barrier, would have been a nodal line.

As a matter of fact, the exact phase of the image depends upon the type of water wave (transverse or longitudinal) and the type of barrier it hits (vertical or slanted, soft or hard). This aspect is not worth going into, however. It is too complicated.

PROBLEM 20

Two sources in a ripple tank are operating at frequencies of 15 cycles per second and 16 cycles per second. Describe the resulting pattern of nodal lines.

The intent of this problem is to again call students' attention to the fact stated in Section 6, that to maintain a fixed interference pattern it is necessary for the phase delay between two sources to remain constant. If the phase delay changes at a constant rate the nodal lines will sweep slowly across the tank beginning on the side of the higher frequency source and sweeping across the tank toward the lower frequency side. During each second a nodal line would move to the position where its neighbor was at the beginning of the second. The nodal lines are no longer hyperbolas, but rather strange curves.

Procedures described in Experiment II-13 lead the students step by step through the concepts and conclusions desired in this problem. The experiment describes generators which will demonstrate the sweeping or fanning motion of the nodal lines.

You should probably not mention to your students unless they bring it up, that for a stationary observer the passage of nodal lines and regions of reinforcement in this situation correspond to the phenomenon of beats in music.

Chapter 19 - Light Waves

Now that the students have produced, seen, and analyzed interference effects in water waves, they are ready for the crowning point of the development of this part of the course --that light shows similar interference effects and therefore behaves like waves. The parallelism seen in the laboratory between the behavior of water waves and light should bring this home vividly.

The first section of this chapter explains why interference effects are not normally seen in light. In a way, this is surprising since we often see two or more light sources illuminating the same region and would expect to see nodal lines in many places. But the random, rapidly-shifting phase delays in light from different sources cause the interference patterns to be blurred out.

Young's classic experiment on interference is then described to show a simple way to get a constant phase delay between two sources. The text leads on through interference of light to the determination of wave length from interference effects and the association of wave length with color. Diffraction is analyzed as a form of interference, and the color effects in interference and diffraction are explained.

Since this chapter is the culmination of this part of the course, it should be covered thoroughly. The laboratory, particularly Experiment II-14, is most valuable, as are the HDL problems, in enabling students to transfer what they learned about water waves to an understanding of the wave-like nature of light.

SCHEDULING CHAPTER 19

	14-week schedule for Part II			9-week schedule for Part II		
Subject	Class Periods	Lab Periods	Exp't	Class Periods	Lab Periods	Exp't
Secs. 1, 2, 3, 4	2	1	II-14	1	1	II-14
Secs. 5, 6, 7, 8	2	1	II-15	1	1	II-15
Secs. 9, 10, 11, 12	1	2	II-16 II-17	1	-	-

RELATED MATERIALS FOR CHAPTER 19

<u>Laboratory</u>. (See yellow pages for suggestions.)

<u>Experiment II-14 Young's Experiment</u>
<u>Experiment II-15 Diffraction of Light by a Single Slit</u>
<u>Experiment II-16 Resolution</u>
<u>Experiment II-17 Measurement of Short Distances by Interference</u>

<u>Home, Desk and Lab</u>. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 19-11, detailed comments and solutions on page 19-12 to 19-23.

Section	Easy	Medium	Hard	Class Discussion
1				1*
2	2, 3*	4, 5*		4, 5*
4				7*
6	9	6	8	8
7		10, 11, 12*		10, 12*
8	13*, 15		14*	14*
9	16*	17, 18*		17
11		19		
12		20, 21		20, 21

Films. At this writing a PSSC film is planned on the wave interference of light. It is not currently available.

Section 1 - Can We See Interference in Light?

PURPOSE To point out the physical factors related to the production and observation of interference in light.

CONTENT a. If the wave model of light is correct, we should be able to obtain, with light, interference patterns similar to those observed in the ripple tank.

b. By analogy with water waves, when we look for interference in light, we expect the angles of the nodal lines to depend on λ/d, and we will need to work with a definite wave length.

c. An experiment, set up to show the two-source interference pattern, fails when two independent light sources are used. This failure may be due to rapid changes of the phase delay between the two sources. Before giving up the wave model of light we should experiment using light sources which are known to be in phase.

EMPHASIS This background discussion should be understood thoroughly. How much time is required here depends strongly on how well the previous chapter has been understood.

DEVELOPMENT This section can be handled best by a class discussion in which students join in "planning" the experiments to be discussed in the remainder of the chapter. The discussion will naturally draw on the experience gained from the ripple tank experiments. Some of these should be re-examined to understand how they could be repeated using light sources.

It should be worthwhile to take a good second look at the two-source patterns for water waves in Figures 18-6 and 18-9, using methods and ideas suitable for application with light sources; for instance, we must imagine the back side of the ripple tank (the side farthest from the sources) replaced by a screen and the two sources of water waves replaced by two light sources which are somehow made to emit light waves in phase. If this is done we can only see the places on the screen where the nodal lines intersect,

and the places where the double crests arrive at the screen. To calculate the wave length (actually the ratio λ/d) we must now apply the geometry outlined in Figures 18-12 and 18-13. Unless you have already done so, by all means you should evaluate λ/d for Figures 18-6 and 18-9 by this method. The simple geometry used should be thoroughly understood since it is used over and over again in the chapter, sometimes with minor changes. Make sure also that everyone understands qualitatively and quantitatively what happens when the ratio λ/d changes.

Students should also reexamine Figures 18-16 and 18-17 which show, better than words, the effect of changing the phase delay between the sources. Make sure students notice that the patterns on the "screen" for $p=0$ and $p=1/2$ are complementary; "dark" and "bright" regions are interchanged so that if the two patterns are superposed, the screen is uniformly illuminated, i.e., no interference pattern. Alternatively we note that if the phase delay is changed smoothly from $p=0$ to $p=1$, each "dark bar" moves smoothly across the screen, until for $p=1$ each bar occupies the position formerly occupied by its neighbor. With a sequence of interference patterns produced with phase delays which increase in uniform steps from $p=0$ to $p=1$, the resulting bright bars fill the screen uniformly; the superposition of interference patterns which have all possible phase delays results in a uniformly illuminated screen. In this circumstance the interference pattern is not discernible.

From such a discussion the need to use sources having a constant phase delay should be obvious. If possible, challenge your students to devise light sources which are guaranteed to be in phase (or to have a fixed phase delay). HDL Problem 1 is a good starting point for this discussion. Note that it is not necessary to the purpose of this section to know <u>why</u> two independent light sources have a phase delay which varies rapidly with time. This interesting question will be taken up shortly in Section 3.

Section 2 - Interference of Light Waves: Young's Experiment

PURPOSE To qualitatively describe a two-slit interference experiment.

CONTENT To provide two light sources which are locked in phase we use a single source and split the light from it into two beams. One way to do this is to arrange a line source of light parallel to two narrow slits in an opaque barrier.

LABORATORY II-14 Young's Experiment.

DEVELOPMENT This section presents a solution to the puzzle posed in the previous section: how do we make two light sources which are known to be in phase?

It will be worthwhile to do Experiment II-14 at this point since the students should have by now all of the necessary background to understand the results both qualitatively and quantitatively. The following sections (3 and 4) can then be used to review and summarize what has been learned in the laboratory.

Section 3 - The Phase of Light Sources: Atoms

PURPOSE To briefly explain how the phase of the sources is related to the stability of interference patterns.

CONTENT In a light source individual atoms send out short bursts of light independently of each other, therefore, the phase delay between two light sources changes very rapidly with time. The interference patterns shift accordingly.

COMMENT Prior to this section we have seen interference of light. The failure to

DEVELOPMENT The material in this section should be developed primarily through the observation in the laboratory of the diffraction patterns of light and through the ripple tank demonstration. If you cannot do the demonstration, the photographs shown in Figure 19-11 of the text should be carefully discussed. The observations in the ripple tank can serve two purposes:

(1) To establish empirically the mathematical relationship derived in the next section by use of the principle of superposition.

(2) To show that Huygen's principle, in which we replace a wave front by a number of sources, is justified.

Unless you have already used Huygen's principle in discussing reflection and refraction you will gain very little by going beyond the limited application of Huygen's principle to the case at hand. In any case you should stress that the present application is in fact based directly on an experiment.

Note that if you succeed in establishing the relationship between $\frac{\lambda}{w}$ and $\sin\theta$ from the ripple tank experiment, a reasonable interpretation of the diffraction of light as the basis of the wave model is available independent of the theoretical predictions developed in the next section. Since some students may find the arguments of the next section rather difficult, this alternative approach may be helpful to them.

Section 6 – A Theory of Diffraction by a Slit

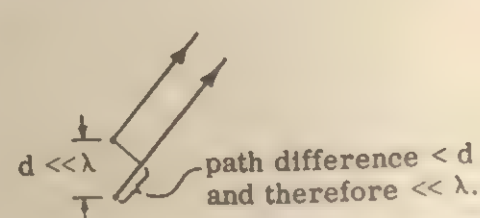
CONTENT The position of the nodes and the (approximate) position of the maxima in the diffraction pattern of a single slit can be determined by "replacing" the slit with an array of sources in phase, and applying the principle of superposition.

DEVELOPMENT You should emphasize quite strongly the two points on which the derivation hinges:

(1) The directions in which there will be complete cancellation can be predicted by grouping the sources in pairs. The pairs are chosen so that the path difference between the members of each pair is $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, etc.

(2) Only directly in front of the slit can the crests from all of the sources coincide. The reason for this is that when the slit is replaced by an array of sources in phase, the sources must be much less than a wave length apart. In this case the path difference between waves from adjacent sources can never exceed a small fraction of a wave length, even at 90° from the normal to the slit.

If the spacing within the array of sources is greater than λ, then at some angle θ, the path difference, $d \sin\theta$ would equal λ, and a bright bar would be seen at the angle θ. This is what happens in a diffraction grating where $d > \lambda$, and several orders of constructive interference, at several angles, are observed. (See comment in the last paragraph of Section 4 of the text.)



Note that the geometry used in this section to find path differences is exactly the same as the geometry used to find path differences with two sources (Section 18-4).

If your class seems to get the arguments of this section, you may wish to apply them to another example or two. One important example is the diffraction grating. Here we actually do have a series of equally spaced sources in phase; each source is a slit, and all slits are illuminated by a single line source. In the diffraction grating however, the distance, d, between neighboring sources (slits) is larger than the wave length λ. Consequently there is complete reinforcement in directions other than straight ahead. This occurs for angles such that the path difference between neighboring slits is an integral number of wave lengths, i.e., for angles determined by $d \sin \theta = m \lambda$, when $m = 0, 1, 2, 3, \ldots$. However the argument pursued in this section tells us that complete cancellation occurs with N slits if $(Nd) \sin \theta = (\text{integer}) \times \lambda$, unless $d \sin \theta$ is also equal to an integral number of wave lengths. This means for instance, that there is a strong maximum at $d \sin \theta = \lambda$, i.e., for $Nd \sin \theta = (N+1)\lambda$, or $(N-1)\lambda$. If N is very large, this means that the bright bar at $d \sin \theta = \lambda$ is very narrow. As in diffraction by a slit, about midway between successive minima there are secondary maxima where partial reinforcement occurs. These secondary maxima are much weaker and are not usually observed. Do not try to go into detail.

CAUTION The method of pairing slits to show cancellation at $Nd \sin \theta = \text{integer} \times \lambda$ only works if N is a power of 2. Cancellation does occur for other values of N but it is more difficult to prove. HDL 12 treats the case $N = 4$.

Clearly the method of pairing works in exactly the same way for the radiation pattern of an antenna composed of stacked, equidistant sources of radio waves, all in phase. If the spacing is less than a wave length, the radiation pattern of such an antenna would be like the diffraction pattern of a slit.

Section 7 – Experimental Checks with Single and Double Slits

CONTENT The observed diffraction patterns produced by slits are in quantitative agreement with the predictions of the wave model of light. The interference pattern from two slits falls within the overlapping central maxima of the diffraction pattern from each individual slit.

COMMENT The role of diffraction in producing a two slit interference pattern can be seen rather clearly in Figure 19-30 (see HDL 10).

Section 8 – Resolution

PURPOSE To point out the effect of diffraction on resolution.

CONTENT The resolution of an optical instrument is a measure of its ability to give separated images of objects that are close together. The ability of optical instruments to resolve fine details is limited by the wave nature of light.

EMPHASIS Sections 8, 9, 10 and 11 describe light phenomena which are interesting in their own right and which also enable showing more broadly the applicability of the wave model for light. This latter point should not escape students' attention.

LABORATORY Experiment II-16, Resolution, can be done with this section.

DEVELOPMENT You should probably limit discussion to the qualitative aspects treated in the text. A crude estimate of the resolving power of a lens used in viewing distant objects can be obtained from an extension of the discussion of HDL 14 (see green pages). In an optical system used to look at near objects (microscope), the situation is somewhat different but the result is quite similar. (See HDL 13.)

The diameter of the pupil of the eye is of the order of $2\text{mm} = 2 \times 10^{-3}$ meters. Taking $\lambda = 5 \times 10^{-7}$ meters we find (see page 294):

$$\theta \approx \sin\theta = \frac{\lambda}{w} = \frac{5 \times 10^{-7} \text{ meters}}{2 \times 10^{-3} \text{ meters}} = 2.5 \times 10^{-4} \text{ radians.}$$

An angular resolution of 2.5×10^{-4} radians implies that a good eye should be able to resolve two lines a distance 2.5×10^{-4} meters apart, i.e., 0.25 mm, at 1 meter. This is not very far from the actual resolution.

Some of your photographic enthusiasts will see quickly that, for pinhole photography, the pinhole can be too small. For distant objects (distant compared with the pinhole to film distance), the optimal diameter of the pinhole is approximately $A_0 \approx 2\sqrt{\lambda q}$, where λ might be taken as an average (say 4×10^{-5} cm), of the wave lengths to which the film is sensitive, and q is the distance in cm between the image and the pinhole.

More precisely, and for object distances that are relatively near the pinhole,

$$A_0 = 2\sqrt{\frac{.9\lambda pq}{p+q}}.$$

Among other sources, a discussion of this can be found in J. E. Mack and M. J. Martin, The Photographic Process, McGraw-Hill Book Co., 1939.

Section 9 – Interference in Thin Films

CONTENT a. The two beams of light resulting from reflection at the two surfaces of a thin film can interfere.

b. Whether the interference results in reinforcement, cancellation, or something in between, depends on the thickness of the film.

c. With a film which is thin compared to the wave length, complete cancellation occurs; this implies that one of the two reflected waves has been turned upside down by the reflection. The analogy of wave reflection from the junction of two springs (in which waves travel at different speeds) suggests that inversion occurs at the surface where the wave is moving from a high speed medium to a low speed medium.

d. With films of finite thickness, cancellation or reinforcement can be predicted by taking into account the path difference between the two reflected waves.

LABORATORY Experiment II-17, Measurement of Short Distances by Interference, can be done with this section.

DEVELOPMENT There are two effects to be taken into account if we are to explain interference in thin films:

a. The beam reflected from the back side of the film travels farther than the beam reflected from the front side by twice the thickness of the film (for normal incidence).

b. One of the reflected beams is turned upside down, the other is not.

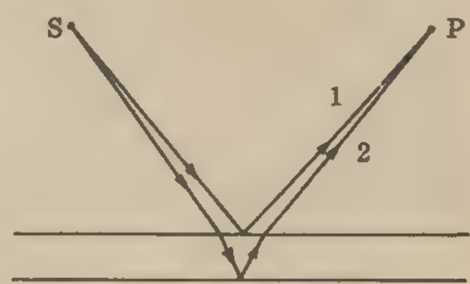
If you wish to simplify matters you can take up the two effects separately by doing the laboratory experiment first. In the laboratory experiment we can be concerned only with the effect of path difference; we simply look at the alternation of strong reflection and no reflection due to the variation in thickness along the wedge-shaped air film. We can understand what happens by noting that if we have complete reinforcement for a given path difference, we also should get reinforcement if the path difference is increased by λ, i.e., if the film thickness is increased by $\lambda/2$. Again, if the two waves reinforce (crest matches crest, trough matches trough) at a given path difference, we get

complete cancellation by increasing the thickness by $\lambda/4$. This changes the path difference by $\lambda/2$, so that the two waves are displaced relative to one another by $\lambda/2$, and now crests match troughs and troughs match crests.

It is in the discussion of how to see that a wave in air could be turned upside down upon reflection from glass that your work with reflected waves on a coil spring will pay off. The argument goes: just as pulses on coil springs are reflected back inverted as the pulse goes from a medium of high velocity to a medium of low velocity, so light waves are reflected upside down in going from a high velocity medium to a low velocity medium.

The fact that if one reflection is right side up, then the other is upside down is a general property of all waves. We know this as an experimental fact from the case at hand, since complete cancellation of the two waves occurs when the path difference is negligibly small. This tells us that one wave is upside down, but not which wave is upside down (see HDL 17). We could probably have guessed that no reflection should occur from a very thin film by going to the limit of thinness; a film of zero thickness means no film and therefore no reflection.

Students may ask how the two parallel rays 1 and 2 in Figure 19-22 can interfere with each other since they do not overlap. The simplest answer is that we are really only concerned with the case of normal incidence, in which case 1 and 2 do coincide in space. Of course this answer is not quite fair since we also get interference at angles other than 90°. A complete answer would involve drawing a figure showing the source of light, and the point at which the interference effects are seen (e.g. a screen). The figure would then look something like that at the right. Things are rather more complicated if we do not have normal incidence (we have a small difference in path outside as well as inside the film). This is why it may be wise to emphasize only the case of normal incidence. The figures in the text are drawn as they are so that it is easy to distinguish the various rays.



Someone may ask how the first and second reflected beams can cancel, since the second reflected beam is obviously of smaller amplitude. The answer is that the first and second reflected beams are so nearly of the same amplitude that cancellation is almost perfect.

Section 10 - Interference in Light Transmitted Through Thin Films

CONTENT a. Interference effects are observed in thin films by transmitted as well as reflected light.

b. It is sufficient to take into account two transmitted beams: a stronger beam which is transmitted directly and a much weaker beam which is transmitted after two reflections in the film. The path difference between the two beams is equal to twice the thickness of the film for normal incidence.

c. When the path difference is 0, λ, 2λ, etc., the two beams reinforce each other. When the path difference is $\lambda/2$, $3\lambda/2$, etc., there is maximum cancellation between the two beams. Transmission maxima coincide with reflection minima, and vice versa, as would be expected.

DEVELOPMENT The material in this section should require very little discussion.

Section 11 - Color Effects in Interference

CONTENT The location of the dark and bright zones in an interference pattern differs for different wave lengths. With white light the mixture of wave lengths produces overlapping interference patterns, thus giving rise to the observed color effects.

DEVELOPMENT This subject should require little discussion. The main point to get across is that the color effects give further evidence that the physical difference between light of different colors is the difference in wave length.

Section 12 - Conclusion

PURPOSE To briefly summarize Part II of the course and to mention a few further interesting questions about light. Some of these questions will be considered later in the course, some will not.
These questions are:

1) What is the nature of the medium through which light propagates?
2) How does the amplitude of light "waves" vary as the intensity of the light changes?
3) What is the nature and significance for light of standing waves?
4) How can the wave and particle aspects of light be encompassed in one theory?

DEVELOPMENT Aside from whatever preferences you may have for or against "reviews" in class, it will be worthwhile to spend a little time discussing in terms of Part II the idea of a model and how the validity of a model is established.

APPENDIX "Discussion of the Lensmaker's Formula"

This appendix is added here because it can be used, if you have time to illustrate that the interference of light, a wave phenomenon, underlies the behavior of lenses.

Chapter 19 - Light Waves

For Home, Desk and Lab - Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on page 19-12 to 19-23.

Section	Easy	Medium	Hard	Class Discussion
1				1*
2	2, 3*	4, 5*		4, 5*
4				7*
6	9	6	8	8
7		10, 11, 12*		10, 12*
8	13*, 15		14*	14*
9	16*	17, 18*		17
11		19		
12		20, 21		20, 21

SHORT ANSWERS

1. Phase delay of the sources changes rapidly.
2. 5×10^{-5} cm.
3. Screen: 1.5 meter.
   Bar Spacing: 0.7 cm.
4. Experimentally, no change.
5. For angles satisfying the equation $\sin\theta = n\lambda/d$.
6. From diffraction pattern $w = \lambda L/x$.
7. 1.9×10^{-15} sec.
   10^6 wave lengths.
8. Diffraction pattern widens.
9.

n \ w	1 mm	10 mm	0.1 mm
1	$\theta = 0.033^\circ$	0.0033°	0.33°
2	$\theta = 0.066^\circ$	0.0066°	0.66°
3	$\theta = 0.100^\circ$	0.0100°	1.00°

10. See discussion on page 19-17.
11. See discussion on page 19-17.
12. a) Nodes for $\sin\theta = \lambda/4d$, $\lambda/2d$, and $3\lambda/4d$.
    Maxima for $\sin\theta = 0$ and λ/d.
    b) Node for $\sin\theta = \lambda/2d$.
    Maxima for $\sin\theta = 0$ and λ/d.
13. a) Angular separation must be greater than θ where $\sin\theta = \lambda/w$.
    b) Fluorescent screen or photographic film.
    c) Lens of quartz or corex.
14. a) See discussion on page 19-20.
    b) $3 \times 10^4\lambda$.
15. Minimum angle of resolution depending on λ/D is reduced for short (blue) wave lengths.
16. See discussion on page 19-21.

17. a) 0.026 cm.
    b) 28 bars/cm.
    c) Minimum.
    d) No.

18. a) 1100 A.
    b) Reflected red and violet produce purple.

19. a) $\theta = 4.3^\circ, 13.0^\circ, 22.0^\circ, 31.7^\circ, 42.5^\circ, 55.6^\circ, 77.0^\circ$.
    b) Nodal patterns obscured by multiple reflections.

20. See discussion on page 19-23.

PROBLEM 1 — Why can't we see interference from two independent light sources?

The question is discussed and answered in Sections 1 and 3 of the text. Briefly: the position of the dark and bright bars depends on the phase delay between sources, as well as on the path difference. If the phase delay varies with time the interference pattern shifts. A rapidly shifting pattern results in uniform illumination of the screen. There is no reason to believe that the phase delay between independent sources should stay constant.

PROBLEM 2 — In an interference pattern produced by white light passing through two narrow slits, the distance between the black bars is 0.32 cm. The distance between slits is about .02 cm, and the distance to the screen on which the bars are observed is 130 cm. Find the average wave length of white light.

Using the formula given for the spacing between dark bars, $\Delta x = L\frac{\lambda}{d}$, we have

$\Delta x = 0.32$ cm
$d = 0.02$ cm
$L = 130$ cm.

$$\lambda = \Delta x \frac{d}{L} = 0.32 \text{ cm} \times \frac{0.02 \text{ cm}}{130 \text{ cm}} = \frac{(3.2 \times 10^{-1})\ (2 \times 10^{-2})}{1.3 \times 10^{2}} \text{ cm} = \underline{5 \times 10^{-5} \text{ cm}.}$$

COMMENTS If students substitute mechanically into the formula they will learn little. Bring out the basic steps in the reasoning:

(1) If the slits are equidistant from the source each slit acts as a source in phase with the other.

(2) A node (dark bar) on the screen will occur if the path difference from the two sources to the screen is $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ . . . . .

(3) From the geometry illustrated in Figures 18-12 and 18-13 (be sure your class understands the geometry), we have: path difference = $d \sin\theta$; and $\sin\theta = x/L$.

(4) If n is very small it is sufficiently accurate to take L as equal to the distance from the plane of the slits to the plane of the screen.

(5) Putting the path difference equal to $\frac{\lambda}{2}, \frac{3\lambda}{2}$, etc., we get the position of successive nodes:

$$\frac{x_1}{L} = \sin\theta_1 = \frac{d \sin\theta_1}{d} = \frac{\lambda/2}{d}, \text{ and } x_1 = \frac{L\lambda}{2d}$$

$$\frac{x_2}{L} = \sin\theta_2 = \frac{d \sin\theta_2}{d} = \frac{3\lambda/2}{d}, \text{ and } x_2 = \frac{3L\lambda}{2d}$$

$$\text{and } x_3 = \frac{5L\lambda}{2d}$$

The distance between successive dark bars is then:

$$\Delta x = x_2 - x_1 = x_3 - x_2 = \ldots\ldots\ldots\ldots = \frac{L}{d}\lambda.$$

PROBLEM 3

A source of red light produces interference through two narrow slits spaced a distance $d = .01$ cm apart. At what distance from the slits should we place a screen so that the first few interference bars are spaced one centimeter apart? What will be the spacing of the bars if we then use violet light?

$$\Delta x_{red} = L \frac{\lambda_{red}}{d}$$

We have $d = 0.01$ cm

$\lambda_{red} = 6.5 \times 10^{-7}$ meters $= 6.5 \times 10^{-5}$ cm.

$\Delta x_{red} = 1$ cm.

$$L = d \frac{\Delta x_{red}}{\lambda_{red}} = \frac{1 \text{ cm}}{6.5 \times 10^{-5} \text{ cm}} \times 10^{-2} \text{ cm} = 1.5 \times 10^{2} \text{ cm} = \underline{1.5 \text{ meters}}.$$

If we use a different wave length the spacing varies directly with the wave length (since the path difference in wave lengths must remain the same).

$$\frac{\Delta x_{violet}}{\Delta x_{red}} = \frac{\lambda_{violet}}{\lambda_{red}}$$

$$\Delta x_{violet} = \frac{\lambda_{violet}}{\lambda_{red}} \times \Delta x_{red} = \frac{4.5 \times 10^{-7} \text{ meter}}{6.5 \times 10^{-7} \text{ meter}} \times 1 \text{ cm} = \underline{0.7 \text{ cm}}.$$

PROBLEM 4

What will happen to the interference pattern in Young's experiment if the source is not exactly on the center line between the slits?

If the source is not exactly on the center line between the slits, a crest from the source reaches the two slits at different times, and the two slits act as sources with a fixed phase delay. We know, from the discussion of Section 5, that this results in displacing the whole interference pattern to one side, without any change in spacing if the angular displacement of the source from the center line is small. Experimentally we observe no change, except, as the source moves, the center of the interference pattern is shifted so as to remain on the line from the source through the midpoint between the slits.

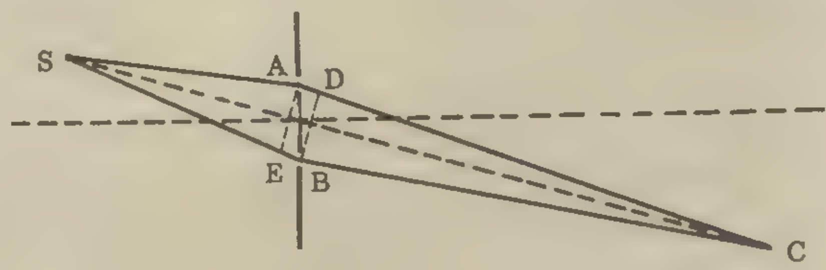
If the angular displacement of the source is large, the effect is similar to decreasing the distance between the slits. Thus more widely spaced nodal lines are obtained.

Alternate solution. Some students may follow a somewhat more concrete and detailed approach:

If the source is not on the center line between the slits, the interference pattern shifts over so as to be centered on a line from the source passing through the center of the pair of slits.

The explanation is as follows: Consider the normal case and consider only the central bright fringe.

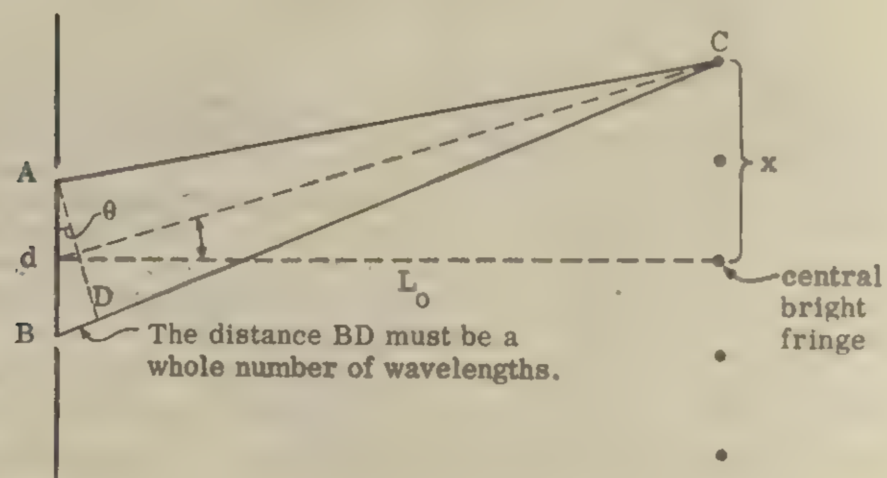
The "central" fringe results from the path SAC being the same length as SBC. Now suppose the source, S, moves upward. Again, the central bright fringe will occur at the point on the screen where the two paths, source to slit to screen, are equal. (Not slit to screen.)



The path AC is longer than BC by the distance AD. However, SB is longer than SA by the distance EB. The point C must move down so that SADC = SEBC.

If all angles are very small (they are, in all cases considered), a position of C on a line from the source through the midpoint between the slits will make SADC = SEBC.

PROBLEM 5 When a source of light of wave length λ is used in a two-slit experiment with narrow slits separated a distance d, at what angles do you expect to find the *maxima* in the light intensity in the interference pattern?



The central maximum in the interference pattern occurs when the path lengths from the two slits to the screen are equal — assuming the slits are at equal distances from the source. Maxima also occur when the paths from the two slits to the screen differ by whole numbers of wave lengths.

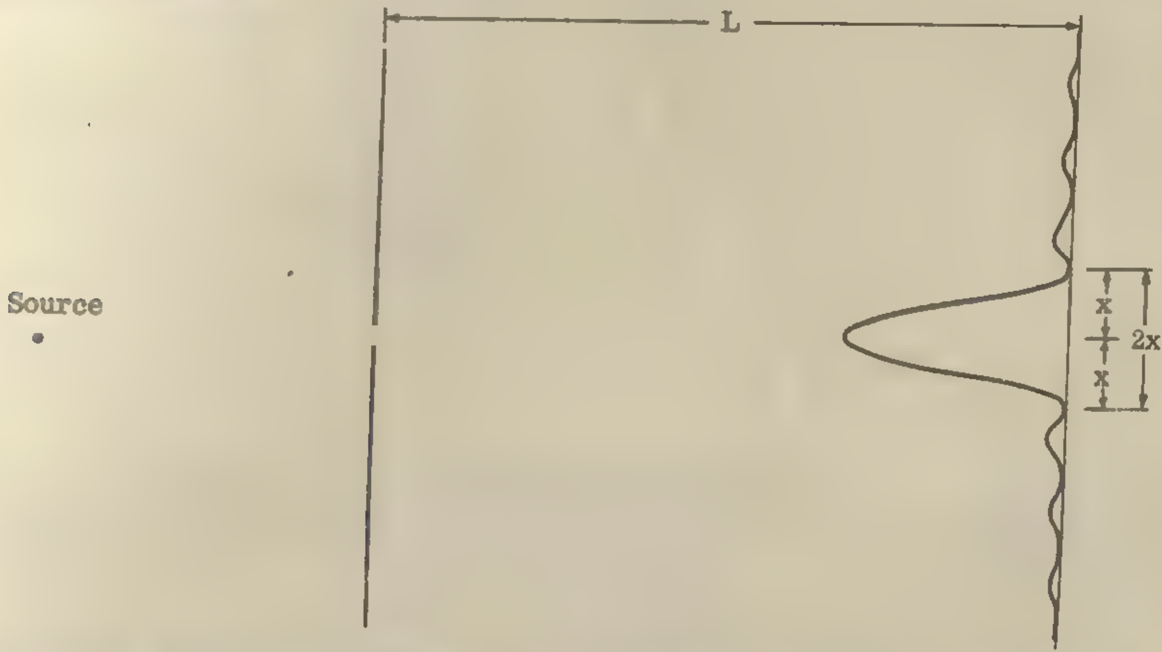
Consider the figure above. The light going from slit B to the maximum at C must have gone a whole number of wave lengths farther than the light from the slit A to the maximum at C. But BC is longer than AC by the distance BD. Hence $BD = n\lambda$, where $n = 0, 1, 2, 3 \ldots$

It can be shown that if C is far away (AC is large compared to AB), $BD = AB \sin\theta$. But $AB = d$, the distance between the slits. Finally then $d \sin\theta = n\lambda$, and $\underline{\sin\theta = n\lambda/d}$ where $n = 0, 1, 2, 3 \ldots$

PROBLEM 6

Suggest an optical method for measuring the width of a narrow slit.

One straightforward method: set up a source (slit or narrow filament) in front of the slit to be measured



and measure the distance x between the central maximum and the first dark space in the pattern appearing on a screen. We now know x and can measure L. To find the slit width w: $\frac{x}{L} \approx \sin\theta = \frac{\lambda}{w}$, and $\underline{w = \frac{\lambda L}{x}}$.

PROBLEM 7

Calculate the period of yellow light. About how many wave lengths are included in a light wave during emission of a light burst by a single atom?

To relate the period, T, to the wave length, λ, we note that the wave moves forward one wave length in one period. Letting c = speed of a light wave we have:

$$c = \frac{\lambda}{T}, \text{ and } T = \frac{\lambda}{c}.$$

We have $\lambda_{yellow} \approx 5.8 \times 10^{-7}$ meters

$$c = 3 \times 10^{8} \text{ meters/second}$$

$$T \approx \underline{1.9 \times 10^{-15} \text{ seconds.}}$$

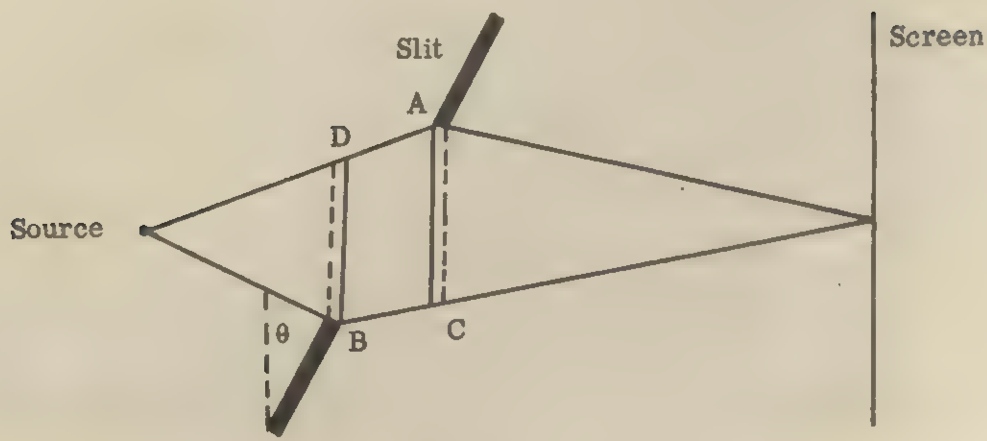
The burst of light from a single atom lasts for a time t_B of the order of 10^{-9} seconds. In this time the number of periods is of the order of:

$$\frac{t_B}{T} \approx \frac{10^{-9} \text{ seconds}}{10^{-15} \text{ seconds/period}} = 10^{6} \text{ periods.}$$

A single atom, therefore, emits a wave train of the order of $\underline{10^{6} \text{ wave lengths}}$ long. This suggests a method for estimating the duration of a light burst. We could set up an interference experiment and keep increasing the path difference between the interfering beams. As long as the path difference is smaller than the length of wave train the wave trains in the split beams overlap. When the path difference exceeds the length of the wave train there is no overlap and the interference pattern disappears.

PROBLEM 8

What happens to the diffraction of a single slit when you turn the slit as in Fig. 19-29 instead of holding the opening perpendicular to the path of the light? Why?



As in Problem 4, the extra distance, DA, the light travels to reach the top of the slit, is compensated for by the extra distance BC, the light travels from the bottom of the slit to the screen. The central maximum does not move.

However, the slit becomes effectively narrower. If it has a width, w, then its effective width is w cos θ. Since the slit becomes narrower the diffraction pattern becomes wider.

Note: The solution given above is correct to first order, and is all that the students should be concerned about. The complete solution of the problem is very complicated.

PROBLEM 9

(a) When yellow light passes through a slit 1 mm wide, at what angles are the first three nodes in the diffraction pattern?
(b) If the slit is 10 times as wide?
(c) If it is $\frac{1}{10}$ as wide?

The nodes in the diffraction pattern occur at angles such that the path difference w sin θ from the two edges of the slit to the screen is equal to an integral number of wave lengths.

$w \sin \theta_n = n\lambda$, and $\sin \theta_n = \frac{n\lambda}{w}$

For $\lambda = 5.8 \times 10^{-7}$ meters, and $w = 10^{-3}$ meters, 10^{-2} meters, 10^{-4} meters, respectively, we get:

For the first node with a 1 mm slit, $\sin \theta = \frac{\lambda}{w} = \frac{5.8 \times 10^{-5}\text{ cm}}{10^{-1}\text{ cm}} = 5.8 \times 10^{-4}$.

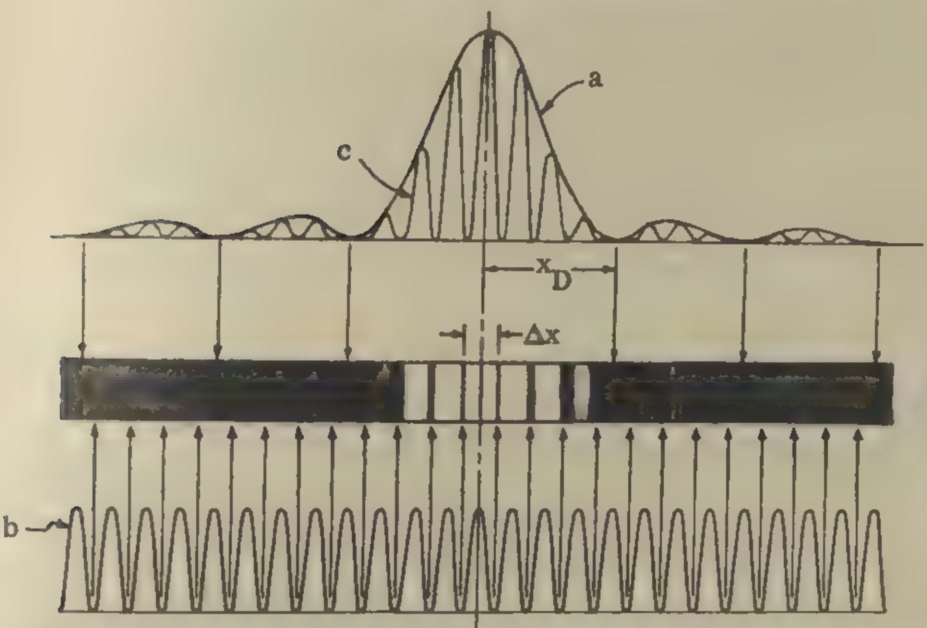
Since our trig tables do not go to this small a value, to get the angle it is necessary to realize that for small angles the angle in radians is equal to the sine of the angle. Angle in radians = 5.8×10^{-4}. Since angles in radians and degrees are proportional,

$\frac{\theta}{5.8 \times 10^{-4}} = \frac{180\text{ degrees}}{\pi\text{ radians}}$ or $\theta = \underline{0.033^\circ}$.

n \ w	1 mm	10 mm	0.1 mm
1	$\theta = 0.033^\circ$	0.0033°	0.33°
2	$\theta = 0.066^\circ$	0.0066°	0.66°
3	$\theta = 0.100^\circ$	0.0100°	1.00°

PROBLEM 10

When two identical fairly wide slits are used to make an interference pattern and when they are separated by a distance comparable to their width, the resulting pattern combines the features of a single slit diffraction pattern and the pattern of interference between the two slits. A photograph of such a pattern is shown in Fig. 19-30. Identify the dark regions arising from the diffracuon pattern of each slit and the dark bars arising from interference between the slits.



Arrows show dark regions due to diffraction by each slit.

Arrows show dark bars due to interference between slits.

Curve (a) represents the intensity distribution to be expected from diffraction by a slit.

Curve (b) represents the intensity distribution to be expected from the interference of the light from two point (or line) sources in phase if they emit light uniformly in all directions.

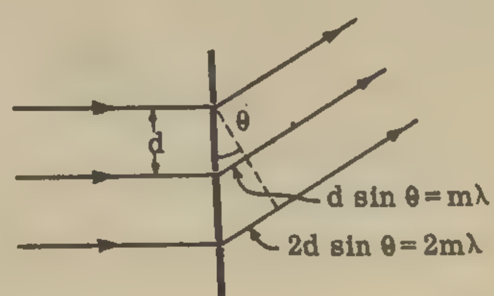
Curve (c) obtained by multiplying (a) and (b) gives the intensity distribution to be expected when we have two line sources which emit light according to the diffraction pattern of a slit rather than uniformly in all directions.

The ratio of slit separation d to slit width w seems to be about 3.8. We would have: $x_D = \frac{L\lambda}{w}$; $\Delta x = \frac{L\lambda}{d}$; $\frac{x_D}{\Delta x} = \frac{d}{w} \approx 3.8$ according to Figure 19-30.

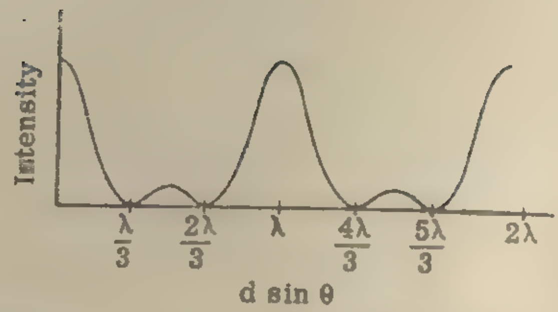
PROBLEM 11

When a source of wave length λ is used with three slits, each separated from its neighbor by the distance d, show that you get maximum intensity at the same angles as for two slits only.

The condition required to obtain complete reinforcement between any number of waves coming from a set of sources all in phase is clearly that the path difference from any two sources be an integral number of wave lengths. If the sources are equally spaced this condition is satisfied when the path difference from adjoining sources is an integral number of wave lengths.



It seems likely that there are secondary maxima where partial reinforcement occurs. In fact the intensity pattern from three slits looks like this.



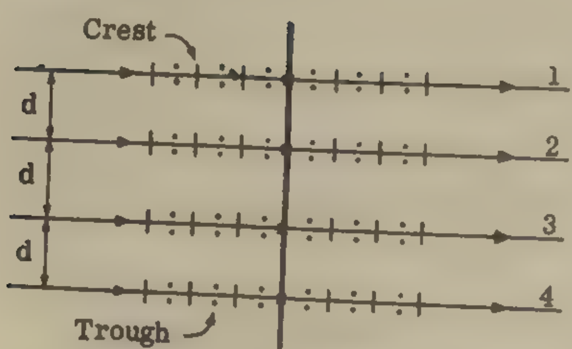
The existence of minima when $Nd \sin\theta = (\text{integer}) \times \lambda$ (N = number of slits), but when $\frac{d \sin\theta}{\lambda}$ is not an integer, is suggested by the discussion in the text (pages 292-3, Figures 19-13, 14). It is true in general, but can be proved by the trick of pairing off slits only when N is a power of 2 (see Problem 12). With more than 2 slits the principal maxima occur at the same angles as with two slits but they are narrower.

PROBLEM 12

(a) For four narrow slits spaced d apart, find out at which of the angles given by $\sin\theta = 0$, $\lambda/4d$, $\lambda/2d$, $3\lambda/4d$, and λ/d you expect maxima and at which of the angles you expect nodes in the intensity of light in the interference pattern. The diagrams (Fig. 19-31) and the idea of pairing may help you.

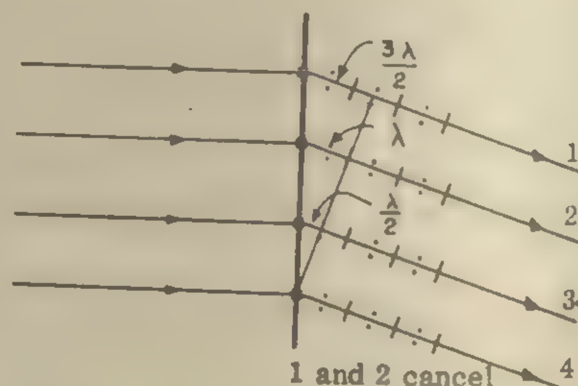
(b) Which of these angles is a node of the two-slit pattern? Which a maximum?

Sin $\theta = 0$:



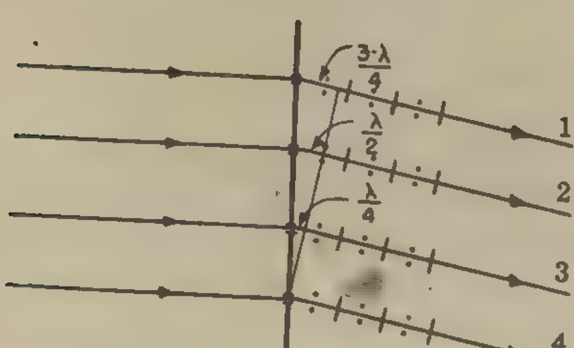
All beams reinforce

Sin $\theta = \lambda/2d$:



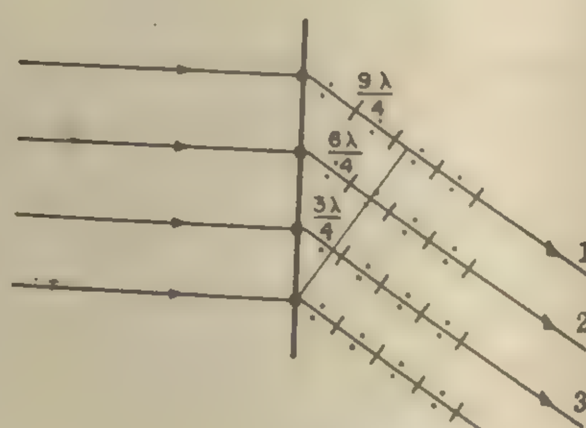
1 and 2 cancel
3 and 4 cancel

Sin $\theta = \lambda/4d$:



1 and 3 cancel
2 and 4 cancel

Sin $\theta = 3\lambda/4d$:



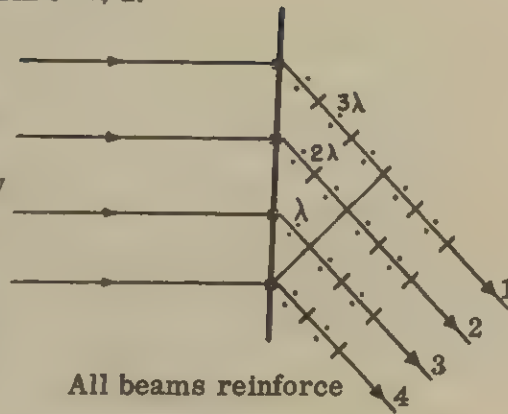
1 and 3 cancel
2 and 4 cancel

We find: $d \sin \theta = 0$: Maximum
$d \sin \theta = \lambda/4$: Node
$d \sin \theta = \lambda/2$: Node
$d \sin \theta = 3\lambda/4$: Node
$d \sin \theta = \lambda$: Maximum

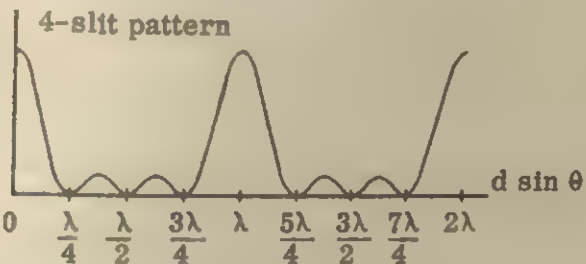
Between nodes partial reinforcement must occur and therefore there must be secondary maxima approximately midway between nodes. The intensity pattern should look like that shown in the figure below.

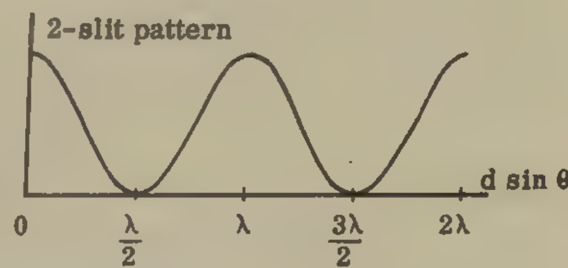
c) The principal maxima of the 4-slit pattern coincide with those of the 2-slit pattern, however, they are twice as sharp. With many thousands of slits (diffraction grating) the maxima at $d \sin \theta = m\lambda$ are exceedingly narrow.

Sin $\theta = \lambda/d$:



All beams reinforce





PROBLEM 13

Microscopes in which the object is illuminated by ultraviolet light can give higher magnifications than microscopes that use visible light.
(a) How do you explain this?
(b) How are the images seen if no visible light is used?
(c) Since glass is opaque to ultraviolet light how can such a microscope be made?

a) For a microscope to give high magnification, the lens diameter (aperture) must be very small so as to permit a small radius of curvature for the lens surface. But, magnification will not be helpful if, because of diffraction effects from the small aperture, two adjacent points cannot be distinguished.

As a way of approximately considering the diffraction effects with a small lens aperture, students might consider the lens aperture as a slit. Light coming through a single slit is spread out into a diffraction pattern with the first minimum occuring at an angle such that $\sin \theta = \lambda/w$. Considering light from two adjacent sources is comparable to considering light coming from two adjacent points on an object. Light coming through a slit from <u>two sources separated by a small angle</u> will produce overlapping diffraction patterns. If the two sources were far apart in angle, two distinct diffraction patterns would be seen and one could clearly say that there were two sources. If the two sources were an angle θ apart, where $\sin \theta = \lambda/w$, the two patterns would overlap and we could not distinguish the two sources clearly. It does not pay to be too fussy about the exact specification, in terms of λ/w, of the sine of the angle that will permit distinguishing between two adjacent sources. For a circular aperture, the requirement is a little different than for a slit.

However, to distinguish between two points, the angular separation of the points must be greater than θ, where $\sin \theta$ is about equal to λ/w, and w is the diameter of the lens. Because w is more or less fixed by the magnification sought, it follows that, to get as small a θ (angular separation of two points that can be distinguished) as possible, we must make λ as small as possible. Hence the use of ultraviolet light.

b) The images are displayed on a fluorescent screen or on photographic film.

c) If lenses are used they must be transparent to the wave length used. Quartz is transparent down to about 2,100 A, fluorite down to about 1,200 A, however, air absorbs strongly below about 1,800 Å. In principle one could build a microscope usi mirrors instead of lenses.

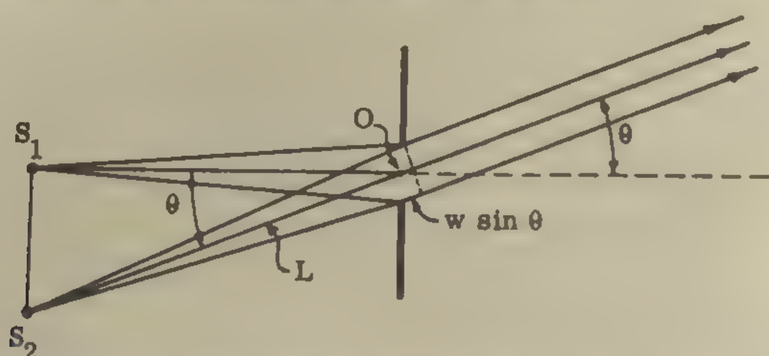
PROBLEM 14

Two images can just be resolved when the central maximum of one falls on the first node of the other.

(a) Show that the resolution of a narrow slit depends upon λ/w where w is the slit width.

(b) About how close together can two line sources be placed and still be resolved if they are viewed through a .01-cm slit (about the smallest size you can make easily) 3 meters away from the sources?

a) Since the phase delay between the two sources varies rapidly there is no interference between the light waves from the two sources. Each produces its own separate diffraction pattern and the two patterns are simply superposed.



The central maximum of the diffraction pattern produced by S_2 is in the direction from S_2 through the center of the slit. If the first node in the diffraction pattern of S_1 occurs in the same direction we must have $w \sin \theta = \lambda$.

Therefore, $\sin \theta = \frac{x}{L} = \frac{\lambda}{w}$

b) For $w \doteq 10^{-2}$ cm, and $L = 3 \times 10^2$ cm; $x = L\frac{\lambda}{w} = \underline{3 \times 10^4 \lambda.}$

If we take λ as, say, 5.7×10^{-5} cm, $w \approx 1.7$ cm.

PROBLEM 15

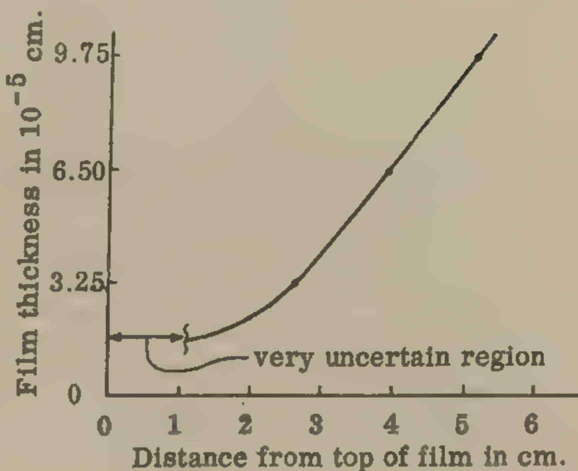
Stars are often photographed through a blue filter. What is the advantage of this?

The angular resolution of a telescope depends on the ratio λ/D of the wave length of light to the diameter of the telescope's objective (lens or mirror). If light of a shorter wave length is used the angular resolution is improved, so that stars which subtend a smaller angle at the earth can be resolved. In Problem 14 we saw that light sources separated by an angle θ can just be resolved through a slit of width w if $\sin \theta = \lambda/w$. The diffraction at the edge of a circular aperture of diameter D is almost the same as with a slit of the same width (actually the angular opening is larger by a factor of about 1.2). A telescope therefore, has an angular resolution of about $\sin \theta = \lambda/D$ ($1.2 \frac{\lambda}{D}$ actually). The Palomar telescope ($D = 5$ meters) has an angular resolution of about 0.028 seconds of arc in white light (average $\lambda = 5.7 \times 10^{-7}$ meters) and of about 0.022 seconds of an arc in blue light ($\lambda = 4.5 \times 10^{-7}$ meters).

PROBLEM 16

From the photograph of a soap film at the top in Fig. 19-20, plot a graph of film thickness against vertical distance down the film.

The photograph in Figure 19-20 shows alternate bright and dark fringes. The upper dark fringe shows that the thickness in that region is less than 1/4 wave length. The next dark fringe is in the region where the film is 1/2 wave length thick. If we take 6.5×10^{-5} cm as the wave length of red light, the graph of thickness against vertical distance down the film is as shown at the right. Distances from the top were measured along a vertical line through the center of the film.



PROBLEM 17

Two pieces of plate glass 10 cm long make an air wedge (Fig. 19-21). The plates are separated at one end by a human hair with a diameter of 0.09 mm. The reflected interference pattern is observed by looking in a direction perpendicular to the surfaces of the plates.

(a) What is the spacing of the bars if the incident light is blue?

(b) How many bright bars are seen per centimeter if the incident light is red?

(c) Is the light reflected from the end where the plates are in contact a maximum or a minimum?

(d) Can you tell from this experiment which reflected waves are turned upside down?

c) One way to handle this problem is to begin with part (c), noting that the light reflected from the region near the end where the plates are in contact must be a minimum. Where there is no air gap there is no reflection. This implies that the two reflected waves cancel. Therefore one of them must have been turned upside down. For a very small air gap the waves reflected from the two sides of the gap would still nearly cancel and there would be very little reflected light.

a) From (c) we see that there will be a black bar near the end where the plates touch. Now start counting bars. The next black bar will be where the air wedge is $\lambda/2$ thick (a total path difference of λ); the next black bar at λ. When we come to the end, we will have counted n bars and the thickness of the wedge of air is n/2 wave lengths. However it is also 0.09 mm thick. If we take the wave length of blue light as 4,750 A, the thickness in wave lengths is $0.09 \text{ mm} \div 4750 \times 10^{-7} = 190\lambda$. We will have counted 380 bars in 10 cm, so the spacing between bars is 10/380 = 0.026 cm.

b) For red light ($\lambda = 6,500$ A) the hair is fewer wave lengths thick. There are fewer bars, and the spacing is greater.

$$\frac{6500 \text{ A}}{4750 \text{ A}} \times 0.026 = 0.036 \text{ cm.}$$

The number of bars seen per cm is $\frac{1}{\text{spacing}}$ = 28 bar/cm.

d) We cannot tell from this experiment which waves are turned upside down.

PROBLEM 18

Lenses are often coated with a thin film to reduce the intensity of reflected light.

(a) If the index of refraction of the coating is 1.3, what is the smallest thickness that will give minimum reflection of yellow light?

(b) Such lenses often show a faint purple color by reflected light. Why?

a) Since the reflections from air to the film of $n = 1.3$, and from the film ($n = 1.3$) to glass ($n = 1.5$) are of the same type and both are inverted, a film thickness of $\lambda/4$ will give a total path difference of $\lambda/2$ and the two reflected waves will interfere destructively. Yellow light has a wave length of 5800A in air. The wave lengths in two different media are related by $n = \dfrac{\lambda_{air}}{\lambda_{film}}$.

$$\lambda_{film} = \frac{\lambda_{air}}{n} = \frac{5800\text{A}}{1.3} = 4460\text{A}.$$

The film should be $\lambda/4 = \underline{1100\text{A}}$ thick.

b) If the film is of the proper thickness to cancel yellow light, red light and violet light will be somewhat reflected. The reflected light will therefore have a reddish-violet or purple tinge.

PROBLEM 19

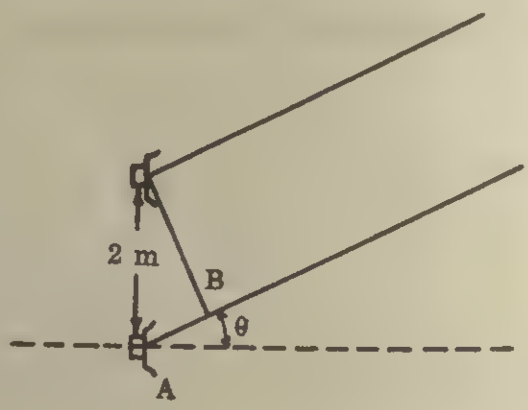
Two 3-inch loudspeakers emit a steady pitch whose frequency is 1,000 vibrations per second. These sources are in phase and are 2 meters apart.

(a) At what angles would you expect to hear no sound? (The speed of sound is about 300 m/sec.)

(b) What do you think would happen if you tried this experiment in a room with hard-surfaced walls?

This problem requires an extension of the students' knowledge of interference of light into the realm of sound.

a) No (or very little) sound will be heard at an angle θ such that the distance AB is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc., because the sound from the speakers will cancel.



For this problem $\lambda = v/f = 300/1000 = 0.3$ meters, $AB = 2 \sin\theta$ meters. No sound will be heard at

$$2 \sin\theta = \frac{0.3}{2}, \frac{0.9}{2}, \frac{1.5}{2}, \frac{2.1}{2}, \text{ etc.}$$

$$\sin\theta = 0.075,\ 0.225,\ 0.375,\ 0.525,\ 0.675,\ 0.825,\ 0.975.$$

$$\theta = 4.3^\circ,\ 13.0^\circ,\ 22.0^\circ,\ 31.7^\circ,\ 42.5^\circ,\ 55.6^\circ,\ 77.0^\circ.$$

b) Because of multiple reflections the sound would travel over many different paths of different lengths in reaching the ear or a microphone. Since it would arrive in many different phases a nodal pattern could not be observed.

PROBLEM 20

Can you suggest reasons why no interference is seen when light reflects from the two surfaces of a windowpane?

There are several reasons why interference fringes are not ordinarily seen in the reflection from a window pane. The principal one is that the window pane is usually viewed in white light. Suppose the pane is 0.3 cm thick. For light of wave length (in glass) 6000A, the path difference is 10,000 wave lengths and this light would interfere destructively and not reflect. Light of wave length 5,999.7A would have a path difference of 10,000.5 wave lengths and would interfere constructively. Thus, many wave lengths evenly distributed over the entire spectrum would reflect and the net results would be, not colored interference fringes, but a white reflection.

If light of one wave length were used we might see interference fringes. However, a single spectral line would have to be selected--not even yellow sodium light will do since it contains two closely spaced lines. Also the surfaces of most window glass probably are too far from being optically flat to show good fringes.

With thin pieces of glass like microscope cover-glasses, and under ideal conditions, interference fringes can be seen.

APPENDIX

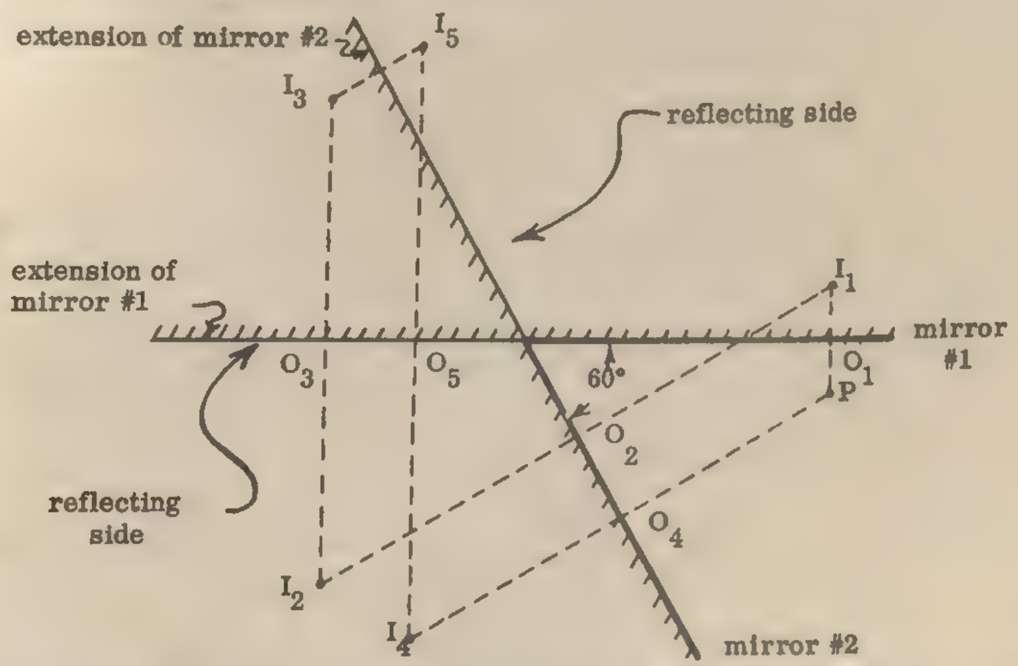
Appendix 1 – Supplement to Chapter 12, Section 5

Multiple Images Formed by Two Mirrors at an Angle

(This information is supplied to reduce the teacher's time and trouble in finding answers to questions which may come up. The subject is not suitable for presentation to the entire class; it is usually too complicated and not important enough to be worth the time it would take to make it understandable to even a small fraction of the class. This material may prove useful for after-school discussions with able students.)

An easy way to locate multiple images is to get "images" of images as outlined below. This procedure would confuse many students and should certainly not be given to the entire class.

The method is shown in the drawing below. Two mirrors are shown at 60°. An object, P, will give 5 images as shown, I_1, I_2, I_3, I_4, I_5. You can find all these images by going through the following steps.



1. Extend the lines of the two mirrors and indicate which side of the extended line corresponds to the reflecting side. For example, in the diagram, the upper right side of the extension of mirror 2, and the lower side of the extension of mirror 1 are the reflecting sides; the shading shows the back sides.

2. Find the image of P in mirror 1 using the standard technique. This gives I_1 where $PO_1 = I_1O_1$; PO_1 is perpendicular to mirror 1.

3. Next find the image of I in mirror 2. When you drop a perpendicular from I to the plane of mirror 2, it strikes the "reflecting side" of mirror 2. This gives I_2 where $I_1O_2 = I_2O_2$.

4. This procedure can now be continued by trying to get the image of I_2 in mirror 1. You drop a perpendicular to mirror 1 which hits it at O_3. Since I_2 is on the reflecting side of the extension of mirror 1, I_3 is a valid image of I_2, and $I_2O_3 = I_3O_3$.

5. Something different happens when you try to find an image of I_3 in the next mirror, which is the extension of mirror 2. This time if you drop a perpendicular to mirror 2, you find you are on the non-reflecting side. Hence, I_3 does not produce an image. I_3 is, therefore, the last image in this sequence.

Although this chain is broken, there may be other images. I_3 is the last of the images which can originate from light leaving P and hitting mirror 1 first. There are still other images which may be formed from light which hits mirror 2 first.

6. The first image of P formed by mirror 2 is labeled I_4. It is analogous to I_1 formed by mirror 1.

7. Similarly, image I_5 is the image of I_4 in mirror 1. It is analogous to I_2, even though it passes through the extension of mirror 1 rather than the mirror itself.

8. A question arises when you try to find the image of I_5 in mirror 2. If you drop a perpendicular you would get another image and a valid one, but this image (which might be called I_6) would exactly coincide with I_3. Since exact drawings or complex geometry would be needed to prove this, you will find it helpful to know that the images all lie on a circle centered at the point where the mirrors meet, and passing through the object. A simple formula can be given only if the angle can be obtained by dividing 180° by an integer. If the mirror angle, θ, is $\frac{180^\circ}{n}$, there are $(2n - 1)$ images.

APPENDIX 2

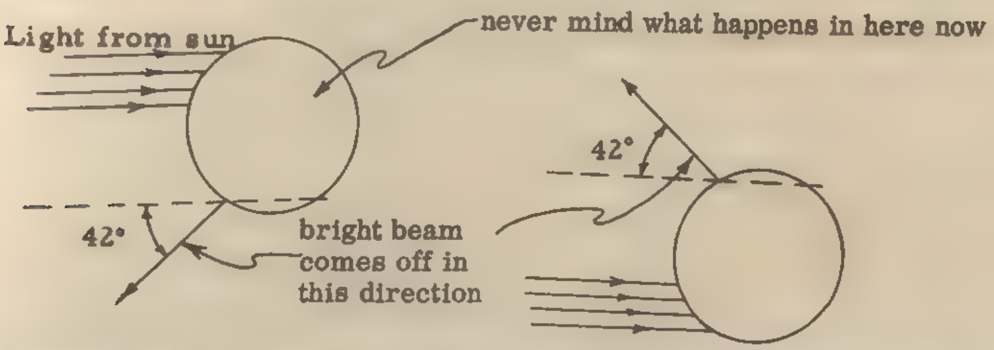
Supplement to Chapter 13: The Rainbow

Why is the rainbow shaped like a bow?

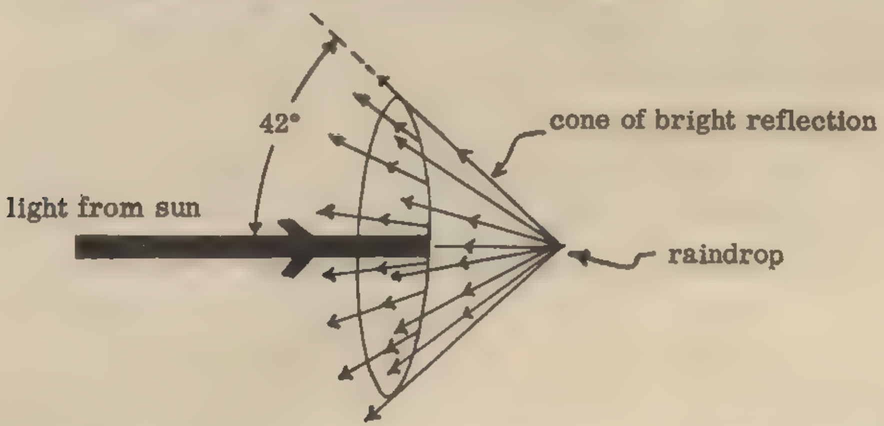
Never mind color now. Think about pure yellow light to see how the bow shape arises.

When a beam of (yellow) light hits a raindrop, some of the light enters the drop, refracts, and reflects. It bounces (never mind which way now) and finally leaves the drop in many directions. For now, all that is important is that in directions which are 42° away from the incoming beam, the light coming back is particularly bright. It is brighter than 41° or 43° or any other nearby direction.

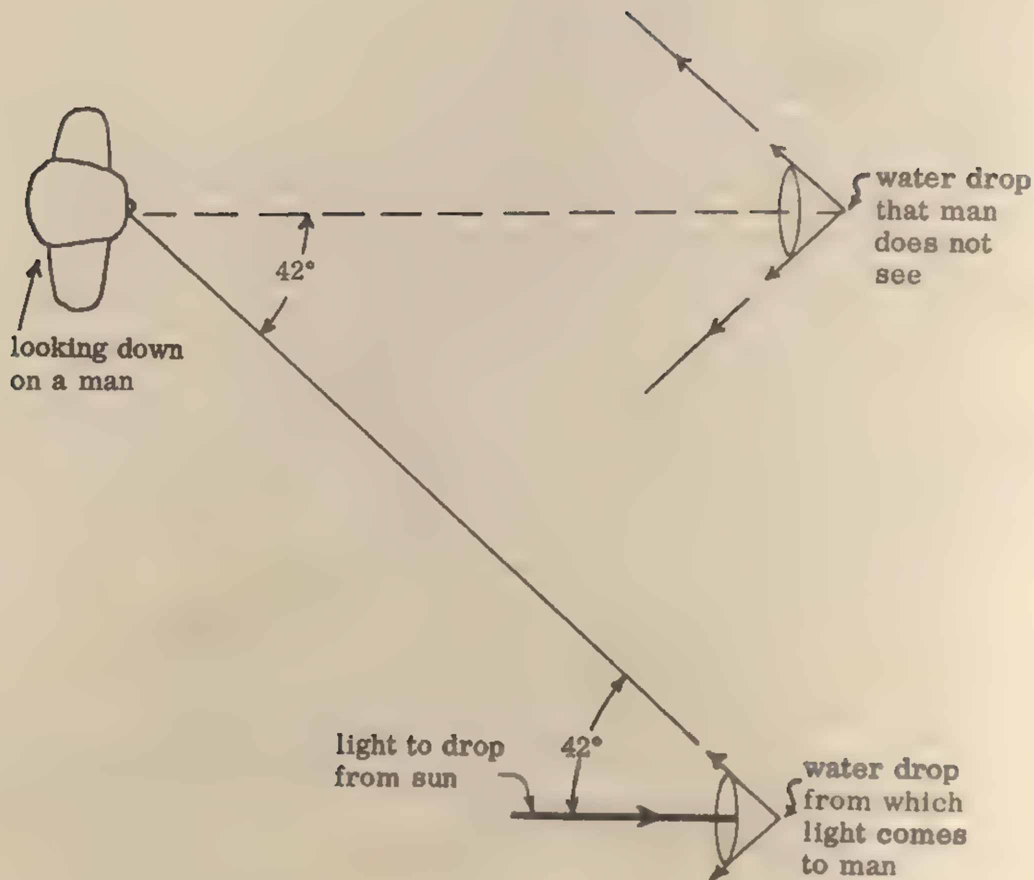
Cross-sectional view of raindrop



The picture above just shows part of what happens. The bright reflection comes back on all lines that are 42° from the incoming beam. The bright reflection from one drop is therefore a cone of light with vertex at the drop.



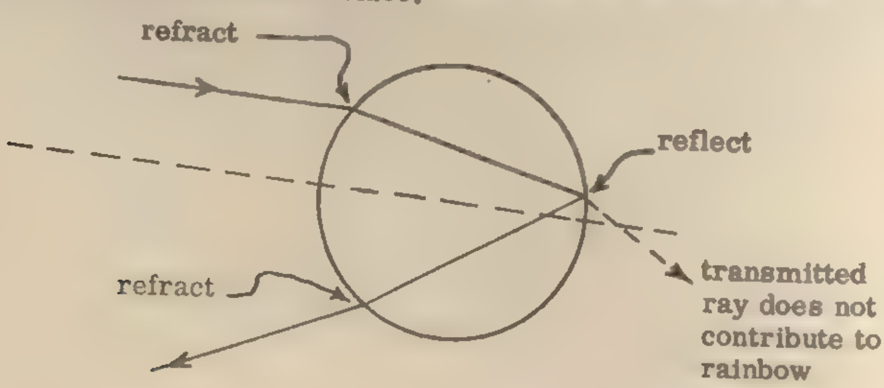
Where must such a cone of light be located in order for a ray moving along the cone surface to head directly toward you, that is, in order for you to see it? A cone whose vertex is directly in front of you would shoot around you on all sides and miss you completely. The vertex of the cone must be 42° off a line straight ahead of you if a ray of the cone is to come to you.



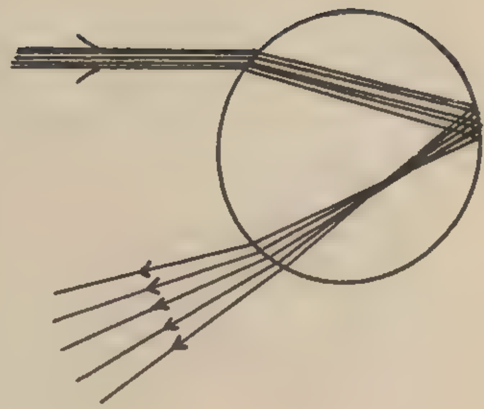
There are many places a raindrop could be so that a ray heads toward you -- any place that is 42° away from the line straight in front of you. All such drops lie on a cone with the vertex at your eye, and appear to you as though they lie on a circle. Since you are on the surface of the earth, no drops send light to you from below ground, and you see at most half of a circle -- a rainbow! If some regions of the sky do not contain the necessary drops, you may see only a small piece or several disconnected pieces of the bow. See Figure 13-19 on page 223 of the text.

<u>Why does a single spherical raindrop produce a bright cone of light</u>?

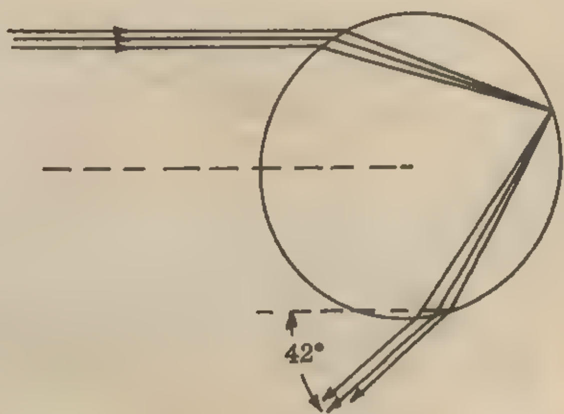
Slice the sphere through its center to get a circle whose plane contains some of the light rays from the sun. One path that a light ray can take is to enter the circle, refracting as it goes from air to water, bounce once at the rear of the circle and refract out when it next hits the surface.



If a narrow parallel beam (much narrower than the width of the drop) of light enters the drop, for most points of entry, it is converted into a diverging beam of light by the time it leaves the drop.



Since the light from such beams spreads out upon leaving the drop, it does not appear much brighter than the rest of the sky to someone looking at the drop. <u>However</u>, there is one <u>ring</u> on the drop (or two points on the drop cross section) where an incoming beam goes out also as a narrow beam.

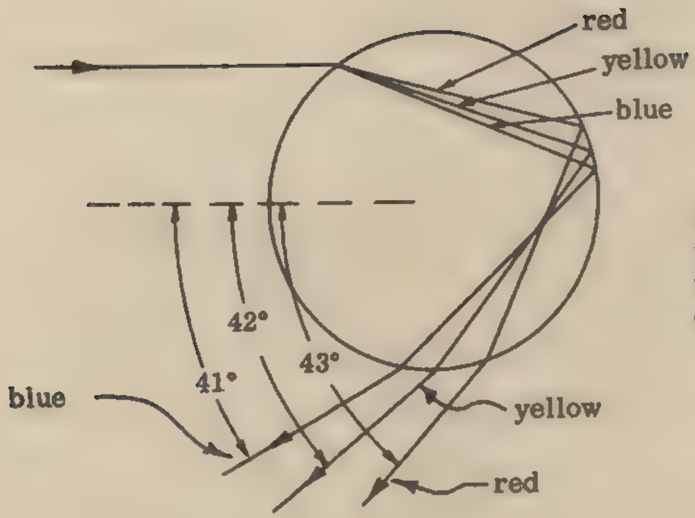


Only a small beam of light into and out of the raindrop accounts for bright 42° cone.

One can check that this is the case by making a sufficient number of accurate scale drawings--all that is involved is refraction and reflection, and you can handle both in a drawing if you work with sufficient care. Since the water drop is a sphere and not a circle, the bright emerging beams leave the drop in a 42° cone as previously mentioned.

<u>Why is the rainbow colored</u>?

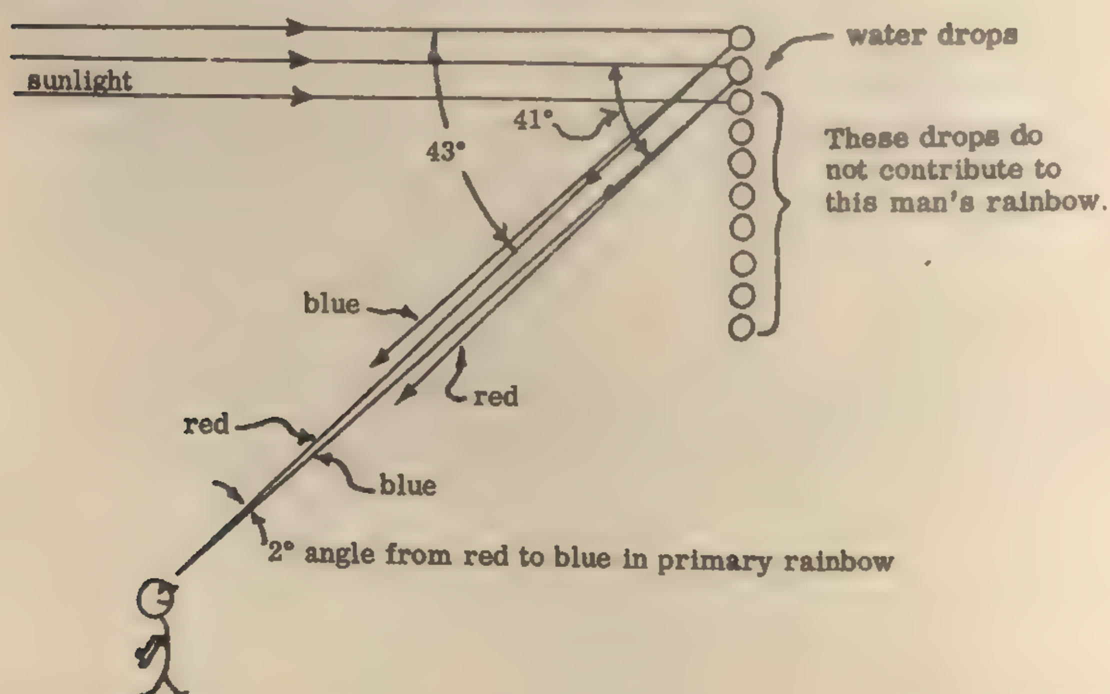
Remember that the entire previous discussion has been for yellow light. The index of refraction of water changes a little bit as the color of light changes. If you repeat the drawings above for different colors, you find that whereas yellow light is concentrated at 42°, red light is concentrated at an angle of 43°, and blue light at an angle of 41° (approximate).



NOTE: Not to scale. The angles between the colors are exaggerated.

Which color appears on top?

Since concentrated red light leaves the drop at an angle of 43° (42.37°, really nearer 42°) and concentrated blue light leaves the drop at an angle of 41° (40.6°), from each drop there is a 43° cone of red light and a 41° cone of blue light with the other colors coming out in cones in between.

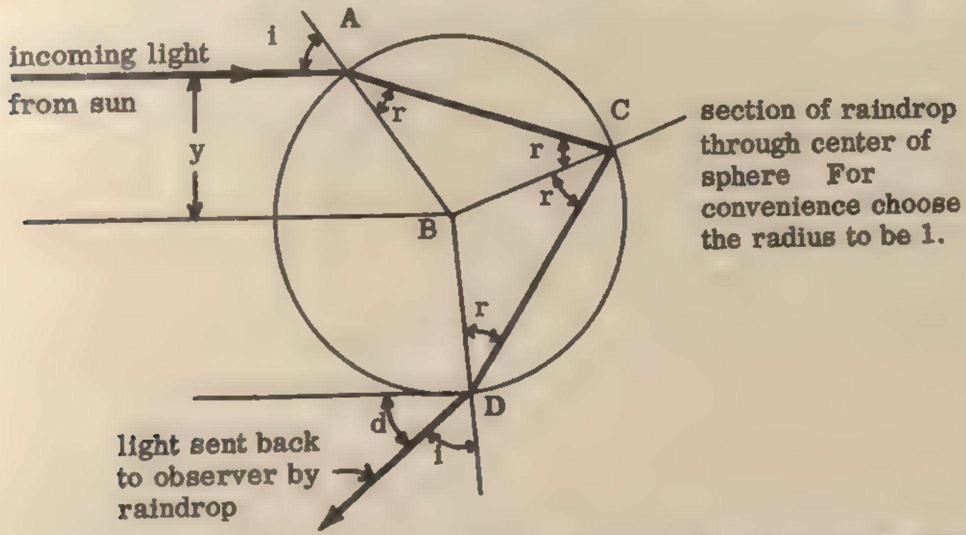


Consequently, we see red light at an angle of 43° from straight ahead (at the outside of the bow) and blue light at an angle of 41° (on the inside). The drops sending any

one of the colors to us lie on part of a cone with vertex at our eye, and so appear to lie on part of a circle. Thus every primary rainbow is about 2° wide.

What is the geometry of rays going through a spherical drop?

If we add three radii and some auxiliary lines to Figure 13-17 on page 222, we obtain



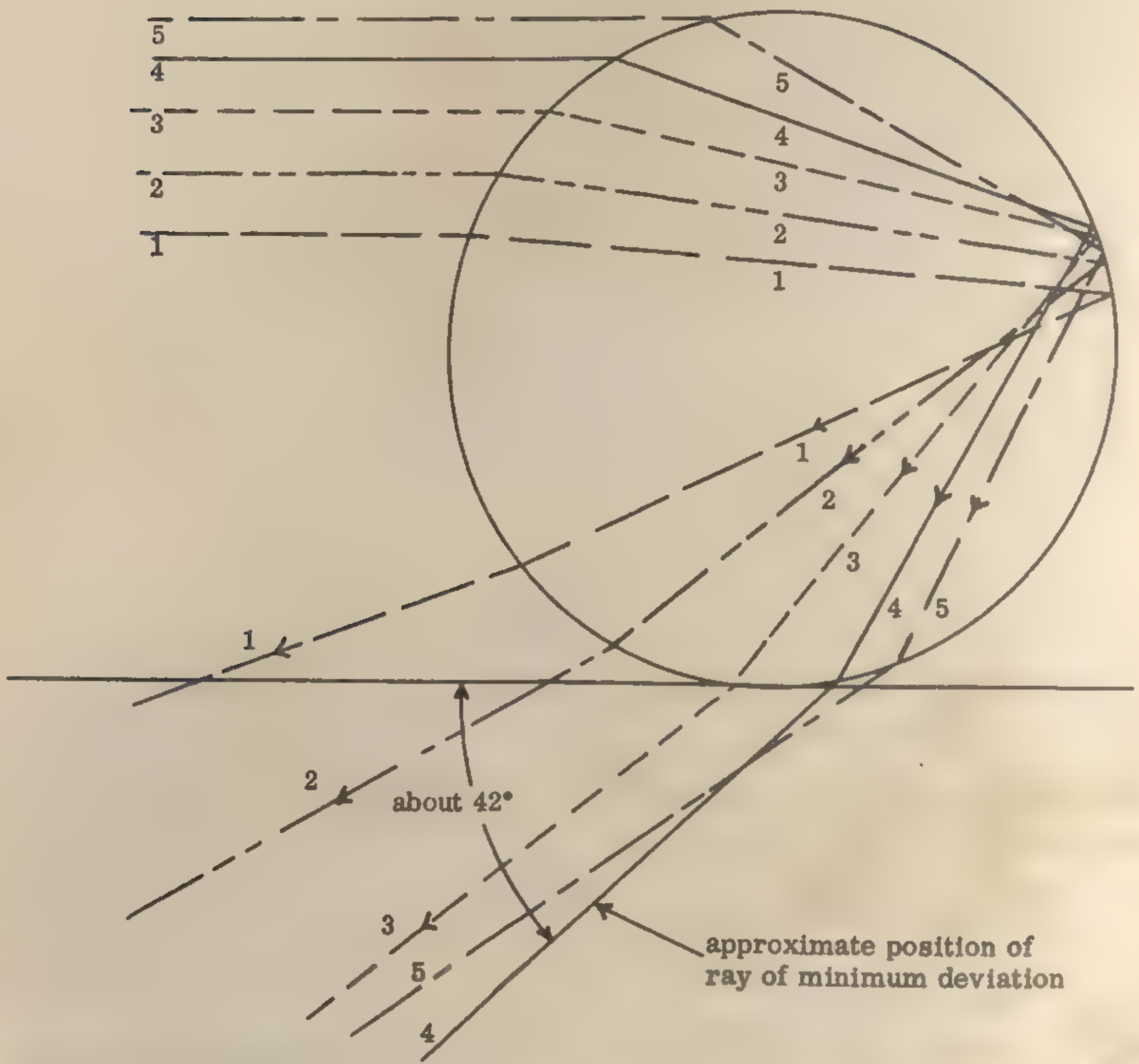
The geometry of such a figure is much simpler than one might expect. Since BA, BC, and BD are radii, they are the same length. Hence, triangles ACB and CDB are isosceles. Because of this, angle BAC is the same size as angle ACB. Furthermore, since a radius is also a normal to the circle, the law of reflection requires that angles ACB and DCB be the same size. Therefore, the four acute angles in the triangles are all the same size as the angle of refraction. The two triangles are congruent. From this congruence, chords AC and CD are the same length, a fact which is useful in simplifying accurate graphical construction. Once you have found point C by using Snell's law at A, you can find point D by swinging an arc of radius AC from point C. You can then construct the emerging angle, i, the same size as the incoming angle and measure angle d. Alternatively, you can compute d using the easily verified equation

$$d = 2r - 2(i - r) = 4r - 2i.$$

There are some simple regularities in the over-all pattern of emerging rays coming from parallel incident rays striking the upper half of the drop. The ray which approaches along the diameter (i.e., the distance y in the figure above is 0), travels through the center and returns along itself. As the distance y is increased, the point D moves counterclockwise around the circle. At first the angle d increases in size. But at a certain point, as the light enters the circle higher and higher, angle d stops increasing and begins to decrease. The maximum for angle d with yellow light is about 42°. This can be verified by scale drawing or by calculus which shows that d is a

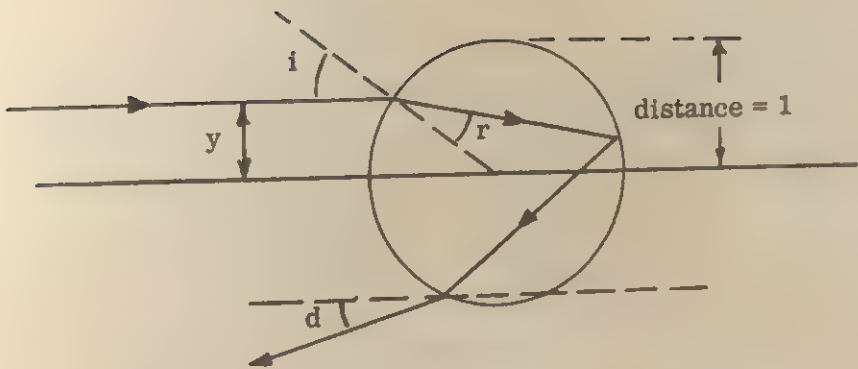
maximum when $\cos i = \sqrt{\dfrac{n^2 - 1}{3}}$.

Here is a diagram of five rays through a drop.



Notice that in this drawing rays 3, 4, and 5 lead to quite different angles, d. These rays do not all contribute to the strong colored light in a rainbow. It is a much narrower band of rays near ray 4 which actually contributes to the rainbow. Remember that the entire rainbow is only about 2° wide so that each color is concentrated in an angular range much smaller than 2°.

(If a student is making a thorough study of rainbows, he should make some scale drawings of the paths of rays in a raindrop.) The ray making the maximum angle with the horizontal is called the ray of <u>minimum deviation</u>. A table of values, more accurate than the drawing above, follows.

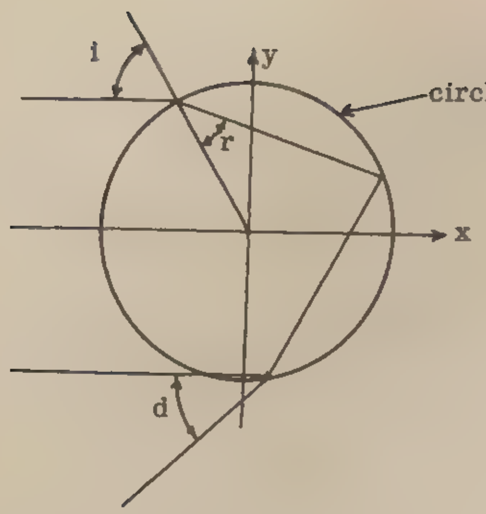


y sin i	i	r	sin r	d
0	0°	0°	0	
0.1	$5^\circ44'$	$4^\circ18'$	0.075	$5^\circ44'$
0.2	$11^\circ32'$	$8^\circ37'$	0.15	$11^\circ24'$
0.3	$17^\circ27'$	$13^\circ00'$	0.225	$17^\circ06'$
0.4	$23^\circ35'$	$17^\circ27'$	0.3	$22^\circ34'$
0.5	$30^\circ00'$	$22^\circ01'$	0.375	$28^\circ04'$
0.6	$36^\circ52'$	$26^\circ45'$	0.45	$33^\circ16'$
0.7	$44^\circ26'$	$31^\circ40'$	0.525	$37^\circ44'$
0.8	$53^\circ08'$	$36^\circ52'$	0.60	$41^\circ12'$
0.8606	$59^\circ23'$	$40^\circ12'$	0.6455	$42^\circ02'$
0.9	$64^\circ09'$	$42^\circ27'$	0.675	$41^\circ30'$
1.0	90°	$48^\circ35'$	0.75	$14^\circ20'$

<u>What is the geometry of different colors in a raindrop?</u>

It is along the ray of minimum deviation that the bright returning light is seen. The sketches in the preceding discussion indicate the reason light is concentrated in the region close to the ray of minimum deviation. At angles less than the angle of the ray of minimum deviation, the intensity of the returning light falls rapidly. (We are not treating here the details of how fast the intensity decreases.)

For $n = 4/3$, the minimum deviation occurs at $y = 0.8606$ or $i = 59^\circ23'$. This index applies to yellow light. A ray diagram for violet light or red light would look similar to one for yellow light. Some key numbers that would simplify the drawing of the minimum deviation rays for red and violet are given in the following table.



Note: If a radius R other than 1 were used, you would replace "y" by y/R and "x" by x/R in this discussion.

	Violet Light	Red Light
Wave length	= 3968 A	= 6563 A
Refractive index	n = 1.3435	n = 1.3311
Values on circle representing drop		
Minimum deviation ray enters	y = 0.855 x = −0.518	y = 0.862 x = −0.507
Ray hits back surface	y = 0.347 x = +0.938	y = 0.361 x = +0.932
Ray emerges	y = −0.987 x = 0.163	y = −0.979 x = 0.206
Angle of incidence i	58°48′	59°31′
Angle of refraction r	39°33′	40°21′
i − r	19°15′	19°10′
d	40°36′	42°22′

Because each color comes back in a narrow band of angles, there is some mixing of the colors. This is another topic we are not treating in detail.

All of the geometry of light in a raindrop above has been done in a cross section through the center of the sphere. Any ray of light that hits the sphere will stay in the plane determined by that ray and the normal to the sphere at the point of entry. All such planes intersect the center of the circle and the discussions above apply. Thus you do get a cone of light for each color with its half vertex angle the size of the angle of minimum deviation for each color.

Intensity of the Angle of Minimum Deviation for Each Color.

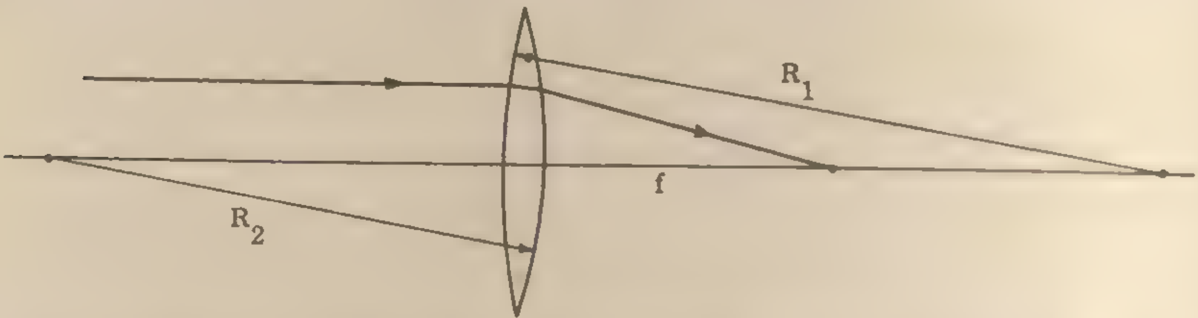
A calculation of the intensity of the rainbow is extremely complicated. (See Physics of the Air, W. J. Humphreys, McGraw-Hill, New York, 1940, page 488ff, edition III.) For drops that are larger than the size of fog particles, the intensity pattern depends not only on the angle, but on the index of refraction, the size of the drop, and the wave length of light. (The intensity is significant only very close to the angle of minimum deviation so that the variation in reflectivity is negligible.) If the drops are the size of fog particles, even more complex calculations are needed which are based on electromagnetic theory.

APPENDIX 3

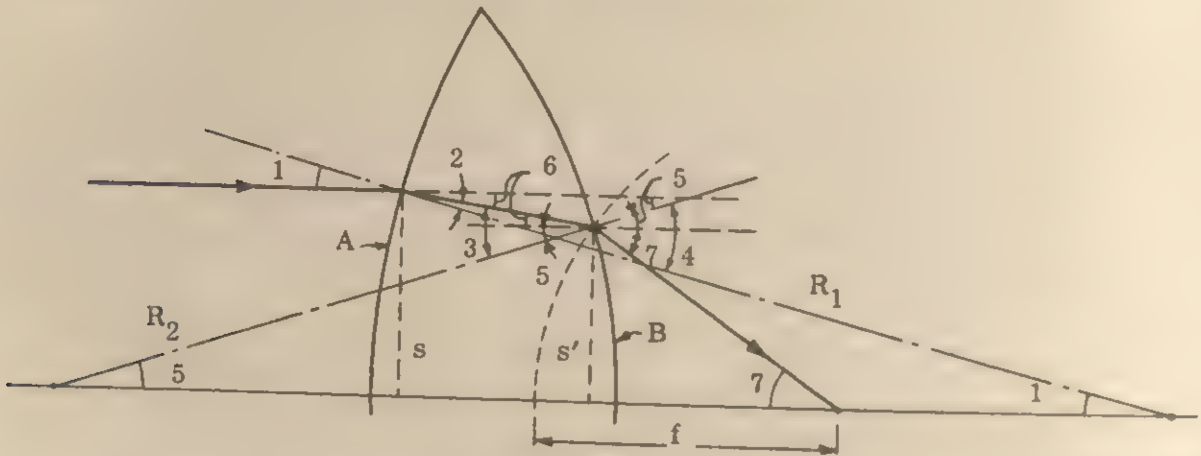
Supplement to Chapter 14, Section 2. Derivation of the Lensmaker's Formula

The following derivation involves no more than Snell's law, geometry, and radian measure.

Since all rays parallel to the principle axis of a lens cross the axis of the lens at a common point (principal focus) it is only necessary to consider one of these rays in deriving this formula.



The above diagram is too small in the lens region through which the ray passes so it is necessary to enlarge and exaggerate the angles in this region.



The refractions of the ray at both surfaces A and B are described respectively by Snell's law as

$$\sin 1 = n \sin 2$$
$$\sin 4 = n \sin 3$$

With a thin lens all of these angles are very small and thus the angles in radians may be substituted for the sines of the angles.

$$\text{(a)} \quad \angle 1 = n \times \angle 2$$
$$\text{(b)} \quad \angle 4 = n \times \angle 3$$

The next step is to reduce these equations to one equation with angles 1, 5, and 7 only. Additional lines have been drawn parallel to the axis of the lens and the equal angles thus formed have been labeled with the same number for simplicity. Adding (a) and (b) gives $\angle 4 + \angle 1 = n(\angle 2 + \angle 3)$, but since $\angle 4 = \angle 5 + \angle 7$ and $\angle 3 = \angle 5 + \angle 6 = \angle 5 + \angle 1 - \angle 2$, this equation (by substitution) is $\angle 5 + \angle 7 + \angle 1 = n(\cancel{\angle 2} + \angle 5 + \angle 1 - \cancel{\angle 2})$.

By rearranging terms and factoring $\angle 7 = (n-1)(\angle 5 + \angle 1)$. From the first drawing we can see that the lines marked s and s′ in the second drawing and the length of the corresponding arcs are all so nearly equal that they can be considered equal. Also since it doesn't really matter to what point in the lens f is measured for a thin lens, a convenient arc was drawn about the focus and f is measured to this arc. Thus, upon substituting the ratios of arcs to radii for the angles in radians the following equation is obtained:

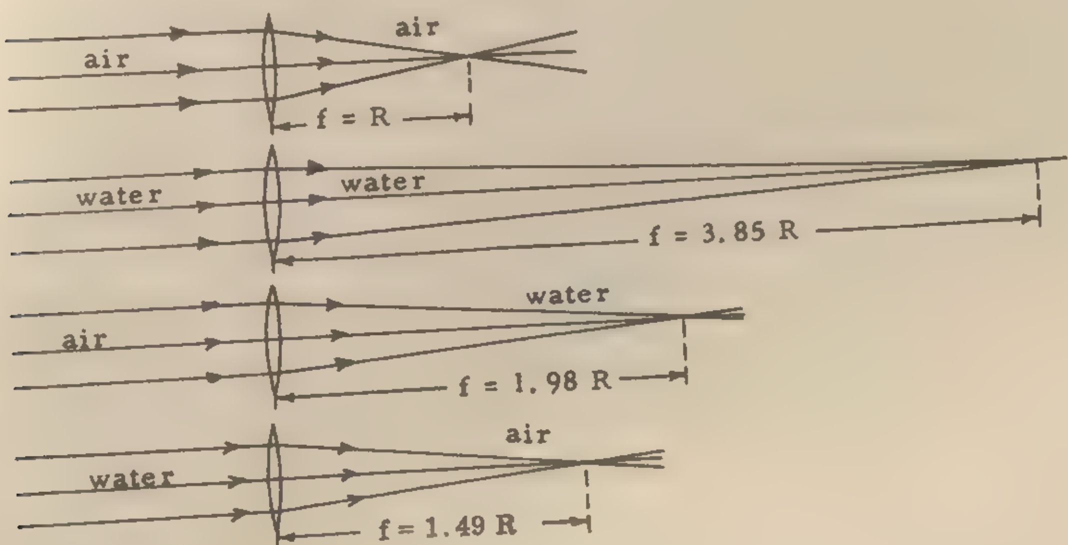
$$\frac{s'}{f} = (n-1)\left(\frac{s'}{R_2} + \frac{s}{R_1}\right)$$

which gives the desired formula upon dividing each term by either of the equalities s or s′.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_2} + \frac{1}{R_1}\right)$$

It should be noted that the index of refraction, n, used in the above derivation is the relative index of the lens material and the medium in which it is immersed. Thus, if there are two different media on either sides of the lens the first factoring in the derivation could not be done. Diagrams which better portray these ideas are shown below.

$(n_{air} = 1.00,\ n_{water} = 1.33,\ \text{and}\ n_{glass} = 1.5)$



APPENDIX 4

Supplement to Chapter 14: Information on Camera Lenses

Like the other appendices, this information is not intended for class presentation, simply as a background.

The two principal factors which govern the use and quality of camera lenses are the "speed" of the lens and the image defects it introduces.

Speed

The "speed" of a photographic lens determines the exposure time required. The illumination of the image is directly proportional to the square of the lens diameter and inversely proportional to the square of the focal length:

1. The light-gathering power of a lens is dependent upon its area which is proportional to the square of its diameter.
2. If we have two lenses of the same diameter, but one has twice the focal length of the other, the light gathered from a given object point will be the same for both lenses. The image of the object point formed by the longer focal length lens will have twice the linear dimensions (and $2^2 \times$ the area) of the image for the shorter focal length lens. Thus each unit area of the image formed by the longer focal length lens will receive 1/4 the light received by a unit area of the image formed by the lens of shorter focal length.

The f-number system is used to indicate the speed of a lens. The f-number is the ratio f/d where f is the focal length and d is the effective diameter of the aperture. Thus, though one lens may have twice the focal length and twice the aperture of another lens, the two have the same f-numbers and would require the same exposure time.

The f-number is controlled for various purposes (including correct exposure) in most cameras by varying the effective diameter of the lens with an iris diaphragm. From their knowledge of geometric optics, students should be able to answer the question, "Why, if a camera is focused on 10 feet, does using a larger f-number (smaller lens diameter) result in sharper focus for objects beyond and less than 10 feet?"

The usual series of f-numbers marked on a camera lens (indicating various settings of the iris diaphragm) is 32, 22, 16, 11, 8, 5.6, 4, 2.8, 2. Each of these "stops" admits twice as much light as the next higher stop number. (The squares of the successive numbers in this series are nearly in the ratio of 2/1.) Hence, f-number 11 with a shutter speed of 1/25 sec gives the same exposure as f-number 8 with 1/50 sec.

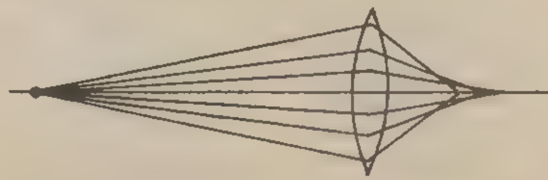
Lens Defects

In the manufacture of lenses it is not practical to make non-spherical surfaces except for specialized applications where cost of manufacture is not a limiting factor. Hence, photographic lenses are composed of elements ground and polished to spherical surfaces. Even though the surfaces are perfectly spherical, there are many image aberrations a lens may produce. Some of your students may be interested in reading accounts of the development of photographic lenses. They can be found in encyclopedias and in the more complete photographic handbooks and textbooks.

A perfect lens would reproduce points, straight lines and planes in the object by points, straight lines and planes in the image. This is difficult to achieve except with monochromatic light and with rays that are at small angles to the axis and to the normal of the lens surface. The more significant lens aberrations are discussed briefly below.

Chromatic aberration, as discussed in the text, occurs because violet light is bent more than red light; the focal length for violet light is shorter than that for red light. Chromatic aberration can be reduced by using two kinds of glass in a two-element (compound) lens. With two selected kinds of glass, a two-component lens can be made to bring any pair of colors to a focus, but colors between the pair have slightly shorter focal lengths, and colors toward the ends of the spectrum have slightly longer focal lengths. Three kinds of glass can be used to bring three colors to the same focal point.

Among the lens defects that represent failures to produce a point image of a point object are spherical aberration, coma and astigmatism.

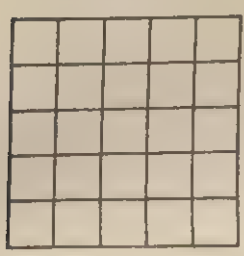


Spherical aberration is illustrated in the diagram at the right. From an object point on the axis, the rays traveling near the center of the lens focus at a different point than rays that travel near the edge of the lens. In correcting lenses for spherical aberration through choice of lens shapes and combinations of glasses, the lens designer tries to bring both marginal and central rays to the same focal point. Even when this is done,with spherical surfaces there is usually some remaining aberration for the rays between central and marginal. This is called zonal aberration. It is often serious in lenses having a greater aperture than about f-number 3. (This is why many "fast" lenses give greatly improved definition when stopped down to about f/3. In large refracting telescopes, one or more of the lens surfaces is retouched by hand to minimize zonal aberration.

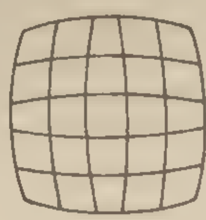
Coma is a kind of spherical aberration for object points that are not on the lens axis. It increases as the object point moves farther from the axis. Coma is basically a variation in magnification or distance of the image point from the lens axis, by the various (central to marginal) zones of the lens. The name of this aberration comes from the fact that the image point is not a circle, but a bright spot which is surrounded by a fainter spot that usually tails off from the axis. Hence the image is comatic (comet-like).

Astigmatism occurs with a perfectly spherical lens surface for object object points that are off the axis. For such object points, a spherical lens surface presents a cylindrical aspect, forming an image of a single object point as two mutually perpendicular lines at different distances from the lens. Radial lines (spokes of a wheel) and tangential lines (wheel rim) in an object are focused at different distances from the lens. With this defect, the best "image point" is midway between the two line images. Astigmatism, like coma, increases as the object point is moved farther from the axis.

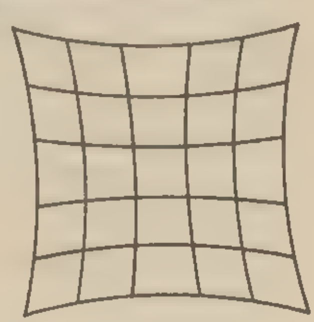
Distortion is a failure to produce a straight line image of a straight line object. It does not occur for lines passing through the lens axis. In distortion, magnification is not uniform over the entire image area. Outer parts may be magnified more or less than central parts. These two cases result in "barrel" and "pincushion" effects.



Square



"Barrel" distortion



"Pincushion" distortion

Since the corners of a square are farther from the axis than the centers of the sides, the corners may be magnified more or less than the sides, thus warping a square into a "barrel" or a "pincushion". With a simple lens, which type of distortion results depends on whether the diaphragm is in front of or behind the lens. In compound lenses, the placement of the diaphragm is the principal means of controlling distortion.

Curvature of field is the case of an object plane which makes a right angle with the axis being imaged as a concave (usually) surface. Curvature of field results when points off the axis are focused closer to the lens than points near or on the axis. Since the film must lie flat, (in most cameras), extensive curvature of field is troublesome. Because of astigmatism, there are actually two curved fields (one for tangential lines and one for radial lines) which come together only at the axis.

Aberrations are inherent in a simple lens. They are not due simply to poor workmanship or faulty design. A simple lens of given focal length and diameter can be made from different kinds of glass, with different combinations of radii of curvature, and in different thicknesses. Various choices may minimize one aberration or another, but it is impossible to free a simple lens from all aberration. Since a given aberration in a negative lens is usually opposite to that aberration in a positive lens, careful combinations of negative and positive lenses can nearly balance out the worst of the troubles. The design of a lens is a complicated compromise between the corrections for the various aberrations. Hence no one aberration is reduced as much as it might be if it were the only one considered.

APPENDIX 5

Supplement to Chapter 14: Information on the Eye

The structure and functioning of the human eye are very complex; the optics of the eye are complicated as well. In connection with class discussion of Chapter 14, an extremely simplified explanation of the gross operation of the eye as a focusing device is sufficient. The following is supplied as background information, and for discussions outside of class.

Structure

The eye is approximately spherical. Its outermost layer is a protective coating (sclera) which becomes the transparent cornea in the front of the eye. Immediately behind the cornea is a water-like substance (the aqueous humor). Behind this is the eye lens which is made of fibrous tissue. Behind the lens and filling the central chamber of the eye, is the vitreous humor (a jelly-like substance). The light-sensitive layer, the retina, covers considerably more than the rear hemisphere of the wall of the eyeball. There are two sorts of sensory cells in the retina, the rods and the cones. Only the cones are involved in color vision. The mechanism of color vision is not well understood.

Geometric Optics

The refractive indices and average radii of curvature of the parts of the eye are:

	n	Radius of Curvature	Approximate Thickness
Cornea	1.376	Front 7.7 mm Back 6.7 mm	0.5 mm
Aqueous humor	1.336		
Lens	about 1.41	Front 10.0 mm Back 6.0 mm	3.6 mm
Vitreous humor	1.336		

For incident parallel light, most of the refraction occurs when the light enters the eye (i.e., at the cornea), not at the lens. (The light rays are refracted at the cornea and are not bent much more before they reach the retina.) The equivalent focal length of this refracting surface is about 31 mm in the eye. Since the retina is an average distance of 24.4 mm behind the cornea, the image formed of a distant object would be only about 6.6 mm behind the retina if there were no eye lens.

The effect of the eye, including the lens, is to produce an image 24.4 mm behind the cornea. The total combination behaves like a thick lens rather than an ideal thin lens. However, the focusing function can be approximated by replacing the cornea and the lens by a single refracting material with an index of refraction of 1.336, a radius curvature of 5.7 mm, and a position in the eye of 1.5 mm behind the cornea, or 22.9 mm in front of the retina. This surface would bring parallel light to focus at a distance of 22.9 mm in the eye.

Accomodation To Distance

The eye focuses divergent light by changing its focal length. The lens is enclosed in a transparent membrane. The membrane is attached to the structure of the eye. When the attachment is pulled forward by the contraction of the ciliary muscles, the lens becomes more convex through its own elasticity. The maximum possible change in focal length depends on age. A ten year old can focus on an object 7 cm away, implying a focal length of about 17 mm for the equivalent refracting surface (compared to 22.9 for the normal relaxed eye). In order to do this the radius of curvature of the lens changes by several millimeters. As one grows older, the lens loses its ability to accommodate.

Approximate age	Closest distance (cm)	Equivalent focal length of maximum accommodation (mm) (unaccommodated = 22.9 mm)
10	7	17.2
15	8.5	17.9
20	10	18.5
30	14	19.6
40	23	20.7
60	100	22.2

Notice that the 25 cm used commonly as the "closest distance of distinct vision" is not really an average. It is used, by convention, as some sort of ideal working distance. If one wished to see something as well as possible, he would move it to the closest distance. But using the eye at the closest distance causes eye strain. Thus the conventional distance of 25 cm is commonly used in computing magnification. Experienced workers using optical instruments often focus their instruments so that the image is quite distant; this reduces the magnification only slightly but causes much less eye strain.

Variation of Sensitivity with Color

Under normal lighting conditions, the eye is most sensitive to green-yellow light, and the sensitivity drops most sharply toward the shorter wave lengths (blue). The sensitivity is negligible below 4000 A or above 7100 A.

The eye is most sensitive (after it has been in the dark for more than 30 minutes) to photons of light whose wave length is 5000 A. It requires about 50 such photons entering the eye within a 0.1 sec interval to produce the sensation of light. About half of these are absorbed on the way to the retina (the eye is not very transparent); of the remaining 25, about 5 produce a photochemical reaction in a dye (visual purple – molecular weight 270,000). It is necessary for about 5 different rods to be stimulated in order to get the sensation of light.

Accommodation to Intensity

The pupil of the eye changes its diameter in response to changes in light intensity. The opening through which light can enter the eye can be changed from about 2 mm to 8 mm; this corresponds to a change by a factor of 16 in the opening area and in the admitted light. However, the eye can accommodate intensity changes of 10^4. This accommodation occurs through a change in the sensitivity of parts of the retina. For example, in very faint light, the rods in the retina are very sensitive; in bright light, however, the cones play a major role. When one goes into a completely dark room from a brightly lighted one, the eye's sensitivity increases by a factor of about 5 in five minutes. After an hour, the eye's sensitivity has increased by a factor of about 10^4.

The eye can detect differences of about 1% in the intensity of adjacent objects.

Aberrations of the Eye

Focusing. Often a relaxed eye does not focus light properly on the retina. The most usual cause is variation of the eye size, (i.e., the distance from the cornea to the retina) although it can be due to defects in the cornea or lens. If the eyeball is too long, the individual is nearsighted (or myopic); he can see close objects well but cannot see distant objects clearly. A diverging lens is used to correct this. On the other hand, if the eyeball is too short, the relaxed eye produces an image of a distant object beyond the back of the eye. Such an individual is farsighted (or hyperopic). Although his eye can accommodate to distant objects, he is unable to focus on close objects without the help of converging lenses. A converging lens corrects this defect.

Astigmatism. Astigmatism in the eye arises from the imperfect sphericity of the focusing system, usually the cornea. (Astigmatism in the eye produces the same imperfection in an image as does astigmatism accompanying the focusing of an off-axis object

point by a perfect spherical lens.) The astigmatic cornea is somewhat cylindrical rather than spherical. At least half the cases of poor eyesight are due to astigmatism rather than nearsightedness or farsightedness. Corrective lenses for astigmatism are cylindrical.

Spherical Aberration and Coma. Some spherical aberration and off-axis aberration exist in the eye, but both the cornea and the lens are somewhat flattened in the regions away from the axis which partially corrects for this.

Curvature of Field. Since the retina is curved, curvature of field is usually of little consequence.

Chromatic Aberration. The total difference in focal length between red and violet light is about 0.5 mm (in a mean focal length of about 23 mm). However, the eye focuses mainly on yellow light when viewing a white object, and the slight lack of focus of red and blue causes no noticeable effect.

Diffraction Effects. When the pupil opening is between about 2 mm and 3 mm in diameter, diffraction sometimes limits the resolution attainable by a perfect eye. Above 3 mm, diffraction decreases but other lens aberrations increase so that the overall resolution is not improved.

APPENDIX 6

Fermat's Principle

Fermat's principle may provide the basis for interesting discussions outside of class with students.

The following discussion is related to the time it takes light to travel from any point A to another point B.

1. Fermat's principle, as he stated it, applies to light traveling in a homogeneous medium or to light reflected or refracted at a single plane surface. The principle states that the path light actually takes between A and B is that which requires less transit time than any neighboring path.

2. Also related to transit time and covered by an extension of Fermat's principle, is the fact that the transit time is the same for all light rays which go through an optical system from one point on an object to the corresponding point on the image.

Fermat's principle (in its restricted sense of the light path being a minimum for plane surfaces) can be used directly to derive the following:

1. In a homogeneous medium, light travels in a straight line.
2. The laws of reflection. (The proof is given below.)
3. The laws of refraction. (The proof in this case is straightforward, but involves either calculus or quite complex algebra and is therefore not reproduced here.)

If you merely mention this principle and all that follows from it, the imagination of many students will be stimulated. Some will wonder from where this powerful principle comes, and whether it has a deeper, even philosophical significance. Other students may wonder how Fermat discovered it. You can start an interesting out-of-class discussion on either of these questions.

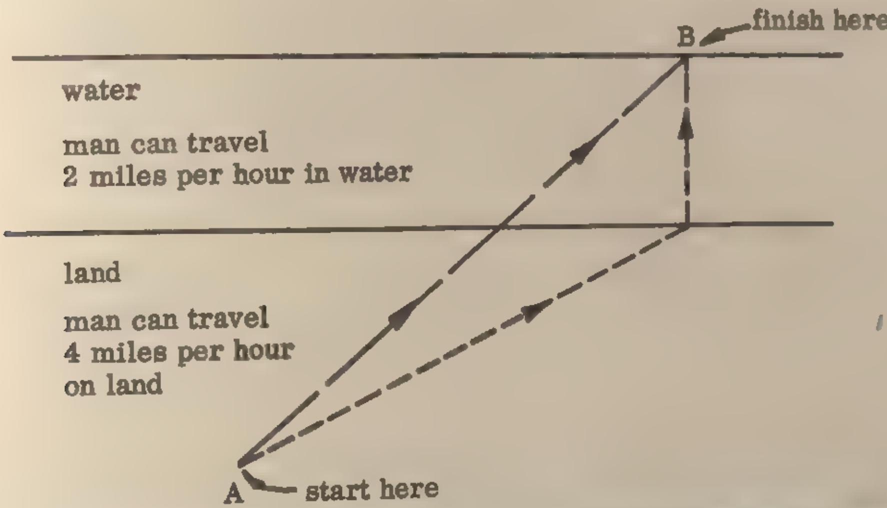
First we might consider whether Fermat's principle seems more correct or more fundamental than the rectilinear propagation of light and the laws of reflection and refraction. Many students will appreciate the simple elegance of one rule which leads to three others that had been only empirical (even though they explained many phenomena). You can continue this discussion by being sure that the students realize that Fermat's principle is not like a model; it is more analogous to the laws of reflection or refraction.

Even though you cannot give a recipe indicating how Fermat discovered this principle, students will enjoy thinking about the kind of question that must have motivated Fermat. The question of what is distinctive about one particular path is typical of a fruitful class of questions in science. One way to get insight into why one thing happens is to think about why other things do not happen. Notice that in Fermat's case he probably did not get ahead by asking why light starts off in a particular direction or why the angle of reflection is equal to the angle of incidence. Instead he asked the rather indirect question: If light is going to go from A to B (perhaps touching a mirror first) why does it choose one particular path? Fermat's principle does not help you decide that light from A reaches any arbitrary point B; it merely states that if light does go, it chooses a certain path among the possible ones.

It is not easy to verify Fermat's principle graphically or analytically unless one is extremely careful and precise. Instead of discussing numerical examples, we can try two other applications:

1. Consider qualitatively the path of a refracted ray going from A in air to B in a medium with large index (relatively low speed). Be sure students see qualitatively that light, to go most quickly, would take a path in which it was outside the medium for a longer time than it would be if it went over the straight path. Note that this discussion makes it possible for students to reason about whether light bends toward or away from the normal.

2. Consider a kinematic problem like "Which path should a boy take if he wants to reach point B from point A, if A is in a region in which he can move twice as fast as he can in the region of B?". (The region A might be ground while the region B might be water.) The simplest way to do this is to think about light, use the "index of refraction" equal to the ratio of the speeds, and try finding the path using Snell's law.



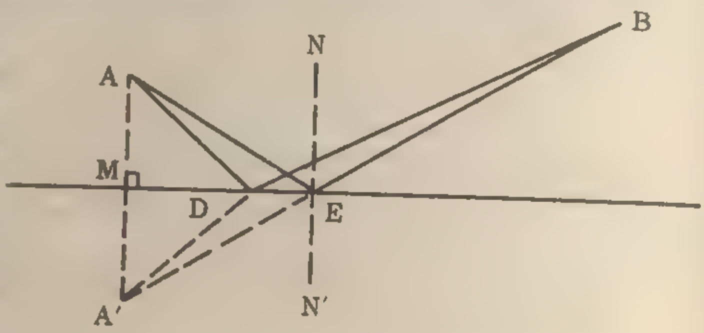
The fastest path is somewhere between the two indicated. Use Snell's law with refractive index 4/2 = 2 to find the point on the river bank where $\frac{\text{sin (angle to normal on land)}}{\text{sin (angle to normal on water)}} = 2.$

There is another point which might be worth making, about the time it takes light to travel in an optical system. If an optical system focuses light from one point on an object to one point on an image (i.e., if the optical system is free from aberrations or distortions), the light which goes from the object point to the corresponding image point takes exactly the same time no matter which of the many possible ray paths it takes, even though the "ruler distance" varies considerably. Notice that you can use this interesting fact to decide which way light will bend when it goes through a lens. Since light goes more slowly through the glass, the ray which goes through the thicker part of the lens spends a long time in glass; during this extra time it spends in glass, the light which goes through the thinner part of the glass moves a greater distance in air. In order to make the two times equal, the ray which spends more time in air must bend toward the ray which goes through the thicker glass.

Proof that Fermat's Principle Leads to the Laws of Reflection.

The first law of reflection, that the incident ray, reflected ray, and the normal are in the same plane, is an easy result of Fermat's principle in Euclidean space.

The second law of reflection, that the angle of reflection is equal to the angle of incidence, must be proved by geometry. Since light, when reflected, stays in one particular medium, the index of refraction and therefore the speed of light may be assumed to be constant throughout the path from A to B. This means that the path over which light takes a minimum time to travel is just the path which is a minimum distance from A to B. The shortest distance from A to B is just the straight line from A to B, but by minimum we mean the minimum path between A and B which touches (reflection) the mirror. This is what we must prove for the equal angle path.



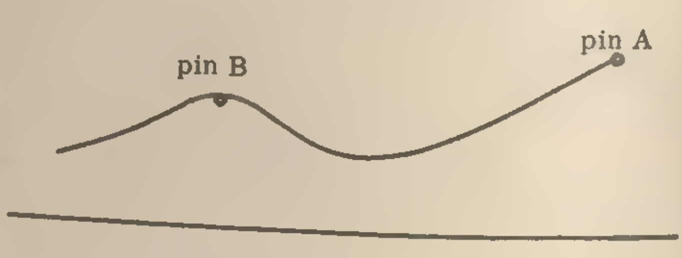
Since the shortest distance between two points is along a straight line between them, we expect the path of light from A to the reflecting surface to be a straight line and the path from the reflecting surface to B to be a straight line. We find the point where the reflected ray hits the surface by making use of the point A′, which is the point such that the line representing the surface is the perpendicular bisector of AA′. Then for any point D on the line representing the surface, AD = A′D by simple geometry. But the distance the light travels is just AD + DB = A′D + DB. The location of D such that the distance A′DB and therefore the distance ADB is a minimum, is the point E on the line between A′ and B. From simple geometry we have ∠AEM = ∠MEA′, and therefore, if NN′ is the normal to the surface at E, ∠AEN = ∠A′EN′. But ∠A′EN′ and ∠NEB are vertical angles and therefore are equal; so, ∠AEN = ∠NEB. This is just the second law of reflection, that the incident ray and the reflected ray form angles of the same size with the normal to the surface at the point of reflection.

An attempt to prove that a path near the "equal angle path" is longer, by using the direct method of taking the difference in length between it and the "equal angle path" as a function, say, of the distance between the point of reflection of the near path and the point of reflection of the "equal angle path", will end in disaster. It involves squaring an inequality three times.

Using calculus in a straightforward way to find the minimum path leads to the equal angle path and also the direct straight line path between A and B.

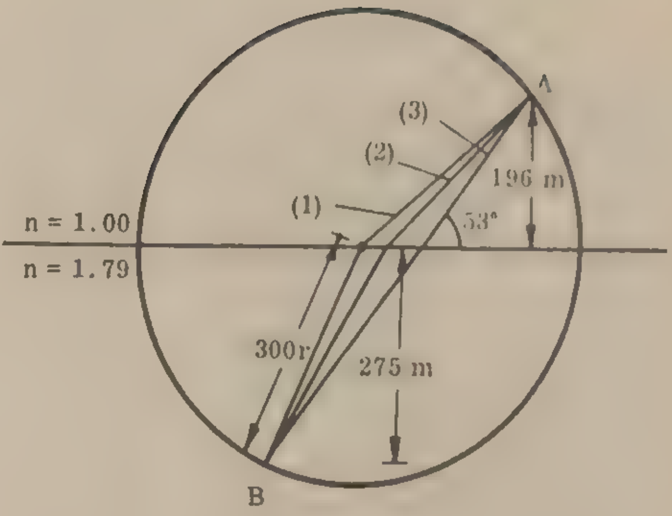
An easy physical demonstration that ∠i = ∠r, when the reflected path is as short as possible, can be accomplished with two pins, and a piece of thread.

Tie the thread to one pin. Loop the free end of the thread around the other pin. Using a pencil, push the thread toward a line while letting the free end of the thread slip past pin B. Pay out as little thread as possible as will allow the pencil to touch somewhere along the line. Mark the point where the pencil touches, construct the normal, and measure the angles of incidence and reflection.



A diagram which illustrates the use of Fermat's principle in the study of refraction and which shows the high precision needed to recognize that the time is a minimum:

With the particular positions of A and B shown at right, path 1 corresponds to Snell's law. It makes an angle of 49.2° with the normal. Path 2 is near the actual path (1) of light as determined by Snell's law, but it makes an angle of 45° with the normal to the surface in air and an angle of 30° with the normal in the dense medium. The time it takes to go from A to B along the true path (1) is:



$$\frac{300\text{ m}}{c = 3 \times 10^{8}\text{ m/sec}} + \frac{300\text{ m}}{c/n = 1.68 \times 10^{8}\text{ m/sec}} = 10^{-6} + 1.79 \times 10^{-6}\text{ sec} = 2.79 \times 10^{-6}\text{ sec.}$$

The time it would take to go from A to B along path (2) would be almost the same:

$$\frac{275/\sin 60^{\circ}\text{ m}}{c/n = 1.68 \times 10^{8}\text{ m/sec}} + \frac{196/\sin 45^{\circ}\text{ m}}{c = 3 \times 10^{8}\text{ m/sec}} = 1.89 \times 10^{-6} + 0.92 \times 10^{-6} = 2.81 \times 10^{-6}\text{ sec.}$$

Even for path (3), the straight line between A and B, the transit time would not be very different:

$$t = \frac{275/\sin 53^{\circ}\text{ m}}{c/n = 1.68 \times 10^{8}\text{ m/sec}} + \frac{196/\sin 53^{\circ}}{3 \times 10^{8}\text{ m/sec}} = 2.05 \times 10^{-6} + 0.82 \times 10^{-6}\text{ sec} - 2.87 \times 10^{-6}\text{ sec.}$$

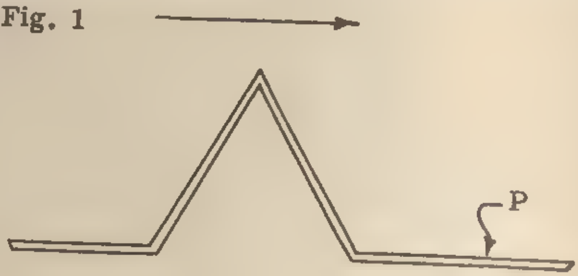
APPENDIX 7

Supplement to Chapter 16: Information on Wave Dynamics

In class discussion of Chapter 16 the question of why a wave moves as it does should be avoided insofar as possible. At this stage, students do not have the necessary preparation in mechanics to talk about even the simplest cases. However, the following discussion may be helpful as background information for the teacher.

As the wave shown on the rope in Figure 1 progresses to the right it passes the point P.

Fig. 1

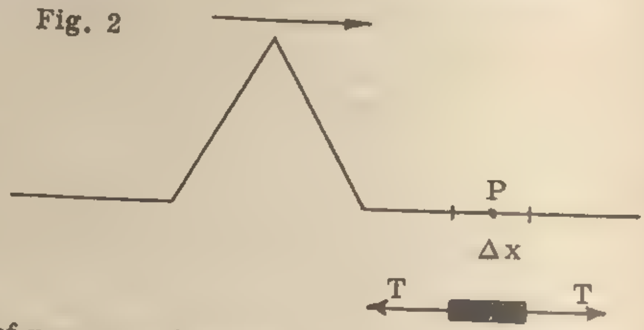


The piece of rope at P will rise and then fall. When it falls back to its original position it suddenly stops. Why doesn't it overshoot and go below its original position?

There are two possible answers to this question. One is that no point on a real rope or tube which is transmitting a wave stops suddenly. All real waves have rounded corners. The particles of the tube are only gradually accelerated and brought to rest. This is only a partially satisfactory answer since the more flexible the spring or tube used, the more nearly the "corners" of a wave can be sharp. However, discussion in class of this point can be avoided by using as examples smooth waves without sharp corners.

The second answer is found through considering the mechanics of a wave as it passes some point P. In Figure 2, consider the small bit of rope, Δx long, about the point P. We assume that the tension in the rope is T (newtons). That means that the rope is pulling the point P with a force T both to the right and to the left, as shown in the inset of Figure 2. The net force on the bit of rope at P is zero. By Galileo's principle of inertia, the piece of rope must be standing still or moving with a uniform velocity. Since the wave has not yet arrived at P we know that it is standing still.

Fig. 2



Next let us consider a later time when the pulse has progressed to the point shown in Figure 3. Here the rope at P is stretched somewhat more than in Figure 2 and hence has a tension T′. We will avoid for the moment how T′ is related to T, the tension in the level part of the rope. Again we see in the inset that the piece of rope at P is pulled just as much one way as the other and again the point P must be at rest or in uniform motion.

Fig. 3

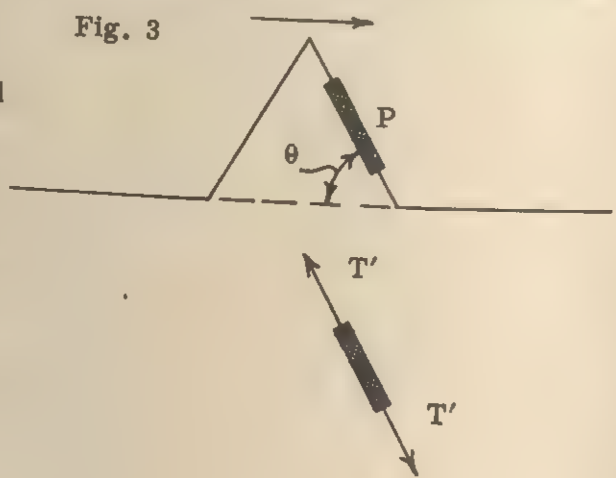
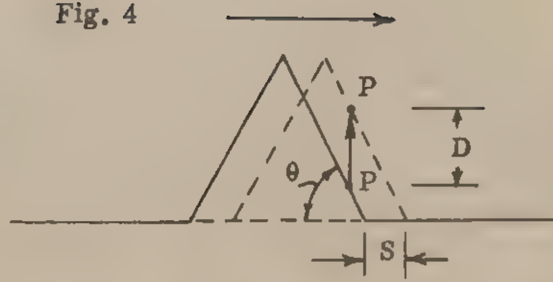
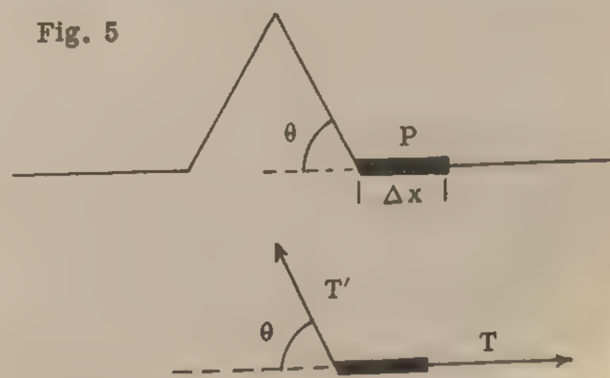


Fig. 4



Referring to Figure 4, we see the wave drawn at one time and dashed at a time t later. During this time the wave has moved a distance S and the point P has moved up a distance D. The velocity of the wave is thus $v = S/t$ and the point P is moving up with a velocity $D/t = V$. We easily divide these two equations to get $V = v\frac{D}{S}$, and since $D/S = \tan\theta$, we see that $V = v \tan\theta$. Furthermore we can see that all points along the front slope of the wave are moving with the same velocity V. Similarly, if the pulse is symmetrical, all points on the back slope of the pulse are moving down with the velocity V. The point P in Figure 3 is thus moving up with a uniform velocity V as it must since the net force on it is zero.

Fig. 5



In Figure 2, the point P was at rest. At the later time in Figure 3 it was moving up with a velocity V. Clearly, at some intermediate time it was accelerated. This acceleration occurred while the corner was passing point P. In Figure 5, the wave has just arrived at the segment of rope at P. The inset shows that the bit of rope is being pulled to the right with a force T. Now, however, the rope to the left does not just "cancel out" this force. If we are dealing with an idealized wave where the point P moves only up and down (this is an idealization because there is usually some longitudinal motion on a real rope), then $T' \cos\theta = T$ since the horizontal components of the force must cancel out. The vertical component of the force is $T' \sin\theta$ which is also equal to $T \tan\theta$. This vertical component of the force is the one that jerks the rope at P from rest into motion when the wave goes by. Of course, if we consider a small enough bit of rope this force does not act for long. It acts only long enough for the wave to progress the distance Δx.

When a force acts on some object for a short time, the impulse method (see Part III) is usually a convenient way of treating the problem. The change in momentum of the object equals the product of the force on the object and the time the force acts. The rope at point P starts from rest, and under the action of the force $T \tan\theta$, finally acquires a velocity $V = v \tan\theta$. Its initial momentum is zero; its final momentum, mass $\times$ V. Now the mass of the rope at P is zero, but the mass of the bit of rope of length Δx is $\mu\Delta x$ where μ is the mass of the rope per unit length. (If the whole uniform rope has a mass M and a length L, then $\mu = M/L$.) Its change in momentum is thus

$$\text{change in momentum} = \text{mass} \times V = \mu\Delta x \, v \tan\theta.$$

The impulse acting on this bit of rope is a force $T' \sin\theta = T \tan\theta$ (see Figure 5) acting for a time $\Delta x/v$ which is just the time it takes for the corner of the wave to pass the bit of rope Δx long. Finally, then

$$T \tan\theta \, \Delta x/v = \mu\Delta x \, v \tan\theta.$$

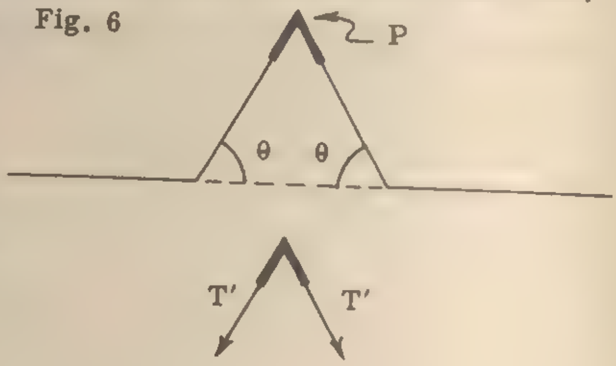
Cancelling, we have $T/v = \mu v$. Notice that Δx cancelled out which means that it didn't matter how long a little piece we considered. Since our choice of Δx was arbitrary, this must happen. Rearranging the last equation we find

$$v^2 = \frac{T}{\mu}; \quad v = \sqrt{\frac{T}{\mu}}$$

This shows that the velocity of a wave is greater when the tension is greater, and smaller when the mass per unit length is greater. The fact that the angle θ cancelled out of the expression is important, for if it had not, waves of different shapes would travel with different velocities, and a wave with a complicated shape involving many values of θ would not maintain its shape as it traveling along the rope.

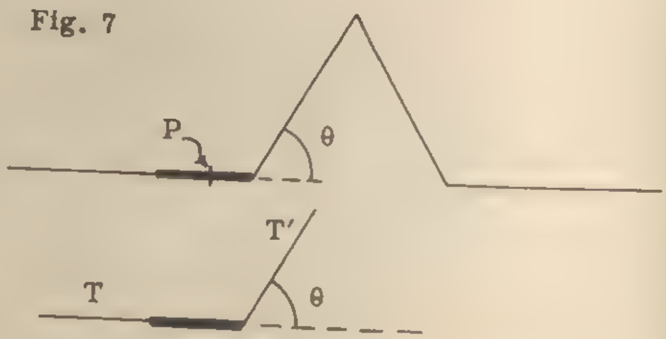
Once we have treated quantitatively the forces acting on the front corner of the wave, we can easily see what happens at other points. Figure 6 shows a time when the point P is the top of the wave. The inset shows the forces acting. Again, the horizontal components cancel out. There are now, however, two vertical components T′ sin θ. The impulse given the bit of rope at P is just twice as great as at the front edge of the wave. The bit of rope P is therefore not only stopped but turned around and given a velocity V downwards

Fig. 6



Then as shown in Figure 7, the point P comes to the back corner where a force T sin θ acts on it in such a direction as to stop the bit of rope at point P. The impulse is just the right amount to bring the bit of rope to a dead stop without overshooting.

Fig. 7



We have considered the mechanics of the passage of a particularly simple type of wave. Net forces on the particles of the rope occur only at the corners. Here, since they act on small bits of rope they create violent accelerations. The accelerations last only for very short times and occur in such a way as to give rise to smooth wave motion. A wave pulse is, in general, curved smoothly. The forces which act on bits of rope are greatest where the curvature of the rope is greatest. A complete treatment of wave motion along a rope involves the solving of partial differential equations (see, for example, R. A. Becker, Introduction to Theoretical Mechanics, Chapter 15.), and we will not consider it here.

APPENDIX 8

Supplement to Chapter 7 - The Speed of Water Waves

The following material is not presented as suggestions for extending classroom or laboratory discussions. It is intended only to serve as an aid to the teacher in answering questions, and in planning laboratory programs.

For any single liquid, the speed of a surface wave depends in a complex way on the frequency of the waves and the depth of the liquid.

The speed of surface waves can be simply calculated only if:

1) There is negligible viscosity (and therefore negligible energy loss) in the liquid.

2) The waves are generated without turbulence (the motions in the liquid must be "simple and smooth").

3) The amplitude, A, of the wave is much less than either the wave length or the depth of the liquid.

If any of these factors--the viscosity, the turbulence, or the wave amplitude--becomes significant, it is extremely complicated to handle the speed theoretically.

Fortunately, all of these effects are usually negligible in ripple tanks, and the analytic formulas are close approximations to experimental results. The main cause for discrepancies between experimental observations and the theoretical descriptions which follow, stems from wave amplitudes which are not negligible with respect to the wave lengths. If the wave amplitude is as much as 7% of the wave length, the speed increases by about 10% of the wave's "small amplitude" speed. Although such an amplitude is commonly used in much of the ripple tank work, it will not affect the students' observations of various wave phenomena. It may explain some of the variations found with different wave generators. It will surely produce differences between precise experimental measurements and the values given below.

The following formulas are all derived for waves of negligible amplitude.

If the water is deep enough(a depth of $1/2\,\lambda$ will introduce less than a 1% error), the speed, v, depends on the sum of two terms as follows:

$$v^2 = \left(\frac{gv}{2\pi f} + \frac{2\pi f T}{v\rho}\right)$$

where g is the acceleration due to gravity
f is the frequency of the wave
T is the surface tension
ρ is the density.

For water, if v is to have the units of cm/sec, the following values should be used for the constants:

$$g = 980\ \text{cm/sec}^2,\quad \rho = 1\ \text{gm/cm}^3,\ \text{and } T = 72.8\ \text{dynes/cm}$$
$$(\text{or } 72.8\ \text{ergs/cm}^2).$$

Using these values, for waves in water we have for the speed v:

$$v^2 = \left(\frac{156\,v}{f} + \frac{457\,f}{v}\right)$$

From this formula, v has a minimum when: v = 23.1 cm/sec and f = 13.5 cycles/sec.

For much smaller values of f, $v = \frac{156}{f}$ cm/sec.

However, in this case λ is large and may become larger than the depth. In the extreme, when λ is much larger than the depth H, the speed depends only on H: $v^2 = gH$.

The graph below implies that for some depth between 0.4 cm and 1 cm the wave speed would be practically constant at about 23 cm/sec, even though f varies from 1 cycle per second to 10 cycles per second. On the other hand, this non-dependence of v on f (i.e., this freedom from dispersion) comes at the expense of a sensitivity of v on H, and H must be kept quite constant for v to be uniform.

For frequencies much higher than 13.5 cycles per second, the speed is given by:

$$v^3 = 457\ f \text{ or } v = 7.7\ f^{1/3}$$

Although these approximate formulas are helpful in getting a qualitative idea of the speed of water waves at various extremes of frequency, they are not adequate for predicting the detailed behavior of ripple tanks because intermediate frequencies and relatively shallow depths are used. The complete formula, including the effect of water depth, is

$$v^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho}\right) \tanh\left(\frac{2\pi H}{\lambda}\right)$$

or

$$v^2 = \left(\frac{gv}{2\pi f} + \frac{2\pi f T}{v\rho}\right) \tanh\left(\frac{2\pi f H}{v}\right)$$

where tanh stands for the hyperbolic tangent, $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

For x large, tanh (x) tends toward 1.

For x small, tanh (x) tends toward x.

For water: $v^2 = \left(156\lambda + \dfrac{457}{\lambda}\right) \tanh\left(\dfrac{2\pi H}{\lambda}\right)$

where the distances, λ and H, are in centimeters and the speed is in centimeters/second.

The graph on the following page gives a series of curves showing the velocity (in centimeters/second) of surface waves on water as a function of frequency (in cycles/second). Each curve applies to a depth, H, of the water.

The following features of the graph and formulas are of particular interest in planning ripple tank experiments:

a. <u>In order to minimize dispersion</u> (i.e., the dependence of speed on frequency), the depth of water should be small. You can choose a value suited to the frequency range you expect to use. For example, if you were interested in the range from 2 cycles per second to 10 cycles per second, H = 0.5 cm or 0.6 cm would be particularly good. If you wanted to use frequencies from 7 cycles per second to 20 cycles per second, H = 0.9 cm might be somewhat better.

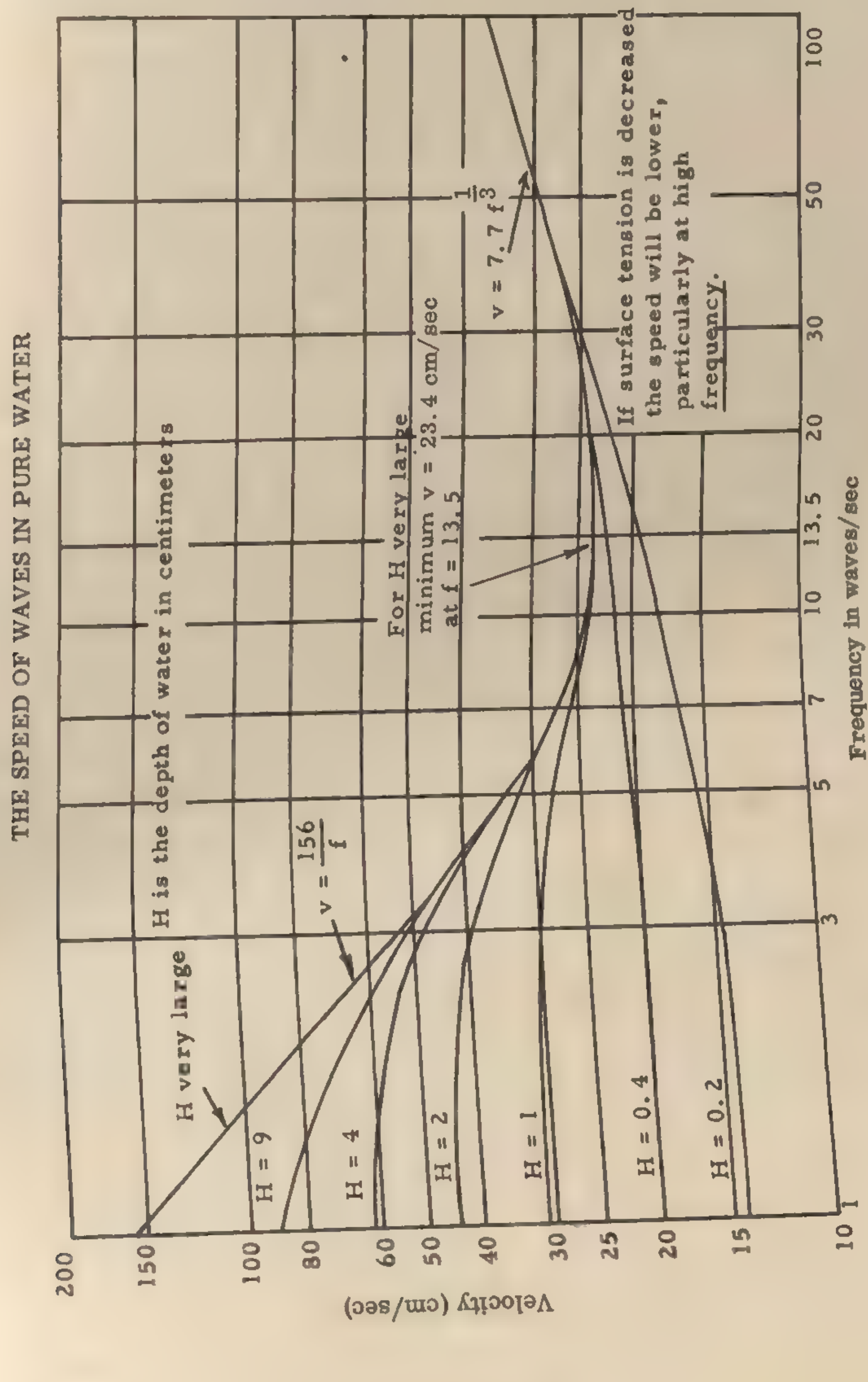
b. <u>In order to demonstrate refraction</u> well, the speed should depend strongly on depth. For this purpose, low frequency waves show a much bigger effect than higher frequency waves.

Usually, best refraction results were obtained when the shallow region was about as shallow as can be conveniently produced. Remember that when you try to get a large, extremely shallow region, it is quite difficult to keep the shallow depth uniform. If you do not keep it uniform, however, you will get a strange "beaching" effect. (See Figure 17-23 on page 271).

c. For other applications, the optimum depth of water in a ripple tank depends in a very complex way on the detailed shape of the waves. If the waves were pure sine waves, they would involve only a single frequency and dispersion would be unimportant. However, the periodic waves almost always contain higher harmonics, and single pulses always contain a large spectrum of frequencies. The way in which dispersion affects an actual pattern will depend on the relative amounts of different frequencies present and the ability of such waves to focus light on the screen. The only practical way to choose is to experiment. If

a wave pattern is not clear, you may adjust the amplitude of the wave, the position of the screen, the details of the wave generator, or the depth of the water.

d. For relatively low frequencies, the surface tension will not be too important. However, for high frequencies a very small quantity of detergent can reduce the surface tension by a large factor, thereby reducing the speed.



TEACHER'S GUIDE FOR EXPERIMENTS, PART II

This part of the text is so interwoven with the laboratory that the experiments can carry the main burden of introducing new concepts. Since there is some laboratory work related to almost every section of the text, the flexibility to alternate between the laboratory and the classroom discussion is especially useful.

The proper scheduling of the experiments in conjunction with specific sections of the text is very important.

Number	Experiment	Best Time	Priority
II-1	Reflection from a Plane Mirror	Before Section 12-4	***
II-2	Images Formed by a Concave Mirror	Before Section 12-9	**
II-3	Refraction	Before Section 13-3	***
II-4	Images Formed by a Converging Lens	Before Section 14-3	**
II-5	The "Refraction" of Particles	Before Section 15-2	***
II-6	The Intensity of Illumination as a Function of Distance	Before Section 15-3	*
II-7	Waves on a Coil Spring	2 half periods during Chapter 16 or one period in the middle of the chapter	***
II-8	Pulses in a Ripple Tank	Introduction to Chapter 17 (Sections 17-1 to 17-3)	***
II-9	Periodic Waves	After Section 17-4	***
II-10	Refraction of Waves	Before Section 17-5	***
II-11	Waves and Obstacles	Before Section 17-7	**
II-12	Waves from Two Point Sources	Part I: Beginning of Chapter 18 Part II: After Section 18-4	***
II-13	Interference and Phase	Before Section 18-5	***
II-14	Young's Experiment	Before Section 19-4	***
II-15	Diffraction of Light by a Single Slit	Immediately after Part 2 of Experiment II-14	**
II-16	Resolution	During Section 19-8	*
II-17	Measurement of Short Distances by Interference	After Section 19-9	*

*** essential

** desirable

* optional

II-1. REFLECTION FROM A PLANE MIRROR

This experiment can be done before the students read or discuss Section 12-4. They will thus discover that the incident and reflected rays make equal angles with the reflecting surface, that the image formed by a plane mirror is as far behind the mirror as the object is in front of it, and that the image and object are the same size. Furthermore, the experience gained in locating images by parallax and ray tracing will be useful in later experiments.

Only nails and pins are used as objects in this experiment; unlike triangles and letters, they are symmetrical, and finding their images in a plane mirror will not raise the matter of reversal.

Nails long enough to project above the mirror should be used in finding the image by parallax. Finding the image is then just a matter of "putting the top" on the image nail. (see Fig. 12-11 of the text.) Another way to hold the mirror is to fit it into a saw-cut in a block of wood [Fig. (a)].

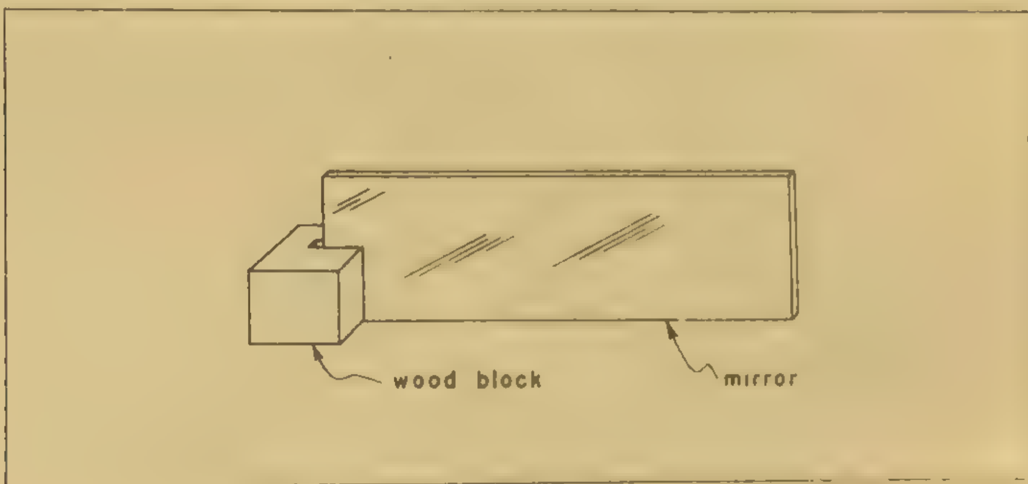


Figure (a)

Because the angles between the rays and the reflecting surface are seen directly on the paper, these angles are the ones we compare. You may delay the introduction of the angles between the rays and the normal until you take up the subject in class or, if you prefer, call the student's attention to it during this experiment.

It is worth emphasizing that the more carefully the lines of sight are established, the more accurate the results will be. Sharp, hard-lead pencils should be used.

The lines of sight drawn to locate an image should form a large angle with each other (30° or more). If the angles are small, the lines of sight are nearly parallel and it is difficult to determine their point of intersection. The rays, angles, reflecting surface, object, image and distances from the mirror should be carefully labelled. A sample diagram is not included in the Laboratory Guide. This enables the student to discover the paths of the reflected rays for himself.

With this apparatus, it is difficult for the student to see that the reflected ray, the normal, and the incident ray lie in the same plane; the proof is therefore left to Section 12-4 and Fig. 12-8 in the text.

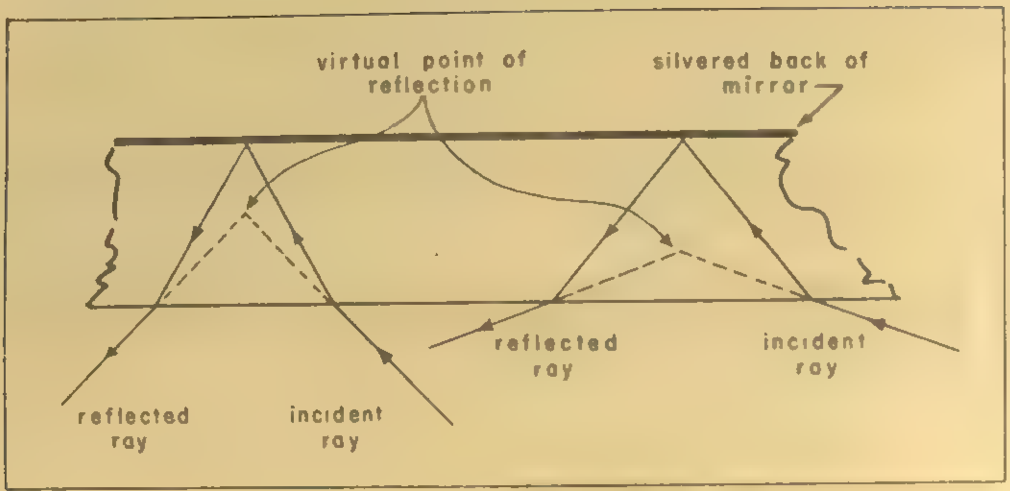


Figure (b)

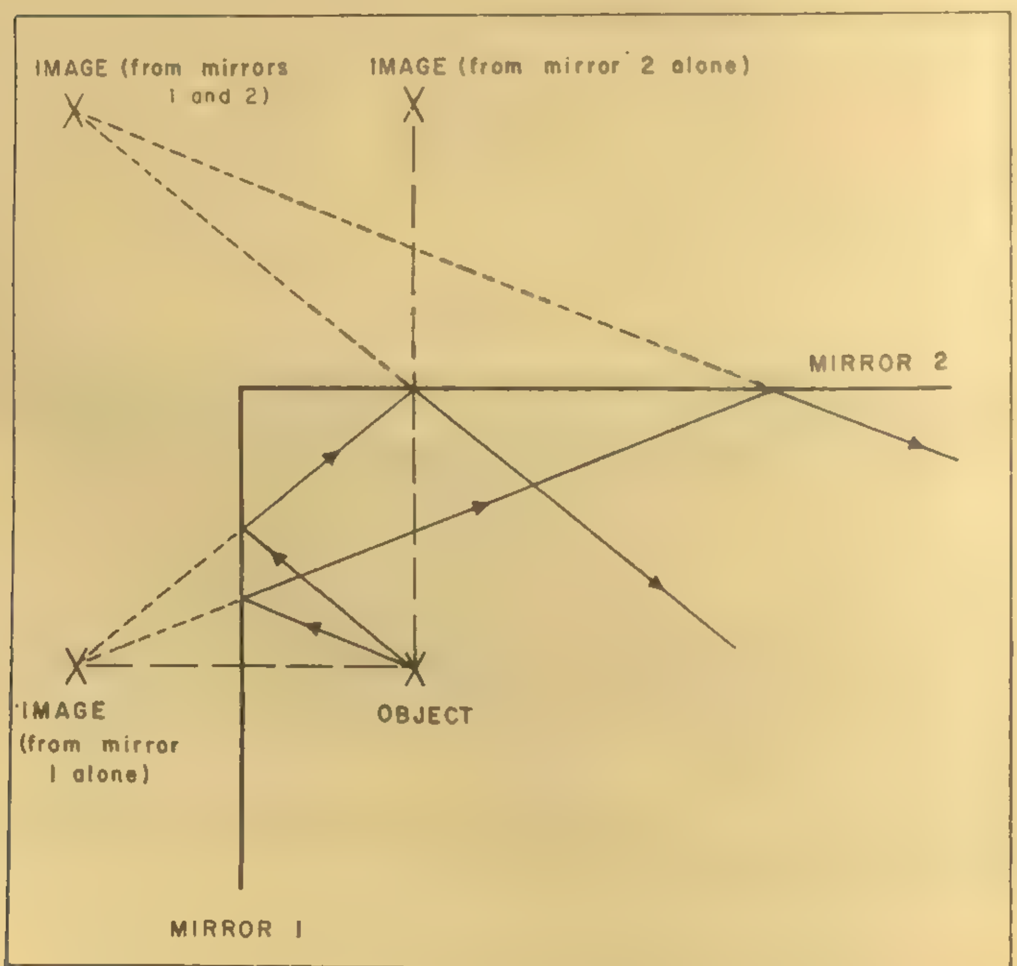


Figure (c)

Answers to Questions

When the head is moved to the left, the nearer of the two pencils will appear to move to the right. This exercise will refresh the student's memory of parallax and make clear to him at the outset that there is nothing mysterious about locating objects or images by parallax. In addition, noting that the nearer of two objects appears to move in the opposite direction to that of the eye will be useful to the student in locating images by helping him to decide which way to move his parallax "indicator."

Many students will think that the image of the nail is in the plane of the mirror. If they come to this conclusion before making measurements, let them have the fun of discovering where the image really is.

Accurate measurements using a mirror with its rear surface silvered will show that both object and image are nearly the same distance from the back reflecting surface. Refraction, however, makes the image distance slightly smaller than the object distance (about 1 mm for a mirror 2 mm thick) as shown in Fig. (b). It is not necessary to discuss this small discrepancy at this time.

The image and object are the same size, although the image appears smaller because it is farther away. When a nail identical to the object is used as a parallax indicator, the indicator and the image are clearly seen to be of the same size.

The three images formed by two mirrors at right angles can be found as shown in Fig. (c).

APPARATUS

1 Plane mirror (about $1\frac{1}{2}" \times 1\frac{1}{2}"$) (Front surface, if available)
2 Nails - flat head (about $1\frac{1}{2}"$ to $2\frac{1}{2}"$ long)
3 Pins (1" long)
3 Sheets of paper ($8\frac{1}{2}" \times 11"$)
1 Sheet of soft cardboard or similar material ($8\frac{1}{2}" \times 11"$)
1 Sharp, hard-lead pencil
1 Protractor
1 Wood block ($2" \times 1" \times 1"$)
1 Rubber band
1 Metric ruler

II-2. IMAGES FORMED BY A CONCAVE MIRROR

In this experiment the student becomes familiar with the images formed by a concave mirror and finds the relation between S_o and S_i. The experiment is similar to Experiment II-4, and we suggest that you assign only one of the two. If reasonably good mirrors are not available, assign Experiment II-4 only; if you have good mirrors, this experiment should be done before the discussion of Section 12-9.

Since students are asked to measure the object and image distances from the principal focus, a small error in measuring the focal length $\underline{f}$ will introduce a large error in the plotted results. The larger the focal length of the mirror used, the more accurate the results will be. However, if the focal length is more than 15 cm, a longer working space will be needed to determine it accurately. An accurate determination of the focal length by parallax can be made by locating the image of an object that is 5 meters or more from the lens.

Students will need a work area about one meter long. A one-meter strip of paper tape from the tape timer (used in Experiment I-5) taped to the table can be used as an "optical bench." The mirror can be supported with modeling clay.

A small porcelain socket is best for holding the flashlight bulb in place, although a dial light socket mounted on a small flat strip of wood also makes a good mounting. The bulb can also be mounted by soldering two connecting wires to the base of the flashlight bulb and then molding a lump of modeling clay around the base of the bulb to hold it upright. The filament of the bulb and the center of the mirror should be the same height above the table.

Finding the image by parallax may be difficult for some students. You can quickly check on proper manipulation by comparing the S_oS_i product with f^2. If the two values are more than about 5% off, it may be necessary to give some individual help in locating the image. Since one of the purposes of the experiment is to have the student discover the relation between S_o and S_i, do not suggest that he check each pair of values this way as he measures them.

Results with an error of up to 5% are not unusual with the apparatus and technique used. Although care and patience will improve results, avoid losing the purpose of the experiment by insisting on a high degree of accuracy. Measuring distances to the nearest half centimeter will give satisfactory results.

A convenient parallax indicator is made of a piece of wire or a common pin stuck into a cork stopper. A pin is just wide enough to about cover the image of the filament but does not overlap too much at most places. This makes accurate location possible. The mounting is not essential, provided one is careful to hold the wire vertically. Be sure the students locate the image of the filament and not the image of the parallax indicator when the filament is between the mirror and the focal point.

The flashlight bulb makes a bright image which stands out from the background. Make sure the students truly sense that the real image is in front of the mirror. The discussion of the distinction between real and virtual images should be delayed until after they have been studied in the textbook.

Answers to Questions

The image is inverted when the object is beyond the principal focus, and right side up when the object is between the principal focus and the mirror. The image is smaller than the object when the object distance is larger than the focal length $\underline{f}$. The image is larger than the object when the object distance is less than $\underline{f}$. The image is the same size as the object when the object distance equals $\underline{f}$.

The plot of S_i vs. $1/S_o$ will give a straight line with a slope equal to $1/f^2$ [Fig. (a)].

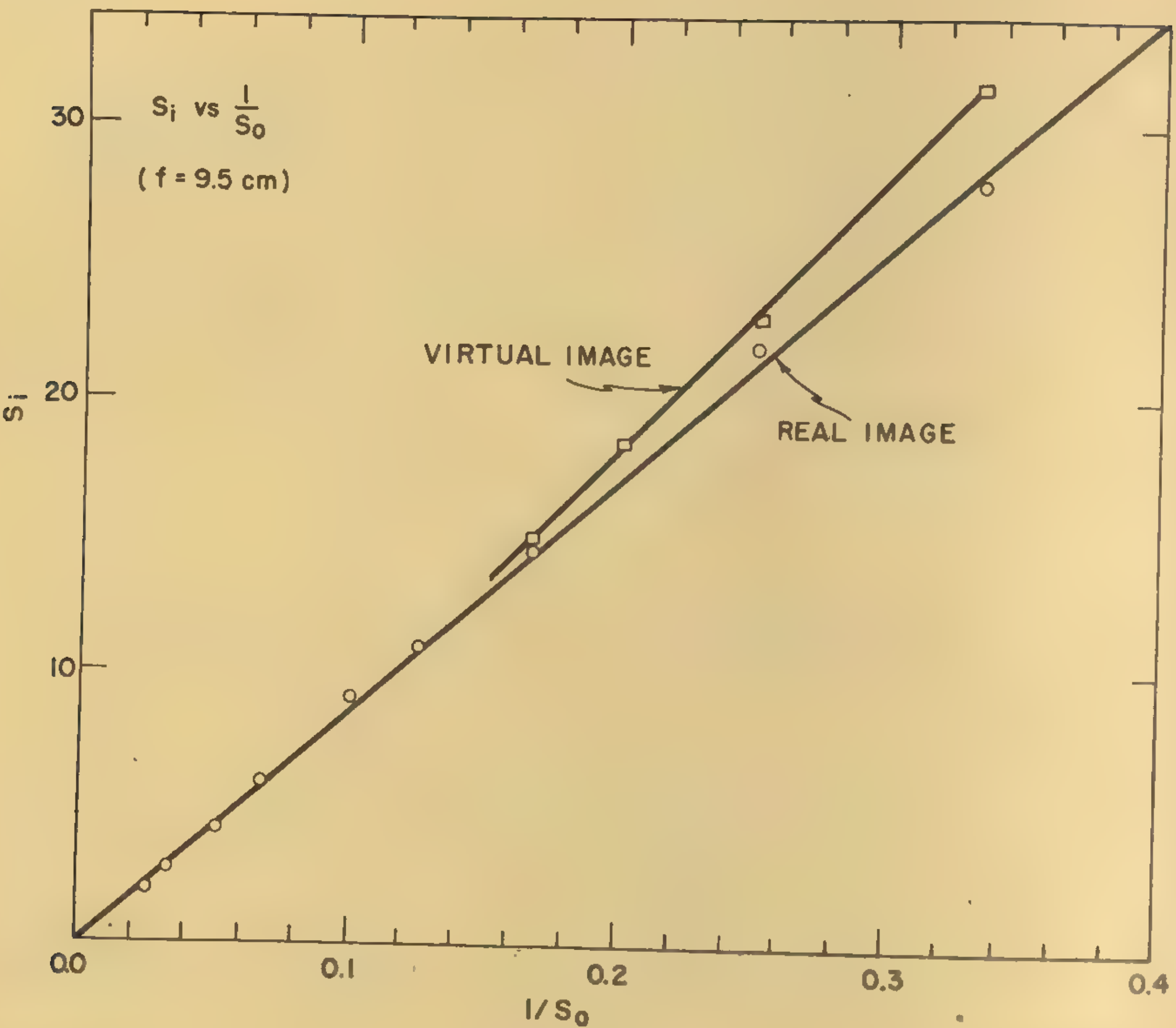


Figure (a)

The image is sharply defined for all positions of the object except when the object is at the focus or close to it. When the object is at the focus, the image is at infinity and cannot be seen.

Have the students select four or five points from their graphs and calculate $S_o S_i$.

APPARATUS

1 Good concave mirror (about 2" dia; 10 cm f.l.)
1 Long strip of paper (about 1.5 meters of timer tape or adding-machine tape)
1 2.5-v. flashlight bulb; #41 and socket (porcelain)
1 #6 Dry cell $1\frac{1}{2}$ volts
2 Connecting wires (about 1 foot long)
1 Lump modeling clay (about 10 gm)
1 Sharp, hard-lead pencil
1 Meter stick
1 Piece of straight wire (about 3" of #22), or a pin
1 Cork (#2 or #4)
1 Sheet rectilinear graph paper

II-3. REFRACTION

The purpose of this experiment is to discover Snell's law. Therefore, the experiment is most appropriately done before the discussion of Section 13-3.

The vertical line on the straight side of the plastic box is more easily seen through the liquid if it is darkened with red pencil. With polar coordinate graph paper, it is possible to read the angles directly from the pin holes to within 0.2°. If other graph paper is used, the incident ray and refracted ray should be constructed with a sharp pencil to avoid unnecessary errors in angle measurement. Corrugated or soft cardboard placed under the graph paper makes pin placement easier.

The object pin should be about 4 cm from the vertical line on the plastic box. Otherwise, at greater distances and large angles of incidence, the image becomes dim and distorted.

We recommend that the students calculate the ratio of the sines of the angles. This method is more accurate and faster than finding the ratio of the semi-chords. In addition, this will give the students a familiarity with the sine function which is used in much of the material in Chapter 13.

You may wish to have different students do the experiment with different liquids. Some liquids that will not discolor or dissolve the plastic box are: glycerine, mineral oil, motor oil ($n \sim 1.5$); salt solution; sugar solution. Do not use carbon tetrachloride, carbon disulfide, turpentine, acids or bases.

If the experiment is performed with care, sin i/sin r is more nearly constant than i/r [Fig. (a)]. But if a student fails to obtain accurate data at large angles of incidence, he will not be able to decide which is the better relation. In borderline cases, suggest to the student that he repeat some of his measurements at large angles.

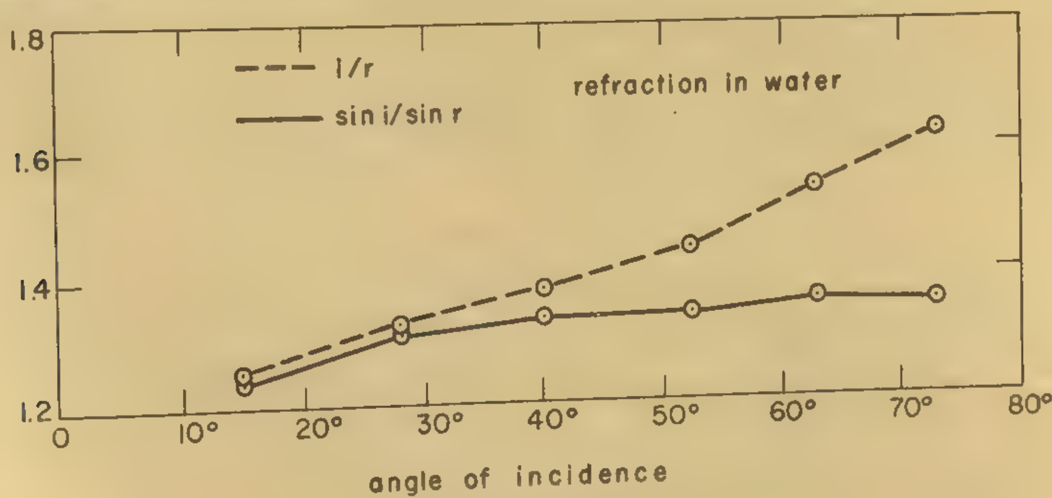


Figure (a)

APPARATUS

1 Semicircular clear plastic box (6 cm rad., 3 cm deep)
4 Sheets rectangular coordinate graph paper, or 2 sheets rectangular and 2 sheets polar coordinate paper
1 Sheet of soft cardboard or similar material about 8" × 11"
2 Pins ~ 1"
Mineral oil (about 75 cm^3) or other suitable liquids
1 Sharp, hard-lead pencil
1 Protractor or a circular scribe (if polar paper is not used)
1 Rectangular block of glass or plastic, 2 sides clear polished (optional)
1 Ruler

II-4. IMAGES FORMED BY A CONVERGING LENS

In this experiment, the student becomes familiar with the images formed by a converging lens and finds the mathematical relation between S_o and S_i. This experiment is similar to Experiment II-2 and may be omitted if the latter has been done. The best time to do it is before the discussion of Section 14-3.

Because students will measure the object and image distances from the principal foci, a small error in measuring the focal length f will introduce a large error in the plotted results. A lens with a focal length of about 20 cm is best. However, if the focal length is more than 15 cm, a longer working space will be needed to determine it accurately. An accurate determination of the focal length by parallax can be made by locating the image of an object that is 5 meters or more from the lens.

Students will need a work area of about 2 meters. A 2-meter strip of paper similar to that used with the timer in Experiment I-5, or adding-machine paper, fastened to the table with tape can be used as an "optical bench."

Two meter sticks, placed parallel to the line on the paper with one end of each corresponding to a principal focus, enable the student to read S_o and S_i directly. Note that S_o is always measured from the principal focus on the object side of the lens, while S_i is always measured from the principal focus on the other side of the lens. To avoid a sign convention, both S_o and S_i are considered distances and are, therefore, always positive (See Experiment II-2 in this Guide). Make sure that the student always looks at the object through the lens. The object should never be between the eye and the lens. If it is, the student will see images formed by reflection from the convex lens surface.

A flashlight bulb is used as the object because it is bright and stands out from the background. A small porcelain socket is best for holding the bulb in place. A dial light socket mounted on a small strip of wood also makes a good mounting. If neither of the above is available, the bulb can be wired by soldering two connecting wires to the base of the bulb, and then molding a lump of modeling clay around the base to hold the bulb upright. The filament of the bulb and the center of the lens should be the same height above the table.

Finding the image by parallax may be difficult for some students. You can quickly check on proper manipulation by comparing the S_oS_i product with f^2. If the two values are off more than 5%, it may be necessary to give some individual help in locating the image. Since one of the purposes of the experiment is to have the student discover the relation between S_o and S_i, do not suggest that the student check each pair of values when he measures them.

A convenient parallax indicator is made of a piece of wire or a pin mounted in a cork stopper. The mounting is not essential, provided one is careful to hold the indicator vertically. Make sure that your students see that the real image stands out in front of the lens and that the virtual image is back of the lens.

Results with an error of up to 5% are not unusual. Care and patience will improve results; although this can be overdone and the purpose of the experiment lost. Measuring S_o and S_i to the nearest half centimeter will give satisfactory results for most object positions.

A half lens makes it much easier to locate the virtual image because it eliminates distortion due to spherical aberration and provides a straight surface to sight over. It is possible to cut a thin lens by scratching it across the diameter with a glass cutter and then breaking it. To avoid injury, smooth the sharp edges with sandpaper.

Answers to Questions

The image is smaller than the object when the object is beyond the principal focus and larger than the object when the object is between the lens and the principal focus. The image will not be visible when the object is at the principal focus or near it.

The image is inverted when the object is beyond the focus, and right side up when the object is between the lens and the focus.

The plot of S_i vs. $1/S_o$ will give a straight line with the slope equal to $1/f^2$.

APPARATUS

1 Converging lens (about 20 cm focal length)
1 Long strip of paper (about 1.5 m × 10 cm)
1 2.5-v. flashlight bulb (#41) and socket
1 #6 dry cell; $1\frac{1}{2}$-v.
2 Connecting wires
1 Lump modeling clay (about 10 gm)
1 Sharp, hard-lead pencil
1 Meter stick
1 Piece of straight wire (about 3" of #22) or a florist pin
1 Cork (#2 or #4)
1 Sheet graph paper

II-5. THE "REFRACTION" OF PARTICLES

The aim of most of our experiments is to have the student discover new physical relations. For example, in Experiment II-3 he discovered Snell's law. Here, we are not concerned per se with the change in direction of steel balls rolling down a slope; rather, we want to find specifically whether the change in direction can be described by Snell's law. In other words, the students are asked to test a hypothesis about light. This experiment is most profitably done before discussing Section 15-2.

The incline and the upper surface of the platform should be of a smooth, rigid material such as masonite. The two pieces, about 12" × 12" and 3" × 12", can be joined together with a masking-tape hinge. The larger section can be placed about $1\frac{1}{2}$" above the table. It may be necessary to tape a sheet of paper to the incline to prevent the ball from sliding.

Both the upper surface and the table should be smooth and level. If the table is rough, another piece of masonite can be placed at the base of the incline. A quick visual check on whether further leveling is necessary is provided by rolling the steel ball slowly across the upper and lower surfaces. Paper wedges can be used to level the two surfaces.

A ramp, to give the ball its initial velocity, can be made from a plastic ruler with a groove down the center. A 3" section and a 1" section of the ruler can be fastened together with glue as shown in Fig. (a). It is necessary to hold the ramp while the ball is accelerating so that the ball gains approximately the same speed on each trial.

The papers taped to the upper and lower surfaces must be carefully placed so that their edges are parallel to each other and remain so for each succeeding trial.

With the short starting ramp, it is possible to get large incident angles; however, it may be necessary to shift the paper on the lower surface to the right or left to pick up the trace.

Different students may find different indices of refraction because the initial speed of the ball or the height of the upper surface is different. The results of a typical run are shown in Fig. (b).

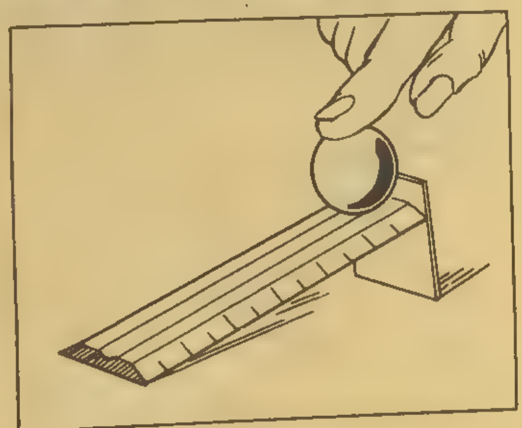


Figure (a)

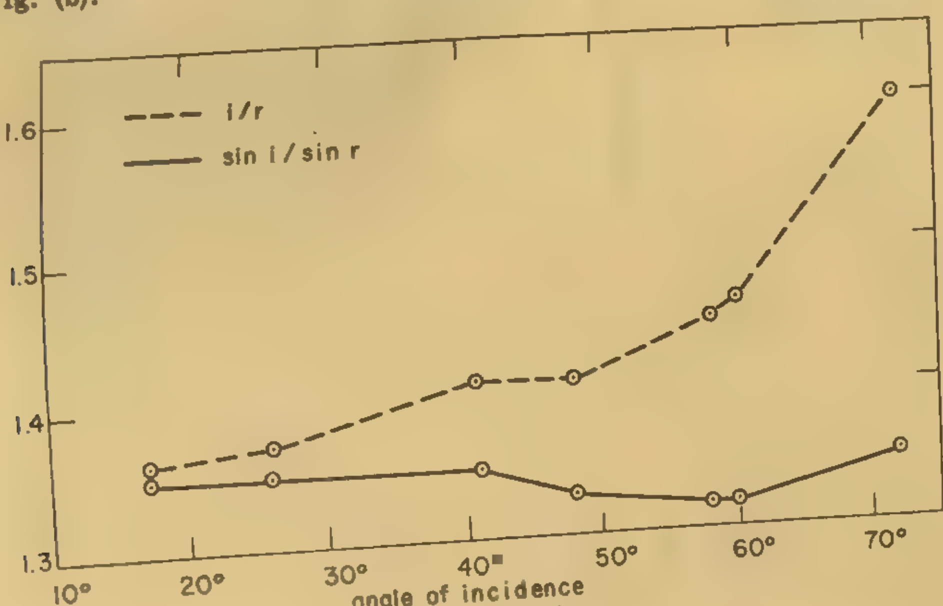


Figure (b)

Answers to Questions

The particle model of light predicts that the speed of light in water is greater than the speed of light in air.

It is possible to make a "lens" that will focus rolling balls if the upper and lower edges of the slope are parabolic in shape.

An Extension of the Experiment

Total internal reflection can be illustrated by rolling a ball up the incline. A piece of masonite must be placed on the table next to the incline to provide a continuous surface. The paper and soft carbon should be arranged on the two surfaces as described earlier. A starting ramp made from a ruler and a block of wood, about one inch higher than the upper surface, will give the ball sufficient speed to get over the top of the incline. The angle of incidence can be increased until the critical angle is reached. When the angle of incidence exceeds the critical angle the ball will not reach the upper surface, but will curve and come back down.

APPARATUS

1 Steel ball, 1" dia.
2 Pieces $\frac{1}{4}$" masonite about 12" × 15" and 3" × 15"
2 to 4 Sheets white paper, $8\frac{1}{2}$" × 11"
2 to 4 Sheets soft carbon paper (Carter No. 1432 for noiseless typewriter)
Grooved ruler (ramp) (Block of wood, 2" × 1" × 1", to support ruler for launching ball or other launching device)
Protractor

Heavy masking tape

II-6. THE INTENSITY OF ILLUMINATION AS A FUNCTION OF DISTANCE

This experiment tests a prediction of the particle model of light, that the intensity of illumination is inversely proportional to the square of the distance. With an appropriate introduction, it may be done before the discussion of Section 15-3. Experiment I-4 is pertinent to the analysis of this experiment.

An alternative arrangement for the screen shown in Figs. 1 and 2 is a library card fastened to two clothespins. The pencil is held in place by inserting the point in a one-hole stopper. If the bulb sockets are mounted permanently on a wooden base instead of with burette clamps fastened to a ringstand as shown in the Laboratory Guide, the wood should be painted with flat black to minimize reflection.

It is convenient to place a meter stick on the table with the screen at one end and the four bulbs (Fig. 2) at the other end. The single bulb, B, can then be moved along the meter stick and positions read directly.

Answers to Questions

To make sure that all the bulbs give the same illumination, compare the shadows illuminated by bulb B and by each bulb at A when both A and B are equidistant from the screen. (Manufacturers' tolerances or damage in shipment and handling will sometimes cause bulbs of the same type to vary in intensity.

The bulbs must be stacked vertically to cast a single shadow. Analyzing the data by calculating the values of Ir^2 or by graphing I versus $1/r^2$ will verify the inverse-square law with an accuracy of about 6% - 8%.

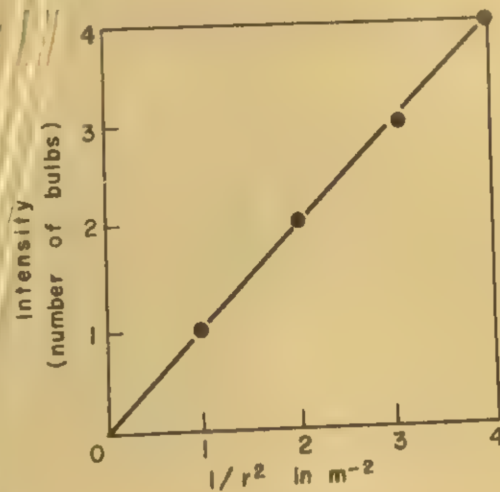


Figure (a)

Background illumination will cause little error. Because of the closeness of the two shadows, they are almost equally illuminated by distant sources.

Students will probably find that a 60-watt bulb has more than 4 times the intensity of a 15-watt bulb, since the efficiency of incandescent bulbs increases with their power.

APPARATUS

5 15-watt, 117-volt light bulbs
5 Sockets with line cords and plugs
2 Ring stands
5 Burette clamps
1 Cardboard screen about 5" square
1 Support for screen (2 split corks, about 1" dia., or 2 clothespins)
1 Pencil
1 One-hole rubber stopper
1 Meter stick
1 Graph paper, mm
1 Extension cord with triple outlet
1 Cube tap

II-7. WAVES ON A COIL SPRING

Chapter 16 presents many new ideas and concepts, which will be better understood if the students have the opportunity to study them in the laboratory. This experiment covers essentially the whole chapter and, therefore, should not be done in one laboratory period in spite of the fact that it is primarily qualitative. If your schedule does not permit two half periods, the experiment is best done about the middle of the chapter.

The various paragraphs are almost independent of one another, so that certain parts of the experiment can be done as class demonstrations. For example, you may wish to demonstrate the part that requires two different coils hooked together.

We use a coil spring in this experiment because pulses propagate slowly enough along it to be easily observed. A smooth floor is essential. The combination of the small round-wire coil and the large flat coil (Fig. 1) gives an optimum splitting of a pulse into a reflected and a transmitted part.

The partial reflection and transmission at the junction of the two coils is best seen when the two coils are stretched to equal length. To achieve this, only about one-third of the large coil should be stretched.

The laboratory guide does not contain instructions on how to shake pulses. We believe that this is best learned by seeing it done. To obtain short pulses a rapid snapping motion of the arm is required. Care should be taken not to overshoot the starting position during the backswing of the arm; this will avoid pulses of complicated shapes. While the large coil is stretched, a twist is introduced which causes a different attenuation of the head and the tail of the pulse, thereby distorting it. To avoid this, have your students attach a thread to one end of the coil as shown in Fig. 2 and hold the thread while stretching the spring. The thread permits the spring to "unwind," thus preventing the build-up of the twist.

To study the reflection of a pulse at the junction of the large coil and the thread, the thread must be several meters long. If a short thread is used, the head of the pulse will reflect from the fixed end of the thread and superpose with the tail of the pulse before the latter reaches the junction.

Answers to Questions

The pulse definitely attenuates as it travels along the spring. Only if the floor is smooth enough, the spring tight enough, and the twist removed can the student be expected to conclude that the shape of the pulse remains basically unchanged.

The speed of the pulse does not change as it moves along the spring; it is independent of the size of the pulse within the experimental accuracy obtainable with a stop watch.

The assumption that the speed of the pulse does not decrease upon each reflection can be checked by comparing the time it takes the pulse to make two or more trips down and back with the time required for one round trip.

The speed of the pulse increases as the tension in the spring increases. Two springs of the same material stretched to different lengths are therefore different media.

When pulses collide, they seem to pass through each other without change.

The maximum displacement of the spring when the pulses meet is approximately the sum of the two separate displacements. The students can determine this to an accuracy of about 10 to 20 per cent.

When a pulse reaches the junction between the two springs it is partly reflected and partly transmitted. Pulses traveling on the small spring are reflected right side up, while pulses on the large spring are reflected upside down.

Pulses on the large spring reflect right side up when they reach the thread; pulses are reflected upside down from a fixed end. The speed of the pulse in the thread is much higher than that in a spring.

Hints for Demonstrations

A long piece of rubber tubing or flexible clothesline can be hung outdoors for demonstration. It has the advantage of small attenuation. Indoors a coil spring can be suspended horizontally from the ceiling as shown in the Teacher's Guide to the text. The spring must nevertheless be under considerable tension to carry the kind of waves we are studying.

APPARATUS

1 Flat-wire coiled spring, 4" long, 3" dia.
  Meter stick
1 Long thread
1 Round-wire coiled spring, 1.75 m long, 2 cm dia.

II-8.

PULSES IN A RIPPLE TANK

This experiment acquaints the student with the ripple tank and is best done as an introduction to the first three sections of Chapter 17. The use of periodic waves and the wave generator have been delayed purposely until the next experiment to allow a gradual transition from the pulses on a line to periodic waves in a plane.

The depth of water in the tank should be about 5 - 7 mm for all experiments unless otherwise stated. (Operation is much better if water and damper covers are changed after a day's use.) Generating a straight pulse requires some practice. Smaller pulses are always present preceding and following the main pulse. The pulses may be curved if the dowel is moved too rapidly; if it is moved too slowly, the pulses will be too weak to be visible. A ruler can be used instead of a dowel.

The opposite end of the tank may be used as a straight reflecting barrier provided the edge of the tank is vertical. By changing the direction of the dowel, the incident angle is varied.

For the latter part of the experiment, bending a rubber tube as shown in the Laboratory Guide gives a satisfactory approximation to a parabola. Circular pulses can be clearly produced by allowing a drop of water to fall into the tank from an eye-dropper.

Answers to Questions

The pulse starts almost from a point and expands in a circular pattern; hence the speed is the same in all directions.

Straight pulses remain straight as they move along the tank, provided they are wide enough to cover the whole tank. When a pulse does not cover the whole tank, it curves at the ends. This is particularly noticeable after the pulse has been reflected.

The angle of reflection appears to be equal to the angle of incidence.

The virtual source of a circular pulse reflected from a straight barrier is as far behind the barrier as the source is in front of the barrier. This can be explained only with the assumption that the angle of incidence equals the angle of reflection. A straight barrier reflects a straight pulse similarly to the way that a plane mirror reflects a parallel beam of light; it reflects a circular pulse similarly to the way that a plane mirror reflects a divergent beam of light.

Straight pulses reflected by a parabola are brought to a focus, which again can be explained only on the basis that the angle of incidence equals the angle of reflection.

The direction of motion of a small segment of a pulse may be indicated by drawing a directed normal to the segment. Connecting the successive normals to a segment results in a picture similar to the ray diagrams studied in Chapter 12 (Fig. 12-16).

When circular pulses are generated at the focus of the parabola, the reflected pulses are straight. There are no other points that will give the same result. To explain these observations, it is necessary to assume that the angles of incidence and reflection are equal.

APPARATUS

1 Ripple tank and dampers
1 Ripple-tank stand
1 150-watt clear-glass light bulb (straight filament), shield and socket
18" Length of thick-walled rubber tubing, diam. about $\frac{3}{4}$"
1 15" to 18" length of $\frac{3}{4}$" dowel
4 Paraffin blocks
Large white paper screen about 2' square
4 Wooden wedges ($\frac{1}{2}$" × 3")

II-9.

PERIODIC WAVES

This important experiment is most appropriately done after studying Section 17-4. Its main purpose is to drive home the use of the relation, $v = f\lambda$. The experiment also demonstrates that the speed of propagation of waves in a ripple tank varies with the depth of the water, thus preparing the ground for Experiment II-10 on refraction.

To obtain clear waves, connect the battery in such a way that the motor will turn in the direction away from the tank. A low and steady frequency can be obtained by setting the rheostat slightly above the stalling point of the motor. This will result in a frequency of about 5 to 7 cycles per second. For low-frequency waves, adjust the nut on the shaft to give higher amplitude. Before taking qualitative measurements some practice may be necessary, to make sure that the generator will operate at low frequency long enough to take several measurements of frequency and wave length. It is important to work with a low frequency. For frequencies of 10 c.p.s. or more, a change in the depth of the water from the standard $\frac{1}{2}$ - $\frac{3}{4}$ cm to 2 cm will not produce an observable change in the speed of the waves. Fig. (a) shows the dependence of the speed of water waves on frequency and depth.

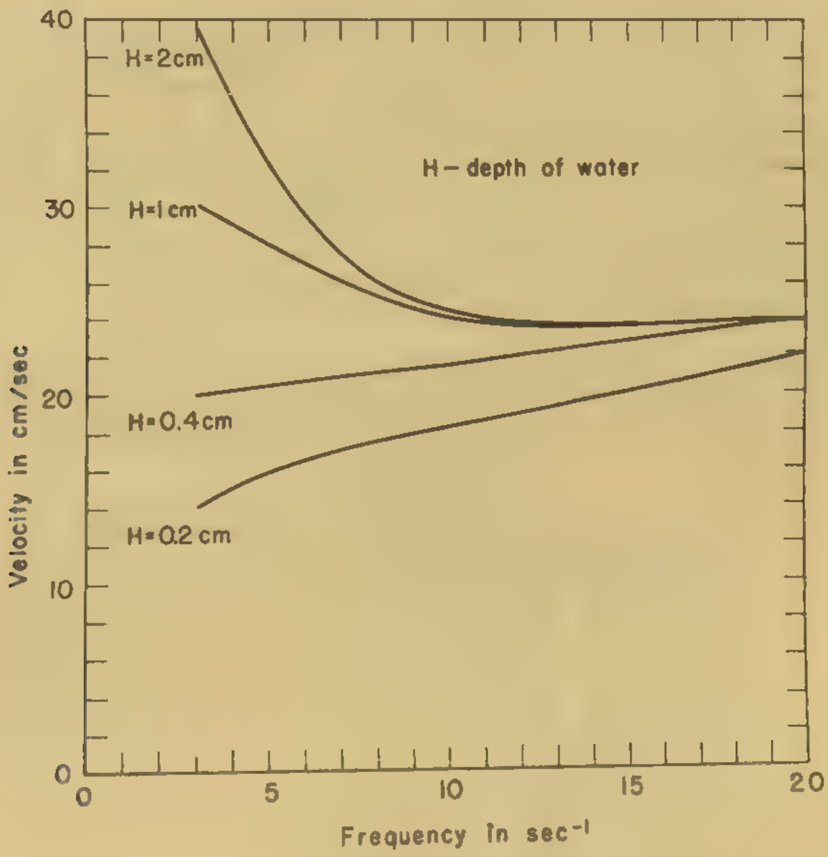


Figure (a)

In the process of generating waves on a coil spring and in later ripple-tank experiments using barriers, standing waves will be observed by most students. You may wish to treat them briefly as a special case of superposition and as another way of measuring wave length. Standing waves are important to Part IV and are discussed in more detail in Section 34-5.

If a small and a large coiled steel spring are tied together (see Experiment II-7), it is best to allow only about one-third of the large spring to extend while the rest of it is held in the hand.

Answers to Questions

When the waves are stopped by the stroboscope with the two slits open, their frequency is twice the frequency of rotation of the stroboscope; with four slits open, the frequency of the waves is four times that of the stroboscope - provided no higher frequency of rotation stops the waves.

The accuracy of the speed determination can be found by first calculating the mean of all measurements, then calculating the deviations from the mean value, and finally, the mean of the absolute value of the deviations. (This procedure is simpler than finding the root mean square). The mean deviation divided by the over-all mean yields the fractional error, which can be expressed in per cent.

Because the images of the waves are magnified on the screen, it is necessary to scale down the measurements of distance accordingly. The students can try to figure out how to determine the scaling factor. There are several ways to do this. One way is to place an opaque block of known length in the tank and compare its length with that of its shadow on the screen. To avoid the indistinct shadows cast by the water meniscus at the edges of the block, the block can be held just below the glass in the ripple tank. Another way is to measure the distance from the light source to the tank and from the light source to the screen and take the ratio.

The distance between two bright bars in the standing wave pattern is one-half the wave length of the traveling wave. The wave length can be determined from the standing wave pattern by finding the distance across, say, ten bright bars and dividing by five.

Both ends of the spring can be considered to be nodes as long as the hand maintaining the standing waves moves with a small amplitude. This will happen only if half the wave length fits an integral number of times into the length of the spring. At this point, it is not important for the students to find the quantitative relation between possible wave lengths of standing waves and the length of the spring; but if they do, this will be very useful in their study of energy levels of atoms in Part IV. You can also mention standing waves in sound-producing instruments.

If you use the two coil springs of Experiment II-7 tied together, the frequencies in both will be the same; the wave length will be less in the small spring than in the large spring; hence the velocity in the small spring is less than that in the large spring.

APPARATUS

1 Complete ripple-tank apparatus; this consists of:
- 1 Ripple tank and dampers
- 1 Stand for ripple tank
- 1 Wave generator
- 1 Support for generator
- 1 150-watt clear glass straight-filament bulb with socket, shield and cord
- 1 Support for lamp
- 1 $1\frac{1}{2}$-v. battery, #6 dry cell
- 1 6-ohm rheostat
- 1 Connecting wire - rheostat to battery
- 1 Paper screen, about 2 ft. square
- 2 Barriers, paraffin blocks

2 Hand stroboscopes
1 Stop watch or clock with sweep second hand
1 Ruler or meter stick
Materials for Exp. II-7

II-10.

REFRACTION OF WAVES

This experiment is basically qualitative and should precede the study of Section 17-5. By taking careful measurements, the quantitative relation between the angles of incidence and refraction can also be established, but with the present equipment, only a few students should be encouraged to do this. The main problem is to keep the frequency constant.

Waves generated at the rate of 5 to 7 cycles per second will be noticeably refracted when passing from the deeper section into the shallower section at an angle. A point source of light should be used to guarantee wave images that are equally sharp regardless of their direction.

During this experiment, it is very important to keep the motor running so that the frequency will remain constant. This will prevent untimely discovery of dispersion. At the end of the experiment, you may wish to call the students' attention to dispersion by suggesting that they observe the change in the angle of reflection when the frequency is increased.

Answers to Questions

The refracted waves are straight. Without taking quantitative measurements, one sees that the angle of refraction is less than the angle of incidence. Both the wave length and speed of waves in the shallower section are less than in the deeper section.

Because the speed of the waves in the shallow water is less, the wave model agrees with the refraction of light better than does the particle model.

The angles of incidence should be chosen from a range of 20° to about 70°. The ratio of sin i/sin r will be nearly constant. However, the accuracy is not sufficient to conclude that the above ratio is closer to a constant than i/r.

Supplement

Several problems in the HDL section of Chapter 17 relate to situations which can easily be realized in the ripple tank. The value of these problems will be greatly enhanced if your students work them out in the laboratory.

Problem 4 A satisfactory ellipse can be made in the following way: first bend a piece of rubber tubing into a circle, connecting the two ends by a short dowel or glass tube. Place the tube in the ripple tank and squeeze it into an elliptical shape by pressing it with two blocks opposite each other.

Problem 5 To obtain satisfactory results be sure that the reflectors are straight and at right angles to each other. Any deviation is amplified by the reflection.

Problem 13 You can use a glass plate resting on the tank on one side and a few washers on the other. It is important to keep the slope of the plate small to obtain gradual bending. A depth of about $\frac{3}{4}$ cm is suitable.

Problems 15 and 16 A lens can be sawed from a piece of lucite. It should be long enough to cover about half the width of the tank or more, but should be rather narrow so that the waves will not attenuate much over it. The exact shape of the lens is not important; two intersecting arcs or an ellipse will be satisfactory. In the latter case, you can use the outside piece of lucite as an elliptical reflector for Problem 4. The depth of the water in the deep part of the tank should be at least 1.5 cm and no more than 2 mm over the lens.

APPARATUS

1 Complete ripple-tank apparatus (see Exp. II-9)
1 Trapezoidal glass plate
  Supports for glass plate (coins, washers, etc.)
1 Hand stroboscope
1 Stop watch or clock with sweep second hand
1 Meter stick or ruler

II-11.

WAVES AND OBSTACLES

The purpose of this experiment is to show that waves bend when they pass by an obstacle and this bending decreases with the wave length. This does not exclude a wave model for light but suggests that the wave length of light is very small. The experiment is best done before studying Section 17-7.

To obtain clean waves at high frequency the edges of the generator must be smooth. Have the students follow the instructions supplied with the kit. Air bubbles can be removed by rubbing a finger along the edge of the generator after it is placed in the water. Before high-frequency waves are used with obstacles, have your students check the straight waves without them. If strange lines and patterns appear, the generator is not smooth enough. This check is essential.

The ends of the paraffin blocks used as barriers should be cut smoothly to form a 45° trapezoid as shown in Fig. (a). The longer side should face the generator. A paraffin block about 5 cm long makes a good obstacle. Under normal conditions its apparent length on the screen will be about 10 cm.

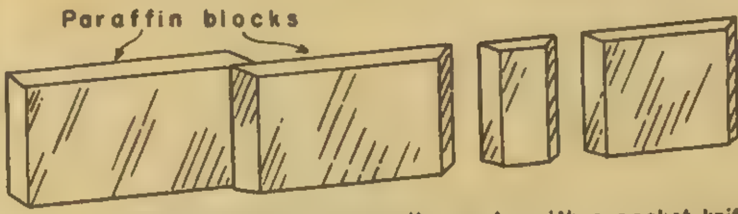


Figure (a)

Answers to Questions

When the wave length is about half the size of the obstacle or larger (both measured on the screen) the waves bend strongly around the obstacle.

Far enough behind the obstacle the waves fuse together and the presence of the obstacle cannot be sensed; the block does not cast a sharp shadow.

As the wave length decreases, the shadow becomes sharper.

The diffraction by a slit shows a similar dependence on wave length. Some patterns are shown in Fig. (b). We have drawn only the central maximum of the pattern. In some cases, students will observe secondary maxima. (This is not the time to elaborate on the detailed structure of a single slit diffraction pattern. This is done in Chapter 19). At this time it suffices to mention that the secondary maxima are very weak and that they too appear at smaller angles to the original direction of propagation as the wave length decreases.

For a constant wave length, progressively smaller slits produce more bending. The change in pattern which results from the decrease in the width of the slit can be compensated for by decreasing the wave length. A quantitative study would show that the pattern is determined by the ratio of wave length to width of the slit. It is not wise to attempt to do this in the ripple tank because the quality of the wave images seen on the screen changes with frequency. Details seen at one frequency may not be seen at another even if λ/d is the same.

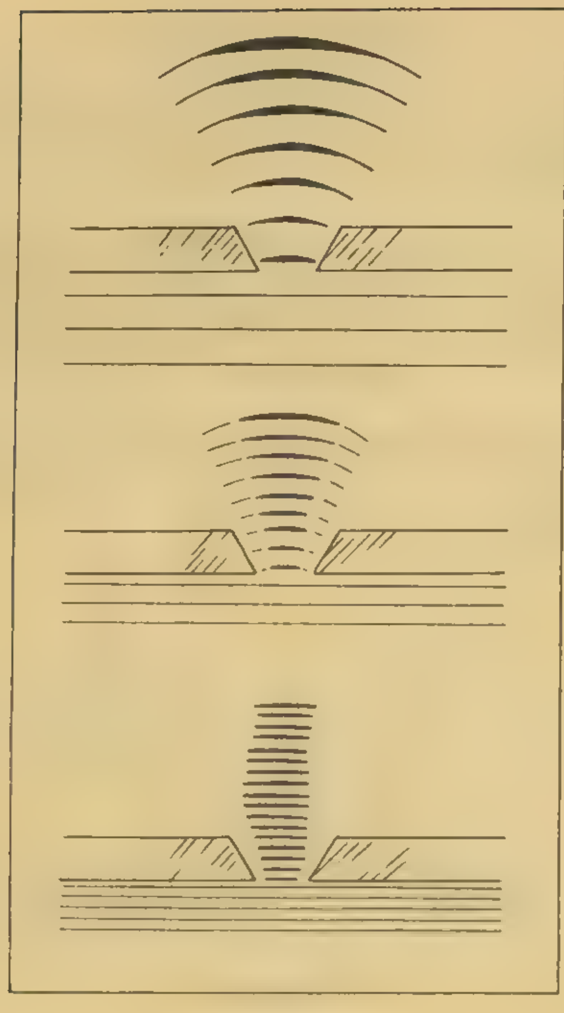


Figure (b)

APPARATUS

1 Complete ripple-tank apparatus (see Exp. II-9)
1 Small barrier, paraffin
4 Barriers, paraffin blocks
1 Hand stroboscope
1 Meter stick or ruler

II-12. WAVES FROM TWO POINT SOURCES

This experiment is divided into two parts: the first part is qualitative and serves as an introduction to Chapter 18; the second part is quantitative and should come after the discussion of Section 18-4.

It takes only a few seconds to look at an interference pattern, but it requires a much longer time to observe the details and to be able to describe the pattern. Learning to describe what one sees is an important part of science. This experiment should not be replaced by a demonstration!

A wave length of about 2 cm on the screen is a good starting point.

We are primarily interested in the pattern far from the sources. Therefore a high amplitude is desirable. In order to observe the waves close to the sources, the amplitude must be reduced by adjusting the nut and by arranging the wave generator so the beads barely touch the water.

Answers to Questions

The main features of the pattern are: waves travel outward from the sources; the waves are separated by paths of no disturbance (nodal lines); far from the sources these lines are straight and appear to radiate from the point midway between the sources; closer to the sources they curve. "Stopping" the waves with a stroboscope will show that the waves separated by lines of no disturbance are out of step: each crest is flanked by two troughs. With small amplitude, a standing wave may be observed between the sources.

When the wave length is decreased, the number of lines of no disturbance increases. The patterns are always symmetrical about the perpendicular bisector of the line between the two sources. We always see moving waves along the bisector. When the source separation is increased the number of nodal lines increases.

Students should be encouraged to draw sketches on the screen as they answer these questions.

In the second part of the experiment the wave length of a periodic wave is calculated from measurements on the interference pattern and the separation of the sources. The result is then checked by direct measurement on the screen to give the students confidence in the method they will use in Experiment II-14 to measure the wave length of red light.

Make sure your students measure $\underline{x}$, $\underline{L}$ and $\underline{d}$ on the screen, giving them the wave length as measured on the screen.

Answers to Questions

The results of the two measurements agree within 5 to 10 per cent. The main sources of error are variations in the frequency of the motor and failure to "stop" the waves long enough to measure their length accurately.

Students need not repeat all the measurements with slits, they need only to obtain the pattern to convince themselves that they can measure the wave length from the pattern.

To obtain wide enough diffraction, the width of the slits has to be less than one wave length. If, on the other hand, the slits are too narrow, not enough intensity will get through. The 5-cm block used in Experiment II-11 can serve as the barrier between the two slits. Fig. (a) shows the arrangements of the slits. An interference pattern can be expected to occur only where the waves from the two slits overlap, which is most clearly visible in the central region.

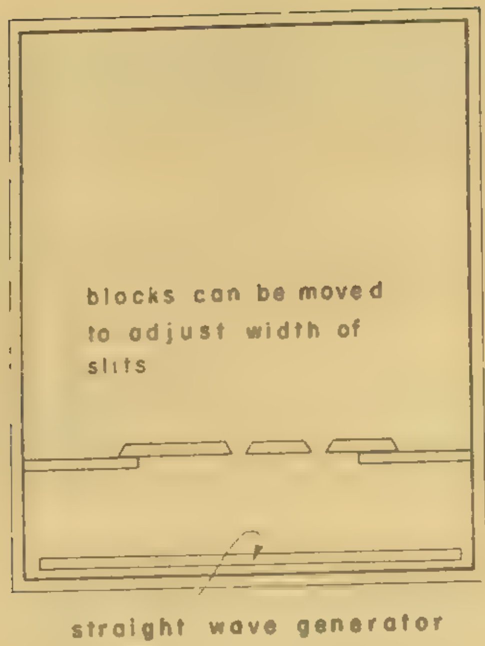


Figure (a)

You may suggest that your students cover one slit and see what happens. They will repeat this with light in Experiment II-14.

APPARATUS

1 Complete ripple-tank apparatus (see Exp. II-9)
1 Hand stroboscope
1 Meter stick or ruler
1 Small barrier, paraffin block
4 Barriers, paraffin blocks

II-13. INTERFERENCE AND PHASE

Understanding the effect of phase on the interference pattern is essential to the discussion of the interference of light. This experiment, therefore, prepares the way for Section 19-3 of the text. It is best done at the beginning of the discussion of Section 18-5.

The experiment consists of two parts using different wave generators. There is an advantage in doing them in the order in which they appear in the Laboratory Guide, but little harm is done if half the class starts with the first part and half with the second part.

Each of the coat-hanger wires shown in Fig. 2 is held between two metal plates (standard repair plates), which are screwed together and clamped to the chair. The distance between the bend in the wire and the metal plate is 4 to 6 in. To increase the frequency, reduce this distance.

Answers to Questions

When the right-hand source is delayed with respect to the left-hand source, the pattern shifts to the right. When, for example, this phase delay is $\frac{1}{2}$, the first nodal line to the left has taken the position of the central maximum in the in-phase pattern; the first nodal line to the right has moved to a position about halfway between the first and second nodal lines in the in-phase pattern. When the phase delay is increased to 1, the sources are again in phase; the first nodal line has moved to the position of the second line, and so on.

Changing the phase while the generators operate would result in a sweeping motion of the nodal lines and their curving away from the direction of the motion. This is actually observed in the next part of the experiment when the two generators do not have exactly the same frequency. ("Two sources with the same frequency but changing phase" really means "two sources with almost the same frequency.")

When the ends of the wires are plucked, the phase delay between the two sources changes abruptly, and with it the interference pattern. If such changes took place very rapidly, we could expect that the interference pattern would be washed out completely.

Fig. (a) shows another way of generating waves with adjustable phase delay. The arrangement of the paraffin blocks is the same as in Fig. (a) of Experiment II-12. The width of the slits must be less than one wave length so they will approximate point sources. Rotating the generator changes the phase of the two sources (slits). If the generator support is rotated very slowly without disturbing the generator, the effect of varying the phase can be observed.

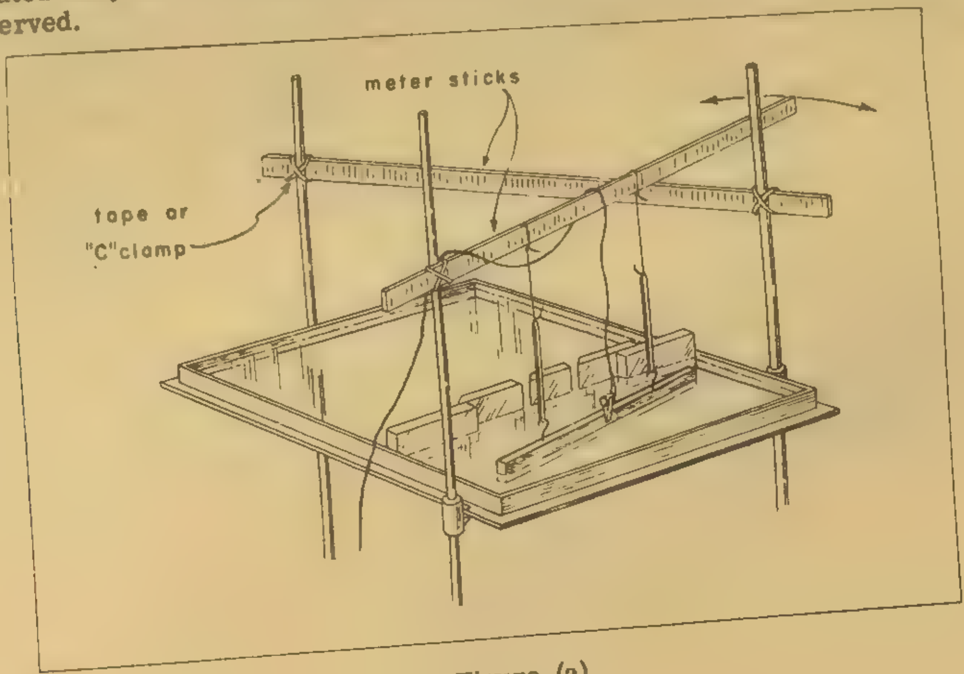


Figure (a)

APPARATUS

1 Complete ripple-tank apparatus (see Exp. II-9)
1 Hand stroboscope
----
1 Adjustable phase wave generator
1 $1\frac{1}{2}$-v. dry cell
----
2 Coat hangers
4 Metal plates
2 Alligator clasps with beads attached
2 C clamps
2 Sliders
----
2 Meter sticks
1 Small barrier, paraffin block
4 Barriers, paraffin blocks

II-14. YOUNG'S EXPERIMENT

This experiment is in a sense the climax of the study of waves. It is best done before the study of Section 19-4.

In preparing the slits, the best results are obtained by using microscope slides coated with a colloidal suspension of graphite in water. A thin, opaque layer of the colloid can be applied with a soft brush. The slits should be made as suggested in Fig. 1 within an hour after the coating has dried.

As an alternative, the slides can be coated with carbon black. This is done by holding the slide above a candle flame until a thin layer of carbon is deposited. Attempts to prepare good slits with slides coated with other kinds of material have not been successful; the coat chips when scratched.

Two clean, double-edged razor blades tilted slightly forward as shown in Fig. 1 will produce slits that are separated by a distance equal to the thickness of one blade. If the same kind of blades are used to produce the slits as were measured in Experiment I-3, it is not necessary to measure their thickness again.

A 40-watt clear-glass showcase bulb is a good light source because of the long, straight filament. If this is not available, a ripple-tank bulb or a weaker clear-glass bulb with a straight filament can be used. In either case, the slits should be parallel to the filament.

It might be worth while to have one ripple tank set up during this experiment. The similarities and differences between measurements of the directions of nodal lines in the ripple tank can then be related to the measurements with light. Note that in this experiment the pupil of the eye subtends the entire interference pattern of the light waves. The analogous situation in the ripple tank would require an "eye" as large as the tank.

Answers to Questions

Light from the bulb passing through the narrow slits is diffracted. The light from the two slits overlaps, producing an interference pattern. Dark bars are seen along the nodal lines and bright bars along the direction of maximum disturbance.

The bars near the end of the pattern will be colored if the colors making up white light are waves of different wave length. Such waves will have maxima and nodal lines in different directions.

When the bulb is covered with red cellophane the other colors are cut out, leaving a sharper pattern of red and black bars. More bars are clearly seen.

From Fig. 2, it is seen that θ_n equals half the distance between $2n$ nodal lines divided by the distance between the ruler and the slits. (Since θ_n is small, the distance from the slits to the ruler along any of the nodal lines is the same, within experimental error.)

The wave length of red light can be determined to an accuracy of about 10 to 20 per cent. The main sources of error are the measurements of x and the separation of the slits. It should be remembered that even the most accurate measurements give a value dependent on the range of wave lengths passed by the filter.

Blue light will show alternating light and dark bars that are closer together than bars from red light; thus, blue light has a shorter wave length.

The interference pattern is spread out when the slide is rotated to form a horizontal angle of about 30° with the line of sight. This is caused by an effective decrease in the slit separation [Fig. (a)].

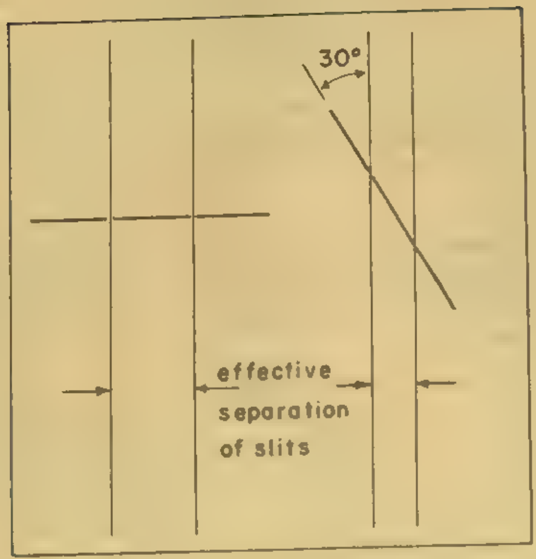


Figure (a)

APPARATUS

1 Showcase lamp, clear glass, 40-watt
1 Socket and cord
2 Microscope slides, 1" × 3"
2 Double-edged razor blades
1 Piece of red cellophane 4" × 4"
1 Piece of blue cellophane 4" × 4"
2 Rubber bands
1 Ring stand
1 Burette clamp
1 Ruler
2 Markers for ruler

Colloidal suspension of graphite or candle for lampblacking
Masking or cellophane tape

II-15. DIFFRACTION OF LIGHT BY A SINGLE SLIT

The first part of this experiment serves as an introduction to the study of single-slit interference. It emphasizes the similarities and differences in the appearance of single and double-slit interference patterns. It is best done immediately after Experiment II-14, perhaps even in the same laboratory period. The second part can be done after the discussion of Section 19-6 and is of less importance.

The slides used in making single slits should be prepared as described in Experiment II-14.

One of a pair of slits can be blocked off by moving the razor blade slowly along the slide until the edge of the blade covers one of the slits.

Answers to Questions

A very narrow slit made with a razor blade shows a broad bright band. A slit made with a needle shows a pattern of bright and dark bars. The central bar is twice as wide as those on both sides and much brighter. (With a narrow slit only the central maximum of the diffraction pattern is visible.)

The dark bars seen with the double slit disappear, leaving the broad diffraction band of a single slit. The places which were dark are now light! A particle model of light could not account for this.

The width of the slit, w, can be calculated from the relation $\frac{\lambda}{w} = \frac{x}{L}$ where x is half the width of the central maximum as read on the scale and L is the distance between the slit and the scale. With wider slits the accuracy of the determination of x can be increased by measuring the distance separating the nodal lines on both sides of the second maxima and dividing the width by four.

The error in the determination of the width of the slit is about 50 per cent.

For comparison, a direct way to measure the width of a single slit is to project the slit on a screen with a slide projector. The magnification can be found by placing a flat, transparent scale in the projector in place of the slide.

Demonstration (for Sections 19-5 and 19-6)

Fig. (a) shows a simple arrangement which you can use to demonstrate that many point sources close together produce the same diffraction pattern as a straight source behind a slit.

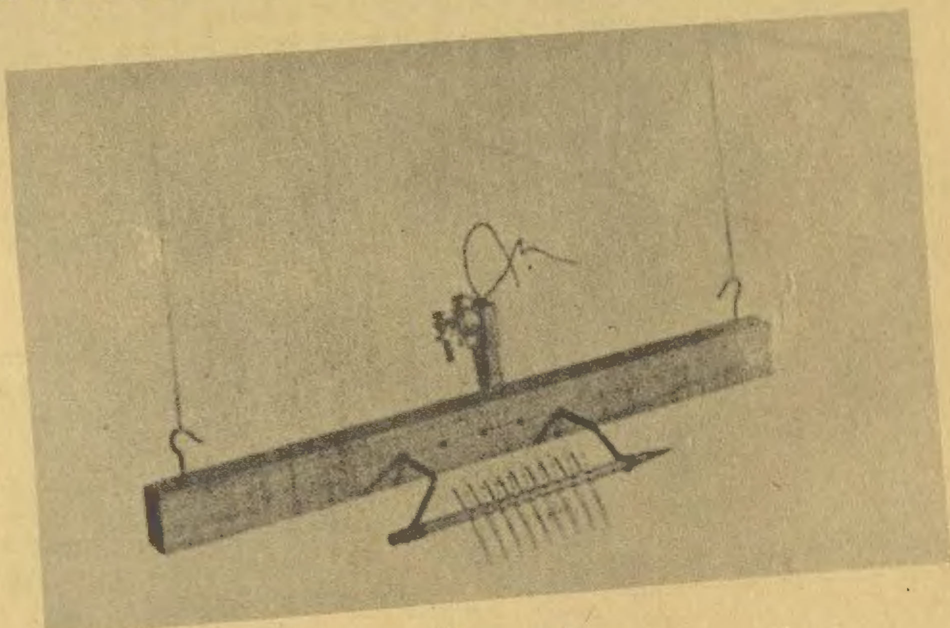


Figure (a)

Arrange the paraffin blocks to form a slit equal in width to the "multipoint source." Show the diffraction pattern with the straight generator, marking the directions of the nodal lines on the screen. Then place the row of point sources in the slit and adjust the wave length to be the same as before. Compare the directions of the nodal lines with those marked on the screen.

APPARATUS

Same as for Experiment II-14
1 Sewing needle or straight pin

Ripple-tank equipment

II-16. RESOLUTION

This experiment shows how diffraction by a small aperture affects the resolution of two closely spaced light sources. The experiment does not require a full laboratory period and is best done while studying Section 19-8.

The best way to make smaller apertures is to place the foil on a smooth, hard surface and push the point of a pin or needle through the foil to different depths.

It is necessary for the filament of the clear-glass bulb to be directly behind the two holes to allow enough light to pass through the hole to make the diffraction rings readily visible. As many as four or five concentric rings around the two holes will be visible through some of the apertures. Speckled patterns or irregular rings which rotate when the aperture is rotated are caused by irregularities in the apertures. It is difficult to prevent this, especially with the smaller apertures, but this will not interfere with the experiment. Students with poor eyesight and no correction may have difficulty resolving the source at a moderate distance without the apertures.

In this experiment there is no interference pattern even when the two sources overlap; the light coming through the two holes originates from different parts of the filament and is, therefore, not locked in phase.

Answers to Questions

The two separate points of light can be resolved with the eye at distances of several meters. The size of the aperture is the size of the pupil of the eye, about 0.5 cm.

The two sources will probably be resolved with one of the middle-sized apertures at a distance of 1 to 2 meters, depending on the size of the aperture. As the distance is increased, the edges of the two sources gradually come together until they begin to overlap.

With increasing distance from the sources, there is a decrease in the angle between the two directions in which we see the centers of the sources. The central maxima of the diffraction pattern of the sources are cones diverging from the aperture. These cones finally overlap when the angle between the direction to the sources becomes smaller than the apex angle of the cone.

As the aperture decreases in size, the angular widths of the diffraction cones of the central maxima increase until the cones overlap.

Red light diffracts more than blue light, because it has a longer wave length. Thus, the diffraction cones of the red light are wider and the sources overlap sooner.

The resolution of the two sources will not be affected by changing their size if their minimum separation is kept constant, because it is the diffraction of the light from the inner edges which determines the resolution.

Supplement

If a monochromatic light source such as a sodium lamp is available, the largest resolving distance from the two sources can be studied quantitatively as a function of the separation of the sources. Plotting the results on a graph will show a linear relationship.

APPARATUS

1 Incandescent clear glass lamp (150-watt ripple-tank lamp)
1 Socket and cord for lamp
1 Ringstand and burette clamp (or other stand for socket and lamp)
1 Thin cardboard rectangle about 3" × 6" (to support aluminum foil)
1 Pin or needle
Red cellophane (or other red filter)
Blue cellophane (or other blue filter)
1 Meter stick or ruler

Aluminum foil
Cellulose tape

II-17. MEASUREMENT OF SHORT DISTANCES BY INTERFERENCE

This experiment, like the second part of Experiment II-15, uses the wave length of light to measure very short distances. It is best done after studying Section 19-9.

Very flat and thoroughly cleaned glass plates about 3 cm wide and 20 cm long are best.

Place one clean plate on top of another under a monochromatic light source. Pairs that show only a few dark and light bands will be flat enough to use.

A monochromatic light source such as a sodium lamp is best. The light from a clear glass fluorescent tube will also give good results. If neither is available, a yellow or green filter placed over an ordinary fluorescent tube will produce countable bands.

Various thin materials such as the thin nylon fiber from the Coulomb's law experiment or Saran wrap can be measured with this device. In the range of 10^{-3} cm this method is as accurate as a precision micrometer caliper.

Answers to Questions

The irregular light and dark bands are caused by interference between light reflected from the lower surface of the top plate and the top surface of the lower plate. This distance will vary from one place to another over the surface.

When one band replaces another, the top plate has been pushed $\lambda/2$ closer to the lower plate.

The separation of the plates differs by one-half wave length between two adjacent bright bands.

The range of thickness that can be measured with these plates is limited by our ability to resolve and count the bands. The lower limit is determined by the flatness and cleanness of the plates.

APPARATUS

2 Glass plates 20 cm × 5 cm × 1 cm thick, flat plate glass
2 Rubber bands
1 Fluorescent lamp (at least 10" long) with fixtures and cord or other monochromatic light source
Thin materials; small pieces 5 cm long, about 1 cm wide, 0.002 to 0.006 cm thick
1 Ruler

Micrometer caliper